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A MANUAL

OF THE

MECHANICS OF ENGINEERING

AND OF THE

CONSTRUCTION OF MACHINES.

DESIGNED AS A TEXT-BOOK FOR TECHNICAL SCHOOLS AND COLLEGES, AND FOR THE USE OF ENGINEERS, ARCHITECTS, ETC.

BY

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IN THREE VOLUMES.

VOL. II.

SECTION II.—APPLICATION OF MECHANICS TO MACHINES.

Part II.—HEAT, STEAM, AND STEAM ENGINES.

WITH 500 WOOD-CUTS IN THE TEXT

TRANSLATED FROM THE FOURTH AUGMENTED AND IMPROVED GERMAN EDITION BY

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WITH ADDITIONS SHOWING

AMERICAN PRACTICE.

BY R. H. BUEL, C.E.

AUTHORIZED TRANSLATION.

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JOHN S. PRELL

Civil & Mechanical Engineer.

SAN FRANCISCO, CAL.

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TRANSLATOR'S PREFACE.

THE present work completes the second volume of Weisbach's "Mechanics," of which the first half ("Hydraulics and Hydraulic Motors") was given to the public about a year ago. There still remains to be translated the *third volume*, which completes the course, and which, in connection with the other two, will, it is thought, furnish our technical schools and colleges with a full and thorough mechanical course, well adapted to the wants of our young students. This third volume will, we hope, soon make its appearance. For a description of its contents, we refer to the preface to the first half of the present volume. It is sufficient to say here, that it is at present undergoing a thorough revision, and in large part has been entirely rewritten, by Prof. Gustav Herrmann, of the Royal Polytechnic School at Aachen. Prof. Herrmann's work is progressing rapidly, at present nearly one third of the work has been completed, and promises when finished to be one of the most valuable mechanical treatises thus far issued from the German press. The work will, it is hoped, be issued in English by the present publishers, simultaneously, or nearly so, with its complete production in Germany, thus bringing within the reach of mechanical students in this country the whole of Prof. Weisbach's great life-work, thoroughly revised and brought down to the latest and best approved practice.

If, therefore, the present volume should seem to any defective in its treatment of certain topics, such as locomotive and marine engines, etc., the explanation will be easily found in the fact that such topics find their proper place and full treatment in the third volume, and could not, in accordance with the plan and scope of the entire work, find a place in these pages. Nevertheless, recognizing the importance of these omissions, we have in press an appendix to the second volume, which will soon be issued, and which will, we trust, in some degree take the place of the third volume until it also can be given to the public. Meanwhile, so far as the present work is concerned, it will be recognized as characteristically full and complete, and simple withal, and will, we trust, be welcomed as still another instalment of the work so well begun by Mr. Coxe. It will be found to be translated with the same care as its predecessor, to be equally free from typographical errors, and will also, we hope, meet with the same kind acceptance.

733630

After the completion of his own labors, the work appeared somewhat incomplete in those directions relating to the application of the steam-engine where recent progress has been very considerable, and the translator has therefore associated with himself Mr. Richard H. Buel, C.E., who has made many additions to the book with especial reference to American practice, and will also take part in the appendix above referred to. Mr. Buel's annotations are distinguished from the body of the work by being, in every case, in fine print and enclosed in brackets. With reference to these, it is but proper to state, that, having commenced their preparation immediately after completing the revision of a number of articles on mechanical engineering for "Appleton's Cyclopædia of Applied Mechanics" (which work is now in course of publication by D. Appleton & Co.), there is naturally much similarity between the present notes and some portions of the annotations referred to, both being compiled, in general, from the same sources of information.

With this much of necessary explanation, we offer the present volume to the engineering public.

A. Jay Du Bois.

Sheffield Scientific School of Yale College,
September 5, 1878.

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PART SECOND.

HEAT, STEAM, AND STEAM ENGINES.

CHAPTER I.

PROPERTIES OF HEAT.

§ **349.**—Just as sound is produced by the measurable vibrations of a body, and conducted by other bodies (as air, water, etc.), so we must assume that heat consists of very minute vibrations of the molecules of a body, and is propagated by a very fine and imponderable fluid, called the ether, which permeates all bodies, and the universe in general. While in the common pendulum, the oscillations are due to a continuous play between the force of gravity and the force of inertia, the oscillations of elastic bodies, and consequently those of their molecules, are due to a similar play between the elasticity and inertia of the bodies or their molecules.

The relation between the height of fall h and the velocity c of a pendulum at the lowest point of its motion, is, as well known,

$$c = \sqrt{2gh}, \qquad \text{or} \qquad h = \frac{c^2}{2g}$$

(see Vol. I., § 320), and if the weight of the pendulum is G, the mechanical effect, which is accomplished by gravity during the descent, as well as the work which is performed by inertia during the ascent, is

$$A = Gh = G\frac{c^2}{2g}.$$

If the pendulum has described only a part z of the total height of fall, and attained the velocity $v = \sqrt{2gz}$, it still possesses, in consequence of its weight, the capacity for work $G(h - z)$, and in consequence of its inertia the capacity for work,

$$G\frac{v^2}{2g} = Gz,$$

therefore the total capacity for work of an oscillating pendulum is

$$G(h - z) + Gz = Gh = G\frac{c^2}{2g},$$

i. e., a constant quantity. If the mass $M = \frac{G}{g}$ of an elastic body has the velocity v when at the distance x from its position of equilibrium, and is driven back to the point of rest by the force $P = px$, then its capacity for work is

$$A = \frac{Px}{2} + G\frac{v^2}{2g} = \frac{px^2}{2} + G\frac{v^2}{2g}.$$

If the velocity of the pendulum at the moment of passing through the point of rest has the value c, the capacity for work will be $= G\frac{c^2}{2g}$.

Now, the work of the repulsive force $P = px$, while the velocity c is changed into v, is

$$\frac{Px}{2} = \frac{px^2}{2} = G\left(\frac{c^2 - v^2}{2g}\right) \text{ (see Vol. I., § 84);}$$

hence we have also

$$A = \frac{G(c^2 - v^2)}{2g} + \frac{G v^2}{2g} = \frac{G c^2}{2g},$$

and also $A = \frac{p a^2}{2}$, if $\pm a$ indicates half of the amplitude of an elastic pendulum.

Therefore, the capacity for work of the body, when the oscillations are produced by elasticity, is *a constant quantity.*

The same law of vibration holds good also for liquids, and even for the rarest fluids, or for the ether itself. If, now, heat is an effect of this condition of vibration, it may be assumed that every body, in consequence of its heat, contains a certain capacity for work, which increases or diminishes with the heat. The quantity of heat of a body may therefore also be measured by its vibration work A. The so-called *undulation theory* explains the phenomena of heat by the vibrations of the ether, while, on the other hand, the now untenable *emanation theory* is based upon the hypothesis of the existence of a peculiar substance or *caloric*, whose greater or lesser accumulation in a body causes the different states of heat.

§ **350.**—Instruments which indicate the heat or the quantity of heat of a body are called *thermometers* (Fr. thermomètres; Ger. Thermometers) and *pyrometers* (Fr. pyromètres; Ger. Pyrometers). The former are used for the purpose of measuring the lower and more moderate, and the latter to ascertain the higher temperatures. In the former it is generally a fluid, in the latter a solid body, which, by its expansion, indicates the amount of heat. The condition of heat indicated by these instruments is called the *temperature.*

By absorbing a great quantity of heat, solid bodies are converted into liquids, and the latter again into a gaseous condition; on the contrary, if fluids are deprived of their heat, they will come back to a solid condition. Heat is

therefore the cause of the three aggregate conditions of bodies. (See Vol. I., § 62.)

If bodies of unequal temperature come into contact with each other, then the heat equilibrium is destroyed in both of them; the heat of the warmer body flows into the cooler body, and, after a certain time, equilibrium is again established, and both bodies assume a common temperature.

This transference of heat from one body to another is called heat *conduction*. If, on the other hand, the heat of a body is transferred to another not in contact with it, by the agency of the ether, such transference is called heat *radiation*.

§ 351.—**Mercurial Thermometer.**—The most important and most commonly employed thermometer is the mercurial thermometer (Fr. thermomètre à mercure; Ger. Quecksilber-thermometer). It consists of a narrow glass tube *A B*, Fig. 592, ending in a bulb or flask-like expansion *A*, partly filled with mercury, and furnished with a scale.

Fig. 592.

Upon bringing the bulb of the instrument into contact with a body whose temperature is to be ascertained, the mercury will, after a certain time, assume the temperature of this body, and the change of volume thereby produced will be indicated by the position of the mercury within the tube. But, in order that all thermometers may correspond with each other, so that they shall indicate, under the same circumstances, the same temperature, it is necessary that their scales have such an extension and division, that any two corresponding points of the same shall indicate two definite temperatures. Generally, for the purpose of graduating the scales, the temperatures of freezing and boiling water are used, and the corresponding points which the mercurial column within the tube reaches are called the freezing and

boiling points. To ascertain these points, the thermometer is at first immersed in melting ice, and then in steam, which continually rises from boiling water, as thereby a greater certainty is attained.

As the boiling point depends also upon the pressure of the atmosphere or upon the height of the barometer, this condition has also to be considered in the determination of this point. It is agreed to take the boiling point at the barometric height of 0·76 meters = 336·9 lines (29·92 inches), or to reduce the observed point to this height, according to a rule to be given hereafter.

The interval between the freezing and boiling points is divided into a certain number of equal parts, and, by continuing these divisions below the freezing point and above the boiling point, the scale may be extended as far as may be desired in either direction. *The Centigrade scale*, which divides the above interval into one hundred equal parts or degrees, is certainly the simplest, but in America and England the scale of *Fahrenheit* is in common use. This scale divides the interval between the freezing and boiling points into 180°, and counts its zero from a point 32 of these degrees below the freezing point, so that the boiling point is numbered 212. The scale of *Reaumur* is also very widely used, which, like the Centigrade scale, takes its zero at the freezing point, but divides the interval from here to the boiling point into 80° only.

REMARK 1.—Special directions for the construction of thermometers may be found in the larger works on Physics, such as Müller's "Lehrbuch der Physik und Meteorologie," or Wühlner's "Lehrbuch der Experimentalphysik."

REMARK 2.—Tables for the conversion of Centigrade, Reaumur, and Fahrenheit degrees may be found in the "Ingénieur." We give here the necessary formulæ: Thus, t degrees Centigrade correspond to $\frac{4}{5}t$ degrees Reaumur, or to $\frac{9}{5}t + 32$° Fahrenheit. Again, t_1 degrees Reaumur correspond to $\frac{5}{4}t_1$ degrees Centigrade or to $\frac{9}{4}t_1 + 32$° Fahrenheit. Finally, t_2 degrees Fahrenheit correspond to $\frac{5}{9}(t_2 - 32°)$ Centigrade or to $\frac{4}{9}(t_2 - 32°)$ Reaumur.

§ **352.—Pyrometer.**—Mercury freezes at a temperature of − 40° C., and boils at a temperature of + 400° C. Since

its expansion near these two points of change of aggregate condition is very irregular, the mercurial thermometer can only be relied upon for the determination of temperatures of from $-36°$ to $360°$. In order, therefore, to measure temperatures beyond these limits, we use *spirit thermometers* for the lower temperatures and *pyrometers* for the higher ones. These latter are used to ascertain the temperature of furnaces, etc.

The simplest means of measuring high temperatures consists in the comparison of the lengths which one and the same metallic bar assumes under different temperatures. As the expansion by heat of solid bodies is not very great, special means are employed to attain the desired degree of accuracy; chiefly by means of unequal-armed levers, which multiply the expansion. Moreover, the construction of a serviceable metallic pyrometer presents special difficulties, because, in most cases, it is impossible to observe with these instruments the effects of heat directly—*i. e.*, in the fire-box itself—and also because these effects extend to all parts of the instrument, not only to the metallic bar, but also to its frame and to its scale. All metallic pyrometers heretofore proposed and employed are, therefore, more or less imperfect. One of the best, although most expensive, imstruments of this kind is Daniell's pyrometer (see Gehler's "Physik. Wörterbuch," Art. Pyrometer). The idea upon which the construction of this instrument is based is as follows: AB, Fig. 593, is a tube of graphite, and CD is a bar of platinum, covered by a short porcelain cylinder E which closely fits the tube. If this apparatus is placed in the furnace, the porcelain piece E, in consequence of the expansion of the platinum bar, is pushed outward, and when the apparatus afterwards is taken out of the fire and has been allowed to cool, then the displacement of the porcelain cylinder will indicate the expansion of the platinum bar, and thereby, indirectly, the temperature. To measure this displacement accurately, a contact lever appa-

FIG. 593.

ratus is also used, which is applied to *A D* both before and after exposure to the fire.

Remark 1.—The pyrometers of Guyton de Morveau, Brogniart, Petersen, Neumann, etc., resemble more or less Daniell's pyrometer. (See Gehler's "Physik. Wörterbuch," Vol. VII.)

Remark 2.—Another well-known instrument for the determination of high degrees of heat is the pyrometer of Wedgwood. It is, on account of its simplicity, still often used, although it is very imperfect. In this instrument, small cones or cylinders of porcelain or potter's clay are employed, which before use are dried by being raised to a red heat and then measured. In order to determine the temperature of the furnace, one or more of these clay cylinders are then exposed to the fire, and left therein for some time to allow them to assume the temperature of the space in which they are contained. Owing to the great heat, the cylinders shrink considerably, and remain still contracted even when they are cooled, and more so the more intense the heat has been to which they were exposed. If we measure the diameter of such a cylinder before and after the heating, we can calculate its contraction, and may consider this as the measure of the heat. But to accomplish this measurement accurately and easily, a scale which constitutes the real pyrometer is employed. This scale consists essentially of two converging graduated metallic bars, soldered upon a plate. If, now, the clay cone or cylinder is interposed between these bars, then its thickness can be read off by the graduation. This instrument is generally divided into 240 parts or degrees. Wedgwood takes for his zero $1077\frac{1}{2}°$ F., and every degree Wedgwood $= 130°$ F. Thus, for instance, $240°$ W. $= 1077\frac{1}{2}° + 240 \times 130 = 32277\frac{1}{2}°$ F. The defects of this instrument have been specially pointed out by Guyton de Morveau. According to him, the zero point of the Wedgwood pyrometer is not $1077\frac{1}{2}°$ F., but $510°$ F., and every degree of the same is not $130°$ F., but $61 \cdot 2°$ F.

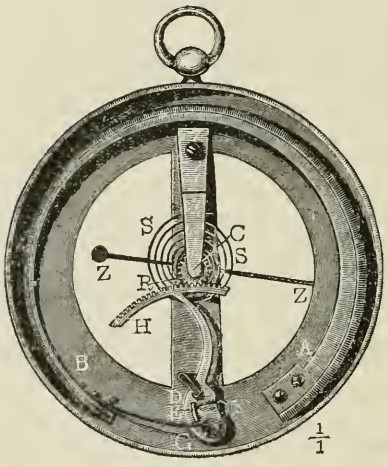
§ 353.—**Metallic Thermometer.**—The most common metallic thermometer or pyrometer for moderately high temperatures consists in a combination of two metallic bars of different degrees of expansion; for instance, of a brass and an iron bar, or a strip of platinum and gold or silver, etc. If these bars lie upon each other and are firmly connected at one end, the difference of their expansions can be observed at the other ends, and hence the corresponding temperature can be calculated. For this purpose, the end of one bar is supplied with a simple graduation, and the other with a corresponding *vernier*. But these thermometers (which were originally introduced by Borda), if they

are made sufficiently accurate, are too bulky for the determination of the temperature of small spaces. Recently, these strips or bars have been soldered or riveted together to prevent their displacement, and they therefore assume a curved shape or increase their curvature when they are exposed to a higher temperature.

The thermometer of Breguet consists of three spiral strips of platinum, silver, and gold, of which the latter serves to unite the other two.

The so-called *Quadrant Thermometer*, which is shown in Fig. 594, consists of a curved spring composed of strips of steel and copper, which is fixed firmly at A, and at its other end B is connected to a lever at E by means of a spring BF. This lever GDH has two unequal arms GD and HD, and turns about a pivot at D. The arm HD carries a curved rack, which engages with a toothed wheel R, which revolves upon the axis C, and thus moves a pointer ZZ, which passes over a dial. Any increase of heat thus causes, by reason of the unequal expansion of the strips, a change of curvature; the end B therefore moves forward or back, and this motion is transferred to the pointer, causing it also to move back and forth over the graduated dial.

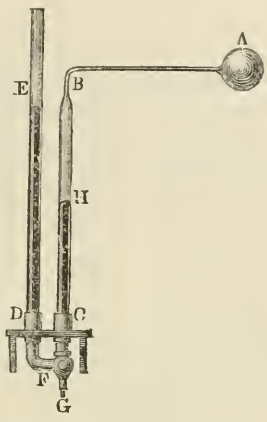
FIG. 594.



REMARK.—Holzmann's metallic thermometer does not essentially differ from the quadrant thermometer described above. (See "Anfangsgründe der Physik," by Scholz, § 294.) Oechsle's metallic thermometer consists of a spiral spring composed of steel and brass strips. Here the outer end of the spring is fixed firmly to the case, and the inner end, by means of a vertical shaft, sets the indicator in motion. (See Dingler's "Journal," Vol. LX.)

§ 354.—Air Pyrometers.—Finally, for the purpose of measuring high temperatures, air pyrometers are used. They consist chiefly of a hollow platinum bulb A, of a narrow tube $A\,B$, Fig. 595, of two wider communicating tubes $B\,C$ and $D\,E$, and of a brass frame $C\,F\,D$, fitted with a cock, whereby not only the communication of these tubes with each other, but also the communication with a small discharge pipe G, is effected.

Fig. 595.



When used, A and $A\,B$ are filled with air and $B\,F\,E$ with mercury, and A is brought into the fire-box, the temperature of which is to be determined. As it becomes heated, the air enclosed in $A\,B$ expands, occupies, in the tube $B\,C$, a space $B\,H$, and presses the dislodged mercury into the tube $D\,E$. The original volume V of the air enclosed in $A\,B$, at the temperature of 0° and at the barometric height b, being known, and the expansion $\overline{B\,H} = V_1$ of this mass of air caused by the heat, and also the manometer height $E\,H = h$ being observed, we can then, by the aid of the well-known coefficient of expansion of the air, calculate the temperature t of the enclosed air. If the original density of the air is $= \gamma$, then the weight of this mass of air will be

$$V\gamma = \left(\frac{V}{1+\delta t} + V_1\right)\frac{b+h}{b}\,\gamma. \quad \text{(See Vol. I., § 392.)}$$

We have, therefore,

$$\frac{b\,V}{b+h} = \frac{V}{1+\delta t} + V_1,$$

and hence the temperature of the fire-box is

$$t = \frac{1}{\delta}\cdot\frac{V\,h + V_1(b+h)}{V\,b - V_1(b+h)}.$$

If we allow the mercury to run out through G until the columns in $B\,C$ and $D\,E$ are of equal height, then we can put $h = 0$, and, consequently,

$$t = \frac{1}{\delta}\,\frac{V_1}{V - V_1}.$$

If, on the contrary, by means of G, mercury is added to such an extent that on heating A the mercury $B\,C$ remains at the same height, and, consequently, the air suffers no expansion, then $V_1 = 0$, and, therefore,

$$t = \frac{1}{\delta}\,\frac{h}{b}.$$

With the pyrometer of Pouillet, the former, and with the pyrometer of Regnault, the latter procedure is adopted. (See "Mémoires de l'Académie Royale des Sciences de l'Institut de France," Tome XXI., 1847. Also "Formules, Tables, etc., par Claudel," Paris, 1854. In regard to Regnault's gas thermometer, see "Annales de Chimie et Physique," September, 1861; also Dingler's "Journal," Vol. 162.)

Remark.—To protect the instrument against heat, it is placed in front of a wooden screen, and, for the purpose of cooling the air and to maintain it at a constant temperature, the tube $B\,C$ can be surrounded by boiling sulphuric ether or alcohol.

Farther, to avoid too great pressures at high temperatures, the reservoir can be filled with rarefied air, and to this end $A\,B$ may be connected with an air-pump. Besides this, the air in A has to be carefully dried by chloride of calcium before it is used.

The application of the above formulæ requires still some correction on account of the expansion of the wall of the vessel, the variation of the barometer, as well as of the temperature in $B\,C$, etc.

[Siemens' pyrometer, in which the temperature is measured by the change in electrical resistance of platinum, is generally regarded as one of the most reliable instruments for measuring changes in temperature, large or small. The portion of the instrument exposed to the direct influence of the heat is a porcelain cylinder, around which platinum wire is coiled, leading wires of platinum passing through holes in the cylinder. This coil of wire is covered by a protecting cylinder, to prevent the deterioration which even platinum suffers from exposure to the flame of a furnace. The electrical resistance is

measured by the decomposition of water in a differential voltameter, consisting of two similar voltameters, arranged, as explained by Mr. Siemens, "in such a manner that the current of the same battery is divided between the two, the one branch comprising the unknown resistance to be measured, and the other a known and constant resistance. The volume of gas V, produced in this second voltameter, having a resistance R in circuit, would be expressed by

$$V_1 = \frac{E\,T}{R_1},$$

and we should have the proportion of

$$V : V_1 :: \frac{E\,T}{R} : \frac{E\,T}{R_1};$$

but E and T being the same in both cases, may be struck out, and the expression will assume the simple form,

$$V : V_1 :: R_1 : R.$$

The constant resistance R of the one circuit being known, it follows that the unknown resistance R_1 is expressed by a constant multiplied by the proportion of gas produced in the two voltameters, irrespective of time, of strength of battery, or temperature, or the state of the barometer." Mr. Siemens has determined, by an extended series of experiments, that the electrical resistance of a heated wire varies in accordance with the formula,

$$R = \alpha t^2 + \beta t + \gamma,$$

t being the temperature measured from the absolute zero, and α, β, γ, specific coefficients for each metal.]

§ 355.—Linear Expansion.—All bodies, with but few exceptions, expand when passing to a higher temperature, and diminish in volume when they lose heat. But this change of volume differs considerably with various bodies, and is only for moderate temperatures of from 0° to 100°,

proportional to the decrease or increase of temperature. At high temperatures, the expansion is relatively greater than at low temperatures, especially when the body is in the solid state. We can divide heat expansions into linear, superficial, and cubical, according as we consider the change in linear dimensions only, or the change of the linear and lateral dimensions, or the change of the whole volume—*i. e.*, of all the three dimensions.

Linear expansion (Fr. dilatation linéaire; Ger. Längenausdehnung) occurs chiefly in solid bodies, and especially in the case of bars, rods, beams, etc. Lavoisier and Laplace have directly observed the linear expansion of different bodies; but Dulong and Petit have measured the cubic expansion, and therefrom calculated the linear expansion. The deviations in the results of both methods are insignificant. In the following table, we give the linear expansion of those bodies which occur most frequently:

	Increase of Heat. Centigrade.	In Common Fractions.	In Decimal Fractions.	Observer.
Platinum	0 to 100°	$\frac{1}{1167}$	0·00085655	Borda.
"	0 " 100°	$\frac{1}{1131}$	0·00088420	Dulong and Petit.
"	0 " 300°	$\frac{1}{368}$	0·00275482	" " "
Glass	0 " 100°	$\frac{1}{1161}$	0·00086133	" " "
"	0 " 200°	$\frac{1}{544}$	0·00184502	" " "
"	0 " 300°	$\frac{1}{329}$	0·00303252	" " "
Steel, not hardened	0 " 100°	$\frac{1}{927}$	0·00107880	Lavoisier and Laplace.
" hardened	0 " 100°	$\frac{1}{807}$	0·00123956	" " "
Cast iron	0 " 100°	$\frac{1}{901}$	0·00111000	Roy.
Bar iron	0 " 100°	$\frac{1}{846}$	0·00118210	Dulong and Petit.
" "	0 " 300°	$\frac{1}{227}$	0·00440528	" " "
Gold	0 " 100°	$\frac{1}{682}$	0·00146606	Lavoisier and Laplace.
Copper	0 " 100°	$\frac{1}{582}$	0·00171820	Dulong and Petit.
"	0 " 300°	$\frac{1}{177}$	0·00564972	" " "
Brass	0 " 100°	$\frac{1}{535}$	0·00186760	Lavoisier and Laplace.
Silver	0 " 100°	$\frac{1}{524}$	0·00190974	" " "
Lead	0 " 100°	$\frac{1}{351}$	0·00284836	" " "
Zinc	0 " 100°	$\frac{1}{340}$	0·00294167	Smeaton.

We see that, of the above bodies, platinum and next to it glass have the least, lead and zinc the greatest, linear expansion—the expansion of the latter being three times

that of the former. We see also, from the results of Dulong and Petit, that the expansion of metals, as well as of glass, increases at high temperatures at a more rapid rate, relatively, than the temperature. Thus, a glass bar will elongate, for an increase of temperature from 0° to 100°, 0·00086133, but from 100° to 200°, 0·00098369, and from 200° to 300°, 0·00118750 longer.

§ 356.—**Coefficient of Expansion.**—The above expansion ratios admit of important practical applications. Assuming that the expansion increases uniformly with the heat, then, from the above results, we can easily calculate the *coefficients of expansion—i. e.*, the proportional increase of length for *each degree* of rise of temperature.

Thus, for example, the coefficient of expansion for iron is

$$\delta = 0{\cdot}00111 \div 100 = 0{\cdot}0000111;$$

for brass,

$$\delta = 0{\cdot}0018676 \div 100 = 0{\cdot}000018676, \text{ etc.}$$

Bessel and Baeyer found, while testing measuring rods, for temperatures from 3° to 17° Reaumur,

$$\text{for the iron rod, } \delta = 0{\cdot}0000148505;$$

$$\text{and for the zinc rod, } \delta = 0{\cdot}0000416372;$$

while subsequently Baeyer found, for temperatures of 7° to 23° R.,

$$\text{for the iron rod, } \delta = 0{\cdot}000014165;$$

$$\text{for the zinc rod, } \delta = 0{\cdot}0000402342.$$

With the Spanish base-measuring apparatus (which was constructed in Paris by Brunner), it has been found, for temperatures from 7° to $40\frac{3}{4}$°,

$$\text{for the platinum rod, } \delta = 0{\cdot}0000090167;$$

$$\text{for the brass rod, } \delta = 0{\cdot}0000189841.$$

(See "Experiencias hechas con El Aparato de Medir Bases," Madrid, 1859.)

If the length of a bar at 0° temperature is l_0, then, at a temperature of t_1°, it will be

$$l_1 = l_0 + \delta\, t_1 \times l_0 = (1 + \delta\, t_1)\, l_0,$$

and at temperature t_2°,

$$l_2 = (1 + \delta\, t_2)\, l_0;$$

hence the ratio of lengths for the same bar, at the temperatures t_1 and t_2, is

$$\frac{l_2}{l_1} = \frac{1 + \delta\, t_2}{1 + \delta\, t_1} \qquad \text{and} \qquad l_2 = \left(\frac{1 + \delta\, t_2}{1 + \delta\, t_1}\right) l_1,$$

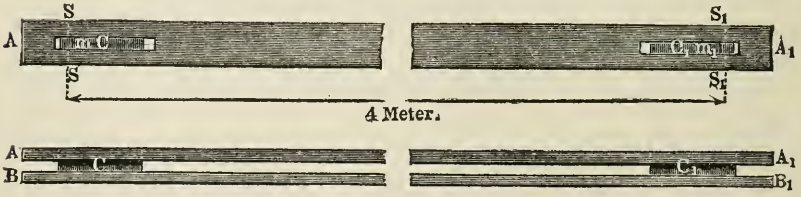
or, by reason of the minuteness of $\delta\, t_1$ and $\delta\, t_2$, approximately,

$$l_2 = [1 + \delta\, (t_2 - t_1)]\, l_1.$$

This formula enables us to reduce the length of a bar at a temperature t_1 to another t_2, or to compare the lengths l_1 and l_2 of one and the same body at different temperatures.

The measuring rod of the Spanish survey consisted of a platinum bar $A\,A_1$, Fig. 596, and a brass bar $B\,B_1$; both fully

Fig. 596.



4 meters in length, 21 millimeters in breadth, and 5 millimeters in thickness. The platinum plates C, C_1, which are firmly united with the brass bar, fit in corresponding notches of the platinum bar, but can move in them a small distance. The ends of the platinum bar and also the plates are furnished with a graduation, from which, by means of a micrometer, the distance between the zero lines S, S_1 of

the platinum measuring rod and the zero lines m, m_1 can be read off upon the platinum plates attached to the brass bar.

If the lines S and m, as also S_1 and m_1, coincide at a certain temperature t, then the common length of both bars is

$$\overline{S S_1} = \overline{m m_1} = l.$$

If the temperature changes to t_1, then the length $S S_1$ of the platinum bar $A A_1$ becomes

$$l_1 = 1 - \delta (t - t_1) l,$$

and the length $m m_1$ of the brass bar $B B_1$ becomes

$$l_2 = 1 - \delta_1 (t - t_1) l,$$

δ being taken as the coefficient of expansion of platina, and δ_1 as that of brass.

By subtraction, we have for the shortening of the brass bar, with respect to the platinum bar,

$$a = l_1 - l_2 = (\delta_1 - \delta) (t - t_1) l.$$

The difference between m and S, as well as between m_1 and S_1, being observed, and their sum a determined, then the difference of temperature can be calculated from the formula,

$$t_1 - t = \frac{a}{(\delta_1 - \delta) l}.$$

And, finally, the length $S S_1$ of the platinum bar, reduced to t degrees is

$$l_1 = [1 - \delta (t - t_1)] l = \left(1 - \frac{\delta}{\delta_1 - \delta} \cdot \frac{a}{l}\right) l,$$

and the reduction itself is

$$l_1 - l = - \frac{\delta}{\delta_1 - \delta} a.$$

Thus we have for $\delta = 0{\cdot}0000090167$ and $\delta_1 = 0{\cdot}0000189841$,

$$l_2 - l = 0{\cdot}90463\ a.$$

§ **357.—Compensation Pendulum.**—An excellent application of these principles is offered by the so-called compensation pendulums (Fr. pendules compensateurs; Ger. Compensationspendel), which are composed of bodies of differing expansion ratios in such a way that they do not change their length when the temperature is altered. As the time of vibration of a pendulum depends upon the length of the same (see Vol. I., § 323), the application of the compensation pendulum to clocks is of very great importance. The simplest pendulums of this kind are provided with a spring *A B A*, Fig. 597, consisting of two metal strips soldered together, and bearing at its extremities two small

Fig. 597.

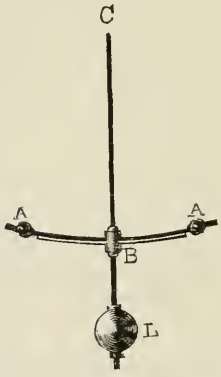
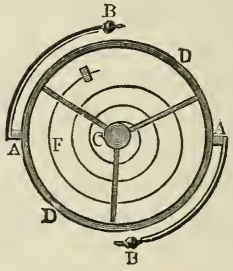


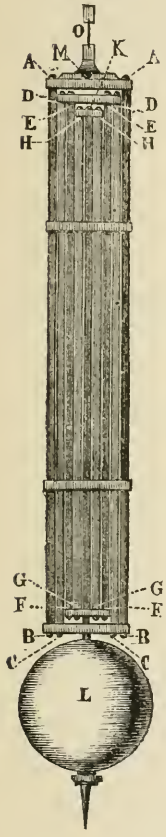
Fig. 598.



balls. If the more expansible metal strip is beneath, then the spring will curve upward, when the temperature increases, and as simultaneously the bar *C L* becomes longer, and therefore the distance of the bob *L* from the point of suspension becomes greater, it is possible so to adjust things that the centre of oscillation shall remain unchanged. (See Vol. I., § 327.) Such compensation strips are also employed on watches and chronometers. As here the period of oscillation depends upon the balance, which is formed by a spiral spring *C F*, Fig. 598, surrounded by a fly-wheel *A A*, the compensation strips *A B* and *A B* are fastened upon the fly-wheel *D D*.

Most frequently the so-called *gridiron pendulum* is employed. This consists of a row of parallel bars of different metals—for instance, iron and zinc, or iron and brass—so connected by cross-bars that the expansion of one bar is neutralized by that of the other. Fig. 599 represents such a pendulum, consisting of five iron bars AB, AB, EF, EF, KL, and of four brass bars CD, CD, GH, GH. The pendulum, to fulfil its purpose, must be so arranged that the downward expansion of the iron bars shall be equal to the upward expansion of the brass bars. Putting the sum of the lengths of the iron bars

FIG. 599.



$$OM + AB + EF + KL = l_1,$$

as also the sum of the lengths of the brass bars,

$$CD + GH = l_2,$$

then we have for the whole length of the pendulum,

$$LO = l_0 = l_1 - l_2,$$

and if we take the coefficient of expansion of the iron $= \delta_1$, and that of the brass $= \delta_2$, and let t be the change of temperature, then the corresponding length of the pendulum can be put,

$$l = l_1 (1 + \delta_1 t) - l_2 (1 + \delta_2 t).$$

Hence the increase of length is

$$l - l_0 = (\delta_1 l_1 - \delta_2 l_2)\, t.$$

In order that this may be zero, we must have

$$\delta_2 l_2 = \delta_1 l_1 \qquad \text{or} \qquad \frac{l_2}{l_1} = \frac{\delta_1}{\delta_2},$$

i. e., the length of the brass must be to that of the iron as the coefficient of expansion of the iron is to the coefficient of expansion of the brass. If the whole length $l = l_1 - l_2$ is given, then we have, according to this, for the length of the iron,

$$l_1 = \frac{\delta_2}{\delta_2 - \delta_1}\, l,$$

and for the length of the brass,

$$l_2 = \frac{\delta_1}{\delta_2 - \delta_1}\, l.$$

REMARK.—Compensation pendulums, especially Graham's pendulum with mercury vessel, are treated in Barlow's "Treatise on Manufactures and Machinery," also in Lamé's "Cours de physique

EXAMPLE 1.—How long must an iron standard be at 16°, in order that when at 0° it may be just 5 ft. in length? We have here to insert in

$$l_2 = [1 + \delta\,(t_2 - t_1)]\, l_1,$$

$$l_1 = 5, \qquad t_2 - t_1 = 16, \qquad \text{and} \qquad \delta = 0{\cdot}000011821,$$

whence we have

$$l_2 = (1 + 0{\cdot}000011821 \times 16)\, 5 = 5{\cdot}0009457 \text{ ft.}$$

EXAMPLE 2.—How long must the iron and brass bars of a gridiron pendulum, 40 inches in length, be?

If we insert $\delta_1 = 0{\cdot}000011821$ and $\delta_2 = 0{\cdot}000018676$, we obtain for the length of the iron bars,

$$l_1 = \frac{18676 \times 40}{18676 - 11821} = \frac{747040}{6855} = 107 \text{ inches,}$$

and for the length of the brass bars,

$$l_2 = \frac{11821 \times 40}{6855} = \frac{472840}{6855} = 69{\cdot}8 \text{ inches.}$$

According to this, we can make each of the smaller brass bars $33\frac{1}{2}$ inches, each of the following iron bars $34\frac{1}{2}$ inches, each of the longer brass bars $35\frac{1}{2}$ inches, and the exterior iron bars $36\frac{1}{2}$ inches long, and there still remain for the middle suspending rod $109 - 71 = 38$ inches.

§ **358.—The Force of Expansion.**—With the aid of the modulus of elasticity E and of the coefficient of expansion δ, the force with which bodies expand by heat and contract by cold can be determined.

The force which expands a prismatic bar of the length l and cross-section F, an amount denoted by λ, is, according to Vol. I., § 204, determined by the formula

$$P = \frac{\lambda}{l} FE.$$

But $\frac{\lambda}{l} = \delta\, t$, therefore we have for the force of expansion or contraction,

$$P = \delta\, t\, .\, FE.$$

Since the modulus of elasticity for the metals is very great, great forces can be produced by heating them, and of this property also important practical applications can be made. Thus, for instance, Molard, at the Conservatoire des Arts et Métiers in Paris, brought back to a vertical position two inclining walls, which threatened to fall, by means of iron rods, which were first alternately heated by means of alcohol flames, and then screwed up before cooling. In covering wooden utensils and tools with iron, especially in fitting on iron rings, etc., the force of heat is very useful, because the iron fitted on in a heated condition effects a very close joint by cooling.

The expansion of a body by heat is changed when external forces also act upon it. If, for instance, a prismatic body whose cross-section is F, and whose length is l, is acted upon by a force of tension P along its axis, and at the same time its temperature raised to t degrees, then the length of the body is increased to

$$\lambda = \frac{P}{FE}\, l + \delta\, t\, l = \left(\frac{P}{FE} + \delta\, t\right) l.$$

(See Vol. I., § 204.) This elongation being known, the force of extension P may be found from the formula,

$$P = \left(\frac{\lambda}{l} - \delta t\right) FE.$$

If $\delta t > \frac{\lambda}{l}$, then P becomes negative and represents a force of compression.

These formulæ assume that the modulus of elasticity E of a body will not be changed by heating. This assumption is, however, not allowable for great changes of temperature, for in such case the modulus of elasticity E, as also the modulus of proof strength T, and the modulus of rupture K, change. If we therefore put the proof strength

$$P = FT,$$

and the breaking strength

$$P_1 = FK,$$

we must use for T and K other values than those which are determined for moderate temperatures.

Assuming that the force of heat produces precisely the same effect upon the body as an external force of extension or compression P, we have

$$\frac{\lambda}{l} = \frac{T}{E},$$

and therefore the proof strength

$$P = (T - \delta t E) F.$$

Accordingly, the proof strength and therefore the elasticity of the body would be = 0, at the temperature

$$t = \frac{T}{\delta E} = \frac{\sigma}{\delta},$$

which nevertheless is not confirmed by experience. Thus, for example, a wrought-iron bar for which $\sigma = \frac{T}{E} = \frac{1}{1500}$ (see Vol. I., § 212), and $\delta = 0.000011821$ (see Vol. II., § 355) would at the temperature

$$t = \frac{1}{1500 \times 0.000011821} = \frac{1}{0.01773} = 56.4^\circ$$

have reached the limits of elasticity. Neither can the breaking strength be put

$$P_1 = (K - \delta\, t\, E)\, F.$$

According to this, the force of cohesion of a body at the temperature

$$t = \frac{K}{\delta\, E}$$

would be zero. Thus, for example, a bar of wrought-iron, for which

$$\frac{K}{E} = \frac{558200}{28000000} = 0.00208,$$

would break at the temperature

$$t = \frac{0.00208}{0.000011821} = \frac{207}{1.1821} = 175^\circ.$$

Example.—With what force does an iron bar of 6 square inches cross-section, heated to 80°, contract, when cooled to 20° ? Here

$$\delta = 0.000011821,\ t = 80 - 20 = 60,\ F = 6,$$

and, according to Vol. I., § 212,

$$E = 28000000,$$

therefore the force of contraction is

$$P = \delta\, t\, .\, FE = 0.00070926 \times 168000000 = 119155 \text{ lbs.}$$

§ **359.**—Recently, numerous experiments have been made upon the variation of the elasticity and strength of metals for varying temperatures. From the experiments upon expansion by Wertheim (see Poggendorff's "Annalen der Physik," Ergänzungsband II., 1845), it appears that the modulus of elasticity of metals, with the exception of iron, gradually diminishes as the temperature increases from 15° to 200° C.; and that, on the contrary, the modulus of elasticity of wrought-iron and steel increases with the temperature from — 15° to 100° C., and only decreases at higher temperatures, so that at 200° it is smaller than at 100° or 0°. According to the experiments of Baudrimont (see "Annales de chimie et de physique," Tom. XXX.), the same holds true for the modulus of breaking strength of metals, and especially of iron.

The experiments of Wertheim have also shown that the modulus of breaking strength of metals is considerably diminished by tempering, while the modulus of elasticity is not greatly changed, and that, on the other hand, the cohesion of previously tempered metals is not considerably diminished by a rise of temperature as high as 200°.

According to Wertheim's experiments, the modulus of elasticity (E) of some of the more common metals is as follows:

Metals.	Temperature.		
	10–15°	100° C.	200° C.
Forged iron	30410000	32070000	25890000 lb.
Cast-steel	28620000	27810000	26220000 "
Copper	15380000	14370000	11500000 "
Silver	10440000	10646000	9320000 "
Lead	2526000	2384000	

Experiments upon the change of strength of iron (wrought-iron) and copper have been made previous to

the above in America. The results of these experiments are published in Vols. XIX. and XX. of the Journal of the Franklin Institute, and can also be found in Vol. I. of Combes' "Traité de l'exploitation des mines."

According to these experiments, when the modulus of breaking strength of copper at 0° is taken as unity, we have the following variations for different temperatures:

0°	16¾°	15°	100°	150°	200°	250°	294°	451°	555½° C.
1·0000	0·9927	0·9825	0·9460	0·9055	0·8487	0·7954	0·7442	0·5056	0·3259

This shows that copper has lost at 280° one quarter and at 555° two thirds of its strength. If the modulus of strength of wrought-iron for 15° to 20° is taken as unity, we have for other temperatures the following:

20°	100°	200°	300°	350°	390°	500°	550°	624°	714° C.
1·000	1·197	1·081	1·040	0·981	0·974	0·760	0·431	0·411	0·346

According to these experiments, the strength of wrought-iron then at first increases as the temperature rises. More information concerning this can be obtained in Bourne's "Treatise on the Steam Engine," article, "Strength of Boilers."

§ **360.—Surface and Cubic Expansion.**—With the exception of crystals and a few other bodies, all objects expand equally in all directions, so that their form at different temperatures remains unchanged. Now, the areas of similar figures are as the squares, and the contents of similar bodies are as the cubes of similar sides; therefore it is possible to compare the contents of one and the same body at different temperatures by means of the length of its sides. If, when exposed to a change of temperature, the

side $A\,B$ of a polygonal plate $A\,C\,E$, Fig. 600, becomes equal to $A_1\,B_1$, then the contents of the expanded plate will be $\left(\frac{A_1\,B_1}{A\,B}\right)^2$ times as great as before, and if the side $A\,B$ of

FIG. 600.

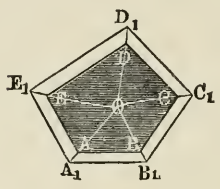
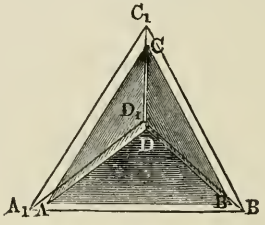


FIG. 601.



a polyhedron $A\,C\,D$, Fig. 601, passes over into $A_1\,B_1$, then its new volume is $\left(\frac{A_1\,B_1}{A\,B}\right)^3$ times that of the original. Assuming this, the coefficients of expansion for surfaces and volumes can be easily calculated from the coefficients of linear expansion. If l_1 and l_2 are the lateral lengths which correspond to the temperatures t_1 and t_2, then we have for the areas F_1 and F_2 the proportion,

$$\left(\frac{F_1}{F_2}\right) = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{1+\delta\,t_1}{1+\delta\,t_2}\right)^2,$$

and also for the cubic contents V_1 and V_2,

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 = \left(\frac{1+\delta\,t_1}{1+\delta\,t_2}\right)^3.$$

Since $\delta\,t_1$ and $\delta\,t_2$ are very small, we easily obtain the more simple approximate expressions,

$$\frac{F_1}{F_2} = \frac{1+2\,\delta\,t_1}{1+2\,\delta\,t_2} = (1+2\,\delta\,t_1)\,(1-2\,\delta\,t_2) = 1+2\,\delta\,(t_1-t_2),$$

and

$$\frac{V_1}{V_2} = \frac{1+3\,\delta\,t_1}{1+3\,\delta\,t_2} = (1+3\,\delta\,t_1)\,(1-3\,\delta\,t_2) = 1+3\,\delta\,(t_1-t_2);$$

or

$$F_2 = [1 + 2\,\delta\,(t_2 - t_1)]\,F_1,$$

and

$$V_2 = [1 + 3\,\delta\,(t_2 - t_1)]\,V_1.$$

We see, therefore, that the coefficient $2\,\delta$ for surface expansion is *twice* and the coefficient $3\,\delta$ for cubic expansion is *three* times as great as the coefficient δ for linear expansion. The latter formulæ find special application in determining the density of a body.

If γ_1 is the density of the body at the temperature t_1 and γ_2 the density at the temperature t_2, then the weight of the body will be $G = V_1\gamma_1 = V_2\gamma_2$; therefore,

$$\frac{\gamma_2}{\gamma_1} = \frac{V_1}{V_2} = 1 + 3\,\delta\,(t_1 - t_2) = 1 - 3\,\delta\,(t_2 - t_1).$$

REMARK.—If cast-iron is heated to glowing (1000° — 1200°), it suffers a *permanent expansion*, which, by repetition or long duration of the heat, may be considerable. According to Ermann and Herter (see Poggendorff's "Annalen der Physik," Band 97), the permanent linear expansion of gray pig iron is from 0·0081 to 0·0097, and of spiegel iron only 0·001114.

EXAMPLE.—In what proportion does the volume and the density of a hollow or massive iron ball change under a variation of temperature from 10° to 70°? For cast-iron,

$$3\,\delta = 3 \times 0{\cdot}00001109 = 0{\cdot}00003327;$$

therefore,

$$3\,\delta\,(t_2 - t_1) = 0{\cdot}00003327\,(70 - 10) = 0{\cdot}0019962.$$

Thus the volume increases about 0·2 per cent, and the density diminishes just as much. If the latter was originally

$$7{\cdot}1 \times 62{\cdot}5 = 444 \text{ lbs.},$$

then at rise of this temperature it will be only

$$444\,(1 - 0{\cdot}0019964) = 440{\cdot}11 \text{ lbs.}$$

§ 361.—Expansion of Liquids.—Liquid bodies, as a rule, expand more under the influence of heat than solid bodies.

As these bodies are enclosed in vessels, and these also, when exposed to heat, expand and become larger, we must distinguish with liquids between the apparent expansion and the real expansion produced by heat. Thus, the former is equal to the difference between the real expansion of the liquid and the real expansion of the containing vessel.

If the contents of a vessel either filled completely, or up to a certain mark, at the temperature t_1 is V_1, and the cubic expansion of the vessel $= \delta_1$, while that of the liquid within $= \delta$, then we have for the temperature t_2 the volume of the vessel after expansion,

$$V_2 = \left(\frac{1 + \delta_1 t_2}{1 + \delta_1 t_1}\right) V_1,$$

and the volume of liquid after expansion,

$$V = \left(\frac{1 + \delta t_2}{1 + \delta t_1}\right) V_1,$$

therefore the real or absolute expansion of the same,

$$V - V_1 = \left(\frac{1 + \delta t_2}{1 + \delta t_1} - 1\right) V_1 = \frac{\delta (t_2 - t_1)}{1 + \delta t_1} V_1,$$

and, on the other hand, the apparent expansion,

$$V - V_2 = \left(\frac{1 + \delta t_2}{1 + \delta t_1} - \frac{1 + \delta_1 t_2}{1 + \delta_1 t_1}\right) V_1 = \frac{(\delta - \delta_1)(t_2 - t_1)}{(1 + \delta t_1)(1 + \delta_1 t_1)} V_1$$

$$= \frac{(\delta - \delta_1)(t_2 - t_1)}{(1 + \delta t_1)(1 + \delta_1 t_2)} V_2.$$

If the expansions are very small, we can approximately put

$$V - V_1 = \delta (t_2 - t_1) V_1$$

and

$$V - V_2 = (\delta - \delta_1)(t_2 - t_1) V_1.$$

We thus find the apparent expansion by inserting in the formula, as the coefficient of expansion, the difference

$(\delta - \delta_1)$ of the coefficients of expansions of the liquid and of the vessel. The absolute expansion of mercury has been ascertained by Dulong and Petit, by comparison of the heights of two communicating mercurial columns of varying temperature, but the apparent expansion within glass tubes was measured by so-called *weight thermometers*, where the temperature is determined by the quantity of mercury driven out by heating. We thus have for the absolute expansion of mercury,

$$\text{By heating, from} \quad 0 - 100°, = \frac{100}{5550} = 0{\cdot}018018$$

$$\text{“} \quad \text{“} \quad \text{“} \quad 100 - 200°, = \frac{100}{5425} = 0{\cdot}018433$$

$$\text{“} \quad \text{“} \quad \text{“} \quad 200 - 300°, = \frac{100}{5300} = 0{\cdot}018868.$$

But the apparent expansion of mercury was found for an increase of heat from

$$0 - 100°, = \frac{100}{6480} = 0{\cdot}015432,$$

whence the corresponding cubic expansion of the glass tube would be

$$= 0{\cdot}018018 - 0{\cdot}015432 = 0{\cdot}002586,$$

which corresponds well with the statement in § 355, as from the above the linear expansion of the glass is

$$= \tfrac{1}{3} \times 0{\cdot}002586 = 0{\cdot}000862,$$

while in the article referred to it is given at 0·00086133. Moreover, according to Regnault and Isidor Pierre (see “Recherches sur la dilatation des liquides, Annales de chimie et de physique,” Tome XV., 1825), the expansion of different kinds of glass varies considerably. The latter especially finds for glass,

$$\delta = 0{\cdot}000019026 \quad \text{to} \quad 0{\cdot}000026025.$$

With the aid of the above-mentioned coefficient of expansion of mercury, $\delta = 0{\cdot}00018018$, the specific gravity ε of the mercury can be calculated for any temperature. Thus,

$$\varepsilon = \frac{13{\cdot}598}{1 + 0{\cdot}00018018 \cdot t}.$$

With aid of the absolute coefficient of expansion of mercury $\delta = 0{\cdot}00018018$, an observed barometric or manometric height can be reduced from a temperature t to another temperature t_1.

Thus the reduced barometric height is

$$h_1 = \frac{\gamma}{\gamma_1} h = \frac{\varepsilon}{\varepsilon_1} h = \left(\frac{1 + \delta t_1}{1 + \delta t}\right) h = \left(\frac{1 + 0{\cdot}00018018\ t_1}{1 + 0{\cdot}00018018\ t}\right) h$$

$$= \left(\frac{5550 + t_1}{5550 + t}\right) h,$$

since for equal pressure the heights of two liquid columns are to each other inversely as their densities γ and γ_1 or their specific weights ε and ε_1.

Remark.—According to Regnault, the volume of mercury at $t°$ is

$$V = (1 + 0{\cdot}000179007\ t + 0{\cdot}0000000252316\ t^2)\ .\ V_0,$$

where V_0 indicates the volume at 0°.

Example.—When a mercurial column enclosed in a glass tube passes from temperature t to t_1, then its height h changes to

$$h_1 = [1 + (\delta - 2\ \delta_1)\ (t_1 - t)]\ h,$$

because the new volume is

$$V_1 = [1 + \delta\ (t_1 - t)]\ V = [1 + \delta\ (t_1 - t)]\ \pi\ r^2\ h,$$

and also

$$= (1 + 2\ \delta_1)\ (t_1 - t)\ \pi\ r^2\ .\ h_1,$$

since the cross-section $\pi\ r^2$ in consequence of surface expansion, assumes the magnitude

$$(1 + 2\ \delta_1)\ (t_1 - t)\ \pi\ r^2.$$

But now

$$\delta = 0{\cdot}00018018 \quad \text{and} \quad 2\,\delta_1 = 2 \times 0{\cdot}0000086133 = 0{\cdot}00001722 66.$$

Hence,

$$h_1 = [1 + (\delta - 2\,\delta_1)(t_1 - t)]\,h = [1 + 0{\cdot}00016295\,(t_1 - t)]\,h.$$

If $t = 10°$, $t_1 = 50°$ and $h = 30$ inches, then we would have

$$h_1 = (1 + 0{\cdot}00016295 \times 40) \times 30 = 30{\cdot}1955 \text{ inches.}$$

§ 362.—**Expansion of Water.**—Other liquids, particularly water, do not expand proportionally to their increase in temperature; the expansions of other liquids also are greater than that of mercury, and especially greater than the expansion of solid bodies. The following comparison gives the expansion ratio of those liquids of most frequent occurrence.

The expansion for an increase of temperature from 0° to 100° is, according to Dalton,

For alcohol of 0·817 spec. gravity $= \frac{1}{9} = 0{\cdot}1112$.
" olive and linseed oil $= \frac{1}{12\frac{1}{5}} = 0{\cdot}08$.
" sulphuric acid of 1·85 spec. gravity $= \frac{1}{16\frac{2}{7}} = 0{\cdot}060$.
" sulphuric ether $= \frac{1}{14} = 0{\cdot}0700$.
" saturated solution of salt $= \frac{1}{20} = 0{\cdot}0050$, according to Hallström.
" water $= \frac{1}{209\frac{1}{2}} = 0{\cdot}04775$.
" mercury $= \frac{1}{55\frac{5}{9}} = 0{\cdot}018018$, according to Dulong and Petit.

Of all liquids, water expands most irregularly, its density even increasing instead of decreasing from 0° to 4° Centigrade, so that at the latter temperature its density is a maximum. It has been endeavored to ascertain the law of expansion of water in many ways, large water-thermometers having been especially employed for this purpose. Endeavors have been made to deduce empirical formulæ from the results of these experiments, and by their aid also to determine the necessary constants for such formulæ. It is supposed that of all these formulæ, the following two of

Hallström most closely conform to the results of experiment.

If V_0 is the volume of water at 0°, and V that at $t°$, then we have for temperatures from 0° to 30°,

$$V = (1 - 0{\cdot}000057577\, t + 0{\cdot}00000075601\, t^2 - 0{\cdot}00000003509\, t^3)\, V_0,$$

and for temperatures between 30° and 100°,

$$V = (1 - 0{\cdot}00000094178\, t + 0{\cdot}00000533661\, t^2 - 0{\cdot}00000000104086\, t^3)\, V_0;$$

accordingly, for $t = 3{\cdot}92°$ the volume is least, and is equal to 0·9998887. Observations show, however, that the minimum volume or the maximum density occurs at 3·9°. According to the latest experiments of Kopp for temperatures between 0° and 25° C., we have

$$V = (1 - 0{\cdot}000061045\, t + 0{\cdot}00000077183\, t^2 - 0{\cdot}00000000103734\, t^3)\, V_0,$$

and consequently the greatest density of water is at 4·08° (see Poggendorff's "Annalen," Band LXXII.). It is generally assumed that this condition of greatest density of water takes place at 4° C. If we put the volume of water

At 4° = 1·00000, then we have, according to Despretz,

" 5° = 1·00001,	
" 6° = 1·00003,	At 40° = 1·00773,
" 8° = 1·00012,	" 50° = 1·01205,
" 10° = 1·00027,	" 60° = 1·01698,
" 12° = 1·00047,	" 70° = 1·02255,
" 15° = 1·00087,	" 80° = 1·02885,
" 20° = 1·00179,	" 90° = 1·03566,
" 25° = 1·00293,	" 100° = 1·04315.
" 30° = 1·00433,	

Remark 1.—According to the French system of measures and weights, the weight of one cubic meter water at 4° C. temperature and 0·76 meters height of barometer is equal to 1 gramme, and, according to the old Prussian

system, the weight of one cubic foot of water at 15° R., and 28 Paris inches height of barometer, is equal to 66 pounds. Assuming this, the weight of the latter at 4° C., since $15° \text{ R.} = \frac{5}{4} \times 15 = 18\frac{3}{4}°$ C., is $= 1{\cdot}00153 \times 66 = 66{\cdot}101$ Prussian pounds. Now, one Prussian foot $= 31{\cdot}38535$ centimeters, and accordingly one cubic foot $= 3091{\cdot}584$ cubic-centimeters; therefore, the value of an old Prussian pound is

$$\frac{30915{\cdot}84}{66{\cdot}101} = 467{\cdot}71 \text{ grammes},$$

as also inversely that of 1 gramme $= 1 \div 467{\cdot}71 = 0{\cdot}0021381$ lbs., or one kilogramme $= 2{\cdot}1381$ Prussian pounds.

REMARK 2.—Experiments on the expansion of water and other fluids have been made by Munke, Stampfer, Hallström, Despretz, and more recently by Kopp and I. Pierre. For further information, see Gehler's "Physicalisches Wörterbuch," Bd. I. and IV., "Jahrb. des k. k. polytech. Inst.," Bd. XVI., and Poggendorff's "Annalen," Bd. I., IX., XXXIV., and LXXII.; also "Annales de chimie et de physique," T. LXX. and XV.

§ **363.—Expansion of Air.**—The expansion of air and other gases by heat is more considerable, and, with regard to the indications of the mercurial thermometer, is more regular than the expansion of liquids. Gay-Lussac found, by using an air thermometer, closed by a short mercurial column, for an increase from 0° to 100° C., the expansion of atmospheric air, as well as several other gases, to be $\frac{3}{8} = 0{\cdot}375$. But Rudberg, experimenting upon air thoroughly dried by chloride of calcium, and heated in a thermometer tube by means of steam to 100°, and determining the expansion by the quantity of mercury which entered during the cooling process, found this expansion ratio to be smaller. The result gave only $0{\cdot}365$. More recently, also, Magnus and Regnault have determined the coefficient of expansion of air, etc., by special methods, with still greater accuracy. Both found, independently of each other, the expansion ratio for perfectly dried atmospheric air, $= \frac{11}{30} = 0{\cdot}3665$.

As to other gases, only those which, under high pressure, can be converted into liquids, give somewhat greater expansion ratios. Especially conspicuous is sulphuric acid gas, which has the large expansion ratio of $0{\cdot}390$. The experiments of Regnault also show that the expansion of air under high pressures is somewhat greater than at lower

and more moderate pressures. Thus, under a pressure of 109·72 millimeters, the expansion ratio is 0·365, it becomes 0·371 at 3655·6 millimeters. The application of these results to the reduction of quantities of gas from one temperature to another, etc., has been already indicated in Vol. I., § 392 and § 393. Upon comparison of the indications of the air and mercurial thermometer, it appears that they do not correspond entirely with each other.

Thus, for instance, Magnus found that 100°, 200°, and 300° of the mercurial thermometer corresponded to 100°, 197·5°, 294·5° of the air thermometer.

REMARK.—The more recent researches concerning the expansion of gases are discussed in Poggendorff's "Annalen," Band L. and LII., as also in Regnault's "Memoiren" (see § 328).

§ **364.**—The formula,

$$\frac{V}{V_1} = \frac{1 + \delta t}{1 + \delta t_1} \cdot \frac{p_1}{p},$$

deduced from the laws of Mariotte and Gay-Lussac (see Vol. I., § 392), becomes, when $t_1 = 0$ and V_0 and p_0 indicate the volume of air and its pressure at 0° of heat,

$$\frac{V}{V_0} = (1 + \delta t)\frac{p_0}{p},$$

or

$$\frac{Vp}{V_0 p_0} = 1 + \delta t.$$

As $\delta = 0{\cdot}00366$, we have also

$$\frac{V_n p_n}{V_0 p_0} = 1{\cdot}366,$$

where V_n and p_n indicate the volume and pressure of the same quantity of air at $t_1 = 100°$ of heat.

For the temperature t of the volume of air V of the pressure p, we have also the formula,

$$t = \frac{Vp - V_0 p_0}{\delta V_0 p_0} = 273° \cdot \left(\frac{Vp - V_0 p_0}{V_0 p_0}\right).$$

The temperature at which the pressure is zero, or the elasticity of the air vanishes, is, therefore,

$$t = -273^\circ.$$

This temperature indicates the so-called ABSOLUTE ZERO, and another temperature τ, measured from this zero point, is called the absolute temperature.

The absolute temperature is, therefore,

$$\tau = 273^\circ + t,$$

and the ordinary relative temperature is

$$t = \tau - 273^\circ.$$

For $t = 100^\circ$, we have, for example, $\tau = 373^\circ$, while for $\tau = 250^\circ$ we have $t = -23^\circ$.

By introduction of the absolute temperature, we obtain simply

$$1 + \delta t = 1 + \delta\left(\tau - \frac{1}{\delta}\right) = \delta\tau,$$

therefore

$$\frac{Vp}{V_0 p_0} = \delta\tau_1,$$

and also for a volume of air V_1 of the pressure p_1, and absolute temperature τ_1,

$$\frac{V_1 p_1}{V_0 p_0} = \delta\tau.$$

Therefore, the above formula assumes the following more simple form:

$$\frac{V}{V_1} = \frac{\tau}{\tau_1}\frac{p_1}{p},$$

or

$$\frac{Vp}{V_1 p_1} = \frac{\tau}{\tau_1}.$$

The formula $\frac{Vp}{V_0 p_0} = \delta\tau$ is applicable to *perfect gases*, to

which hydrogen pre-eminently belongs; for *imperfect gases*, as, for instance, carbonic acid gas, we have to put

$$\frac{V p}{V_0 p_0} = \delta t - A_0 - \frac{A_1}{\tau} - \frac{A_2}{\tau_2}, \text{ etc.}$$

If v, v_1, etc., indicate the volumes of a quantity of gas of the weight unity, then we have in

$$\frac{v p}{\tau} = \frac{v_1 p_1}{\tau_1} = \frac{v_0 p_0}{\tau_1}$$

a constant quantity R, and thus $v p = R \tau$; or, if further γ, γ_1, etc., denote the weights of the unit of volume of gas, and hence $v \gamma = v_1 \gamma_1 = 1$,

$$\frac{p}{\gamma \tau} = \frac{p_1}{\gamma_1 \tau_1} = \frac{p_0}{\gamma_0 \tau_0} = R.$$

If R and R_1 are the constants for two different gases, then we have

$$\frac{R_1}{R} = \frac{p_1}{\gamma_1 \tau_1} \cdot \frac{\gamma \tau}{p};$$

therefore, for $\tau = \tau_1$ and $p = p_1$,

$$\frac{R_1}{R} = \frac{\gamma}{\gamma_1} = \frac{1}{\varepsilon},$$

where ε denotes the specific weight of the second kind of gas with reference to the first. For atmospheric air, we have for $t_0 = 0°$ of heat and 0·76 meters barometric height, the pressure per square meter, $p = 10334$ kilogrammes, and the weight of a cubic meter $\gamma = 1{\cdot}29318$ kilogrammes; consequently, we can put here

$$R = \frac{p}{\gamma \tau} = \frac{10334}{1{\cdot}29318 \cdot 273} = 29{\cdot}272.$$

For oxygen, we have

$$\varepsilon = \frac{1{\cdot}42980}{1{\cdot}29318} = 1{\cdot}10563; \quad \text{therefore} \quad R_1 = \frac{R}{\varepsilon} = 26{\cdot}475.$$

For nitrogen,

$$\varepsilon = 0{\cdot}97137; \quad \text{therefore} \quad R_1 = 30{\cdot}134.$$

For hydrogen,

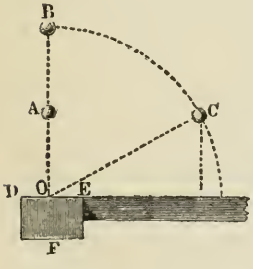
$$\varepsilon = 0{\cdot}06926\,; \quad \text{therefore} \quad R_1 = 422{\cdot}61.$$

§ **365.—Radiant Heat.**—Heat is communicated to other bodies, either by radiation or by contact, and we call the heat communicated in the first manner, radiant heat (Fr. chaleur rayonnante; Ger. strahlende Wärme). The essential difference between these two modes of heat transmission is that radiant heat passes through space, through air, water, or other intervening bodies without producing any effect upon them, and passes into a third body, while, by contact, the medium is first heated, and then from this the heat is conveyed to the third body.

The radiation of heat takes place according to the same laws as the radiation of light. Thus heat, like light, is propagated in straight lines, which are called rays (Fr. rayons de chaleur; Ger. Wärmestrahlen). Radiant heat also is in an inverse ratio to the square of the distance, so that from one and the same source of heat, a body, when removed to a double or threefold distance, receives only one fourth or one ninth part as much heat as before. Further, the intensity of radiant heat increases as the sine of the angle made by the rays with the heat-radiating surface.

Thus the body A, Fig. 602, is four times as much heated by the heat-radiating body DEF as the body B, which is twice as distant from that body, and the body B absorbs twice as much of the radiating heat as the body C, which is at the same distance, because the mean direction of the rays of heat which strike C makes, with the radiating surface DE, an angle COE of 30°, whose sine is $\frac{1}{2}$.

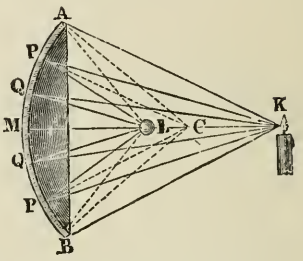
FIG. 602.



In like manner, the rays of heat are reflected after the same law as the rays of light; here also the angle of reflection is equal to the angle of incidence.

The rays of heat KP, QK, which fall upon a spherical mirror AMB, Fig. 603, are thus reflected from the same in such direction PL, QL, that the angle of reflection CPL equals the angle of incidence CPK, as also the angle of reflection CQL equals the angle of incidence CQK, and therefore all rays of heat which strike near the centre M of the mirror are brought to a focus at the same point L.

Fig. 603.



Finally, it is found that the heat rays are refracted, in consideration of the refraction, or deviated by passing from one body into another, according to the same laws as the light rays.

§ **366.**—The capacity of bodies to radiate heat depends upon the temperature of the body and upon the extent and condition of its surface. Generally, the surfaces of very dense bodies radiate less heat than the surfaces of bodies not so dense; but especially rough surfaces have a greater power of radiation than smooth, polished surfaces. According to the experiments of Melloni, if we denote the heat-radiating capacity of a surface covered with soot by 100, that of a white-lead surface is also 100, while that of a surface covered with Indian ink is = 85, that of a gum-lac surface = 72, and that of a metal surface only 12; besides this, the capacity of heat radiation depends also somewhat upon the thickness of the layer which forms the surface of the body.

The heat-absorbing capacity of bodies, or the capacity of bodies to absorb radiant heat, differs with different bodies and corresponds with the radiating power; thus, blackened and rough bodies absorb heat easier than bodies with smooth and polished surfaces. The capacity of bodies to reflect rays of heat, or the so-called power of reflection, is the complement of the radiating or absorbing capacity; the more rays of heat a body absorbs, the fewer it will

naturally reflect. For this reason, surfaces covered with soot reflect hardly any heat, while polished metal surfaces reflect the most heat. Moreover, not all the rays of heat are regularly reflected according to the above-mentioned law, but a portion of them are irregularly reflected in all directions, or, as we say, the rays of heat close to the surfaces of most bodies are diffused. If, according to Leslie, we take the power of reflection of polished brass = 100, then we shall have for silver 90, for steel 70, for glass 10, and for a surface covered with soot 0.

The *diathermal* power, finally, varies greatly for different bodies. Some bodies detain the rays of heat completely and allow none to pass through, while others allow the passage of the rays of heat, just as transparent bodies do the light. The latter are called *diathermanous* and the former *athermanous* bodies. Air is a diathermanous body, and next to it comes rock-salt. Moreover, not only the transparent but also some opaque bodies, as, for instance, black glass, mica, etc., are diathermanous. The diathermic property depends also upon the nature of the source of heat, and rock-salt seems to form the only exception to this rule. Finally, thinner plates allow more rays of heat to pass through them than thick ones, which retain more heat the thicker they are.

Remark.—For more accurate information with regard to these last-mentioned properties of heat, and especially for a more detailed account of the researches of Melloni, the student is referred to standard works on physics, such as the Treatises of Müller, Mousson, Wüllner, etc. See also "Die Wärmemeszkunst," by C. Schinz. The more recent investigations of Provostaye and Desains are to be found in the "Annal. de chimie et de physique," Tome XXX., 1850.

§ **367.—Conduction of Heat.**—The propagation of heat in one and the same body, as also the communication of heat by contact, is called conduction of heat (Fr. conductibilité de la chaleur; Ger. Wärmeleitung). The ease and rapidity of this mode of heat communication differ greatly with different bodies; some bodies possess a great conducting power, and others a lesser one; in the former, the heat

is diffused rapidly, and in the latter very slowly. The former, therefore, are called good and the latter poor conductors of heat. The metals are all good conductors of heat, but some much better than others. Wood, straw, feathers, silk, wool, hair, coal, ashes, etc., and, in general, all spongy bodies, are bad conductors. By grinding and pulverizing, good conductors are converted into bad ones, and the latter, by the same process, into still worse ones. According to the observations of Depretz upon bars heated at one end, if the conducting power of gold, measured by the difference of temperatures at both ends of the bar, is taken at 1000, that of platinum will be 981, of silver 973, of copper 898, of iron 374, of zinc 363, of tin 303, and of lead 180. The conducting power of marble is generally taken at 23, and that of calcined stones only 12, although with less certainty.

The results found by Wiedemann and Franz differ considerably from the above. (See Poggendorff's "Annalen der Physik," Band 89.)

Thus, if the conducting power of silver is taken at 100, then we have

for Copper	73·6
" Gold	53·2
" Tin	14·5
" Iron	11·9
" Steel	11·6
" Lead	8·5
" Platinum	8·4
" Rose's Metal	2·8
" Bismuth	1·8

Liquids are indeed bad conductors of heat, but they absorb heat rapidly, because they are set in motion by the unequal expansion, and thereby the colder particles are brought nearer to the source of heat. In order to convince one's self of the poor heat-conducting power of liquids, a thin layer of sulphuric ether, poured upon the liquid, may be ignited, and the height of a thermometer (immersed

somewhat below this layer in the liquid) observed. According to Despretz, who tried to heat equally a column of water by repeated additions of warm water, the conducting capacity of water is only about from 9 to 10.

Air and gases in general are bad conductors of heat, but their conducting power cannot be observed with precision by the thermometer, on account of the induced currents and their greater radiating power. The poor conducting power may be inferred from the fact that bodies which are surrounded on all sides by a layer of air are very slowly heated and cooled.

§ **368.—Cooling Power.**—The rapidity with which hot bodies part with their heat or become cool varies greatly also. If a hot body is surrounded by a solid body, then the cooling takes place chiefly by reason of the conducting power of the latter; but if the surrounding body is a liquid, then the cooling is effected partly by conduction of heat, and partly and chiefly by the internal motions of the liquid; if, further, the hot body is surrounded by a gas, then the rapidity of cooling depends also upon the radiation of heat; and if, finally, the hot body is placed in a vacuum, then it is the radiation alone which causes its loss of heat. In general, it can be asserted that the rate of cooling depends upon the difference of temperature, and upon the kind and extent of the surface of the cooling body. It may be assumed that the loss of heat is proportional to the surface, and for moderate surplus of temperature, to that also. Later researches of Dulong and Petit have shown that the former law, originally established by Newton, cannot be generally applied, and especially not for great differences of temperature. The laws of cooling are very complicated; Dulong and Petit have tried to ascertain the same for hot bodies in vacuo and in air, by suspending large mercurial thermometers, which were previously heated, in a copper globe, surrounded externally by water of a certain temperature, and observing the fall of these thermometers.

The following table contains the principal results of these observations:

Excess of Temperature.	THERMOMETER BULB BARE.			THERMOMETER BULB SILVERED.		THERMOMETER BULB COVERED WITH SOOT.	
	Complete Cooling.	Cooling by Radiation.	Cooling by Contact.	Complete Cooling.	Cooling by Radiation.	Complete Cooling.	Cooling by Radiation.
260°	24·42°	16·32°	8·10°	10·96°	2·86°	32·02°	23·92°
240	21·12	13·71	7·41	9·82	2·41	27·48	20·07
220	17·92	11·31	6·61	8·59	1·98	23·10	16·49
200	15·30	9·38	5·92	7·57	1·65	19·66	13·74
180	13·04	7·85	5·19	6·57	1·38	16·28	11·09
160	10·70	6·20	4·50	5·59	1·09	13·57	9·07
140	8·75	5·02	3·73	4·61	0·88	11·06	7·33
120	6·82	3·71	3·11	3·80	0·69	8·85	5·74
100	5·56	3·03	2·53	3·06	0·53	6·94	4·41
80	4·15	2·22	1·93	2·32	0·39	5·17	3·24
60	2·86	1·53	1·33	1·60	0·27	3·67	2·24
40	1·74	0·95	0·79	0·96	0·17	2·20	1·41
20	0·77	0·43	0·34	0·42	0·08	1·00	0·66
10	0·37	0·22	0·15	0·19	0·04	0·48	0·33

We see from this table, which gives the degrees cooled per minute, that the observations do not agree with the law of Newton just stated. Thus the second column of the table gives us for the differences,

$$40°,\ 80°,\ 120°,\ 160°,\ 200°,\ 240°,$$

between the temperature of the thermometer which is exposed to cooling and that of the external water-envelope, the cooling per minute,

$$1·74°,\ 4·15°,\ 6·82°,\ 10·70°,\ 15·30°,\ 21·12°;$$

but, according to Newton, we should have

$$1·74°,\ 2\times 1·74°,\ =\ 3·48,\ 3\times 1·74°\ =\ 5·22°,$$

$$4\times 1·74°\ =\ 6·96°,\ 5\times 1·74°\ =\ 8·70°,\ 6\times 1·74°\ =\ 10·44°.$$

Only for slight excess of temperature, at the most of 40°, can it be assumed as approximately true that the rapidity of cooling is proportional to this excess.

A comparison of the figures in the different vertical columns with each other shows plainly that the cooling by radiation of a shining metallic surface is small in comparison with cooling by contact, but that, on the other hand, the cooling by radiation of a surface covered with soot constitutes the greatest part of the total cooling. The values for cooling by contact, which are given in the fourth column of the table, have been found by subtraction of the values in the second and third columns, observed either with empty or air-filled globes, and are applicable to all kinds of surfaces. Besides this, the rapidity of cooling depends also naturally upon the extent of the surface of the body exposed to cooling. The cooling of a body may be well compared to the discharge of water from a vessel; the pressure height or head corresponds to the difference of temperature, and the orifice of discharge answers to the extent of the cooling surface. As we may have discharge under constant and under decreasing head, so we have also cooling at constant and cooling at decreasing temperature.

Just as in the emptying of a prismatic vessel, the time of discharge increases directly with the volume and inversely as the area of the orifice, so the time of cooling is directly proportional to the cooling mass and inversely to its extent of surface. The observations of Dulong and Petit substantiate this, as, according to them, the periods of cooling are proportional to the diameters of the thermometer bulbs.

According to the experiments of Dulong and Petit, the rapidity of cooling by radiation, or in a vacuum—*i. e.*, the loss of heat during one unit of time—is given by the formula,

$$v_1 = \mu_1 a^{t_1} (a^t - 1),$$

where μ_1 and a are experimental constants, t_1 the temperature of the surrounding body, and t the excess of temperature. The constant a depends only on the graduation of the thermometer. Thus, for the centigrade division it

is 1·0077, and for the scale of Reaumer $(1\cdot0077)^{\frac{5}{4}} = 1\cdot0096$. But μ_1 depends upon the power of radiation and upon the extent of the cooling surface. The member $\mu_1 a^{t_1}$, which is to be subtracted from $\mu_1 a^{t_1} \times a^t = \mu a^{t_1 + t}$, measures the heat reflected back by the surface of the blackened copper globe, and would entirely disappear if the cooling should take place in an unlimited space. For the rapidity of cooling corresponding to contact with air, we have

$$v'_2 = n p^c t^{1\cdot233} = \mu_2 t^{1\cdot233},$$

where $\mu_2 = n p^c$, and n indicates a constant, depending upon the extent of the cooling surface and upon the nature of the cooling medium, c a constant only depending upon the latter, p the elasticity of this medium, and t, as before, the excess of temperature. Accordingly, the total rapidity of cooling is to be put,

$$v = v'_1 + v'_2 = \mu_1 a^{t_1} (a^t - 1) + \mu_2 t^{1\cdot233}.$$

The powers $a^t = (1\cdot0077)^t$ and $t^{0\cdot233}$ can be determined in ordinary cases by means of the following table:

Temperature t degrees.	Power $1\cdot0077^t$.	Power $t^{0\cdot233}$.	Temperature t degrees.	Power $1\cdot0077^t$.	Power $t^{0\cdot233}$.
10	1·080	1·710	110	2·325	2·990
20	1·165	2·010	120	2·510	3·051
30	1·259	2·209	130	2·711	3·108
40	1·359	2·362	140	2·927	3·163
50	1·467	2·488	150	3·160	3·214
60	1·584	2·596	160	3·412	3·263
70	1·711	2·691	170	3·684	3·309
80	1·847	2·776	180	3·978	3·353
90	1·994	2·853	190	4·295	3·396
100	2·153	2·924	200	4·637	3·437

For heat radiation, the coefficient μ_1 has, for the period of one hour and for one square meter extent of surface, the following values:

Polished Silver	16	Rusted Sheet-iron	419
Silver-paper	52	New Cast-iron	395
Polished Brass	32	Rusted Cast-iron	419
Gold-paper	28	Glass	373
Red Copper	20	Coal-dust	427
Zinc	30	Paper	470
Tin	27	Soot	500
Polished Sheet-iron	56	Building Stones	449
Leaded Sheet-iron	81	Wood	449
Black Sheet-iron	345	Water	662

The coefficient μ_2 for the conduction of heat through the air depends upon the form and dimensions of the heated bodies. Thus for a horizontal cylinder of the radius r meters,

$$\mu_2 = 1 \cdot 136 + \frac{0 \cdot 0211}{r}.$$

REMARK.—For more detailed information on this subject, see "Recherches sur la mesure de températures," etc., by Dulong and Petit, in "Journal de l'école polytechnique," J. XI.; also, "Traité de la chaleur," by Péclet, and Gehler's "Physikalisches Wörterbuch," Band X.

§ **369.**—For practical use, more convenient approximate formulæ for the rapidity of cooling are given by Péclet in Vol. II. of his previously mentioned work. He places the rapidity of cooling

$$v = A\, t\, (1 + \alpha\, t),$$

and assumes for temperatures from 10° to 260°, for glass surfaces,

$$\alpha = 0 \cdot 0065\ ;$$

for silver surfaces,

$$\alpha = 0 \cdot 0051\ ;$$

for soot surfaces,

$$\alpha = 0 \cdot 0066.$$

For temperatures from 0° to 20°, however, in the first case,

$$\alpha = 0 \cdot 0039\ ;$$

in the second,

$$= 0{\cdot}011\,;$$

and in the third,

$$= 0{\cdot}0043.$$

As to the value of the coefficient A, he refers the same to the loss of heat per hour per square meter, and gives for water enclosed by a polished metal surface,

$$A = 4{\cdot}38\,;$$

by a glass or varnish wall,

$$A = 6{\cdot}40\,;$$

by a sheet or cast-iron wall,

$$A = 7{\cdot}70\,;$$

by a soot-covered wall,

$$A = 8{\cdot}48.$$

Generally, we may take for walls of lime or brick,

$$A = 9,$$

and for a wooden wall,

$$A = 8.$$

Péclet considers the case of a vessel filled with hot water, surrounded at a certain distance from the wall of the vessel by a covering, and the intermediate space filled with air. If, then, F and F_1 are the extent of surface of the vessel and of the envelope, and if t and t_1 represent the excess of temperature in regard to the external air, we have

$$F\,(t - t_1)\,(1 + \alpha\,(t - t_1)) = F_1\,t_1\,(1 + \alpha\,t_1),$$

or approximately,

$$F\,(t - t_1) = F_1\,t_1.$$

Therefore

$$t_1 = \frac{F\,t}{F + F_1}.$$

Hence, the rapidity of cooling for one square meter is

$$v = A\, t_1\, (1 + \alpha\, t_1) = \frac{F}{F + F_1}\, A\, t \left(1 + \frac{\alpha\, F}{F + F_1}\, t\right),$$

and for the cooling of the surface F_1, as also of the whole vessel,

$$F_1\, v = \frac{F F_1}{F + F_1}\, A\, t \left(1 + \frac{\alpha\, F\, t}{F + F_1}\right).$$

Without the envelope, the cooling of the vessel would be

$$F\, A\, t\, (1 + \alpha\, t),$$

and therefore greater, because

$$\frac{F_1}{F + F_1}$$

is a proper fraction.

If the intermediate space between boiler and envelope is but small, or if it contained no air, then the heat would be transferred from the boiler to the cover by radiation only, and we would have to introduce for this cooling a different coefficient from that for the cooling on the surface of the cover F_1. Denoting these coefficients by A and A_1, we obtain

$$A\, F\, (t - t_1) = A_1\, F_1\, t_1;$$

therefore

$$t_1 = \frac{A\, F}{A\, F + A_1\, F_1}\, t,$$

and accordingly the rapidity of cooling for one square meter is

$$v = A_1\, t_1\, (1 + \alpha\, t_1) = \frac{A\, A_1\, F\, t}{A\, F + A_1\, F_1} \left(1 + \alpha \cdot \frac{A\, F}{A\, F + A_1\, F_1}\, t\right);$$

and for the whole surface F_1,

$$F_1\, v = \frac{A\, A_1\, F\, F_1}{A\, F + A_1\, F_1}\, t \left(1 + \alpha \cdot \frac{A\, F}{A\, F + A_1\, F_1}\, t\right).$$

Example.—A wrought-iron boiler of 15 square meters' extent of surface contains water at 100°, and is surrounded externally by air at 20°, what cooling does the water undergo? Here

$$\alpha = 0{\cdot}0066, \quad A = 7{\cdot}70, \qquad \text{and} \qquad t = 100° - 20° = 80°,$$

therefore the rapidity of cooling is

$$v = A\,t\,(1 + \alpha\,t) = 7{\cdot}7 \times 80\,(1 + 0{\cdot}0066 \times 80) = 616 \times 1{\cdot}528 = 941°,$$

and hence the cooling per hour for the whole surface of 15 square meters is

$$F\,v = 15 \times 941 = 14115°;$$

i.e., the boiler is hourly deprived by cooling of 14115° of heat, which must be replaced by heat from another source, if the temperature of 100° is to remain unchanged.

If the boiler were surrounded by a jacket of 25 square meters in superficial area, which encloses a certain quantity of air, then we should have for the loss of heat, only

$$F_1\,v = \frac{F\,F_1}{F + F_1}\,A\,t\left(1 + \frac{\alpha\,F\,t}{F + F_1}\right) = \frac{15 \times 25}{40}\,616\left(1 + 0{\cdot}0066 \times \frac{15 \times 80}{40}\right)$$

$$= 5775 \times 1{\cdot}198 = 6918°.$$

If, finally, the intermediate space between boiler and jacket is devoid of air, and consequently the heat of the boiler can only escape by radiation, we should have

$$A = 0{\cdot}2 \times A_1 = 0{\cdot}2 \times 7{\cdot}7 = 1{\cdot}54,$$

and therefore

$$F_1\,v = \frac{1{\cdot}54 \times 7{\cdot}7 \times 15 \times 25 \times 80}{1{\cdot}54 \times 15 + 7{\cdot}7 \times 25}\left(1 + 0{\cdot}0066 \cdot \frac{1{\cdot}54 \times 15 \times 80}{1{\cdot}54 \times 15 + 7{\cdot}7 \times 25}\right)$$

$$= \frac{355740}{215{\cdot}6} \times 1{\cdot}0563 = 1743°.$$

In this case, then, only about $\frac{1}{8}$ as much loss of heat takes place as in the case of the unprotected boiler.

§ **370.**—Péclet gives also another formula and the necessary constants for the determination of cooling in the case of *bad conductors of heat*. If we denote by C the quantity of heat which hourly passes through a plate of one square meter in area and one meter in thickness, when the difference of temperature of both surfaces amounts to 1°, and

let v denote the heat which passes hourly through a plate of the thickness e, whose area is F, the temperatures being t and t_1, we can put

$$v = \left(\frac{t - t_1}{e}\right) F C,$$

where C has the following values:

For Copper	$C = 64{\cdot}00$	For Pine-wood (wood normal to fibres)	$C = 0{\cdot}09$
" Iron	" $= 29{\cdot}00$	" Oak (wood normal to fibres).....	" $= 0{\cdot}25$
" Zinc	" $= 28{\cdot}00$	" Cork	" $= 0{\cdot}14$
" Lead	" $= 14{\cdot}00$	" Ind. Rubber..	" $= 0{\cdot}17$
" Coke	" $= 4{\cdot}96$	" Brick-dust ..	" $= 0{\cdot}15$
" Marble	" $= 3{\cdot}13$	" Wood-ashes..	" $= 0{\cdot}06$
" Limestone (common).	" $= 1{\cdot}82$	" Linen	" $= 0{\cdot}05$
" Glass.....	" $= 0{\cdot}82$	" Cotton.....	" $= 0{\cdot}04$
" Burned Clay	" $= 0{\cdot}60$	" Paper (gray and unsized).	" $= 0{\cdot}03$
" Gypsum....	" $= 0{\cdot}48$		
" Pine-wood (parallel to the fibres)..	" $= 0{\cdot}17$		

If a plate of the surface F is in contact on one side with a body of the temperature t, and on the other side with a body of the temperature t_1, and thereby the temperature of the plate passes gradually through the thickness e from τ into τ_1, then, for the loss of heat, we have

$$Q = F A (t - \tau) = F C \left(\frac{\tau - \tau_1}{e}\right) = F A_1 (\tau_1 - t_1),$$

and also

$$A (t - \tau) = C \left(\frac{\tau - \tau_1}{e}\right) = A_1 (\tau_1 - t_1).$$

Eliminating

$$\tau_1 = \tau - \frac{A e}{C} (t - \tau),$$

we have

$$A\,(t-\tau) = A_1\left(\tau - t_1 - \frac{A\,e}{C}(t-\tau)\right).$$

For the temperature of the plate on one side, therefore,

$$\tau = \frac{A\,t + A_1\,t_1 + \dfrac{A\,A_1\,e\,t}{C}}{A + A_1 + \dfrac{A\,A_1\,e}{C}},$$

or

$$\tau = \frac{A\,A_1\,t + (A\,t + A_1\,t_1)\dfrac{C}{e}}{A\,A_1 + (A + A_1)\dfrac{C}{e}},$$

and for the temperature of the other side,

$$\tau_1 = \frac{A\,A_1\,t_1 + (A\,t + A_1\,t_1)\dfrac{C}{e}}{A\,A_1 + (A + A_1)\dfrac{C}{e}},$$

and hence the escaped quantity of heat is

$$Q = \frac{F\,C}{e}(\tau - \tau_1) = \frac{F\,A\,A_1\dfrac{C}{e}(t - t_1)}{A\,A_1 + (A + A_1)\dfrac{C}{e}}, \text{ or } = F \cdot A_2(t - t_1),$$

if we put

$$\frac{A\,A_1\,C}{A\,A_1\,e + (A + A_1)\,C} = \frac{C}{e + \left(\dfrac{1}{A} + \dfrac{1}{A_1}\right)C} = A_2.$$

If the constants A and A_1 for the entrance and exit are equal to each other, then we have simply

$$A_2 = \frac{C}{e + \dfrac{2\,C}{A}};$$

and if besides the thickness of the plate e is small, then $A_2 = \frac{A}{2}$, and, therefore,

$$Q = \tfrac{1}{2} F A (t - t_1).$$

If $\tau = t$, *i. e.*, if the temperature of the plate on one side is equal to that of the body touching it, as is the case for water, for instance, then we have $A = \infty$, and hence

$$A_2 = \frac{C}{e + \frac{C}{A_1}} = \frac{A_1 C}{A_1 e + C},$$

and

$$Q = \frac{F A_1 C (t - t_1)}{A_1 e + C}.$$

EXAMPLE.—If the boiler, filled with water of 100° (the same as in the example of the previous paragraph), is surrounded with a brick wall of $\frac{1}{4}$ meter in thickness, then its cooling is hourly,

$$F v = \frac{F A_1 C (t - t_1)}{A_1 e + C} = \frac{15 \times 9 \times 0{\cdot}68 \times 80}{9 \times \frac{1}{4} + 0{\cdot}68} = \frac{7344}{2{\cdot}93} = 2506°.$$

§ **371.—Melting.**—The expansion which bodies undergo under the action of heat has a certain limit, for, beyond a certain degree of expansion, a body will change its aggregate condition. Thus, solid bodies are converted into liquids and liquids assume the gaseous form. Thus, by the action of heat, ice is converted into water, and this again at a higher temperature (100°) is converted into steam. The conversion of a body from a solid form into a liquid is called melting (Fr. fusion; Ger. Schmelzung), and the conversion of a solid or liquid into the aeriform state is called evaporation (Fr. vaporation; Ger. Verdampfung). The temperature at which a solid body melts is called the melting point (Fr. point de fusion; Ger. Schmelzpunkt). Evaporation takes place at nearly all temperatures, but is very trifling at low temperatures; therefore we have no fixed point of evaporation. Inversely, by deprivation of

heat, aerifrom bodies, particularly when they are exposed at the same time to pressure, can be converted into liquids, and the latter again into solid bodies.

In the following, the melting points (or freezing points) of those bodies of most importance are given:

Platinum	melts at	+ 2500° C.
Wrought-iron	" "	+ 1500 to 1600° C.
Steel	" "	+ 1300 to 1400
Cast-iron	" "	+ 1050 to 1200
Gold	" "	+ 1100 to 1200
Copper	" "	+ 1100 to 1200
Silver	" "	+ 1000
Bronze	" "	+ 900
Antimony	" "	+ 500
Zinc	" "	+ 400
Lead	" "	+ 330
Bismuth	" "	+ 260
Tin	" "	+ 230
Sulphur	" "	+ 109
Yellow Wax	" "	+ 61
Phosphorus	" "	+ 43
Soap	" "	+ 33
Ice	" "	+ 0
Turpentine	" "	+ 10
Mercury	" "	+ 39

REMARK 1.—According to Pouillet, the following temperatures have been observed for iron at different stages of incandescence:

Faint red	525° C.
Dark red	700
Faint cherry	800
Cherry	900
Bright cherry	1000
Dark orange	1100
Bright orange	1200
White heat	1300
Bright white	1400
Dazzling white	1500

2. By means of the alloys (Fr. alliages; Ger. Legirungen), we can prepare a scale of fusibility which may be used in pyrometric experiments.

Low temperatures can be determined by the melting points of compositions of lead, tin, and bismuth; but to measure high temperatures we employ (according to Prinsep, Saussure, and Plattner) alloys of platinum and gold.

An alloy of	1	part lead,	1	part tin,	4	parts bismuth,				melts at	94°
Rose's metal	" 5	" "	3	" "	8	" "				" "	100
So also	" 2	" "	3	" "	5	" "				" "	100
Further,	" 1	" "	4	" "	5	" "				" "	118·9
	" 1	" "	—	" "	1	" "				" "	141·2
	" 1	" "	1	" "	—	" "				" "	241
	" —	" "	2	" "	1	" "				" "	167·7
	" 1	" "	3	" "	—	" "				" "	167·7
	" —	" "	3	" "	1	" "				" "	200

We see that these compositions are more fusible than the simple metals. With the alloys of platinum or gold, this is different; such an alloy is more difficult of fusion than gold the more platinum it contains, so that we can determine in advance the melting point of the composition from the proportions of the metals which form it. (See Merbach, "Die Anwendung der erwärmten Geblaeseluft im Gebiete der Metallurgie," Leipzig, 1840.)

Sea-water, on account of its saline composition, freezes only at $-2\cdot5°$.

On melting points and on the temperatures necessary to form fluid combinations (or compositions), see Schinz, in Dingler's Journal, Band 182, Heft 3.

3. In the melting of solid bodies, as also in the freezing of fluid bodies, there is in general a change of density. For instance, water expands at the point of freezing $\frac{1}{11}$ of its volume, and forms ice of the specific gravity of 0·93. The force of this expansion is so great that large bombshells can be burst by it. Most of the metals, as mercury, lead, zinc, silver, etc., contract in solidifying, but some, as bismuth and cast-iron, expand.

In practice, the shrinkage of metals is of great importance in casting. (See Karmarsch's Treatise in XIX. Band (1837) of the "Jahrbücher des polytechnischen Instituts in Wien.") This change of volume depends upon the contraction or expansion during solidification and upon the contraction due to cooling. According as these changes act together or in opposition to each other, the resulting shrinkage is greater or less. The shrinkage per linear unit is

For Cast-iron, $= \frac{1}{95}$ to $\frac{1}{48}$.
" Brass, $= \frac{1}{60}$ to $\frac{1}{65}$.
" Bellmetal $=$ (100 copper + 18 tin) $= \frac{1}{63}$.
" Gunmetal $=$ (100 copper + 12½ tin) $= \frac{1}{130}$ to $\frac{1}{139}$.
" Zinc $= \frac{1}{80}$.
" Lead, $= \frac{1}{29}$.
" Tin, $= \frac{1}{147}$, and
" Bismuth, $= \frac{1}{260}$.

§ **372.—Evaporation.**—Fluid bodies, and even also some solid bodies, assume, under the influence of heat, a gaseous form. This transformation takes place at all temperatures and pressures, but more actively under the action of heat and slight pressure than cold and high pressure. We may distinguish, therefore, *evaporation* from *ebullition* or *boiling*. While by the former we understand the formation of steam at the surface, we understand by boiling or ebullition the formation of steam within the whole fluid mass. The boiling point, or the temperature at which boiling takes place, is not only different for different liquids, but depends also upon the pressure of the air which surrounds the liquid. At the moment of boiling, the force of expansion of the steam is equal to the pressure of the atmosphere. According to observations, the boiling points for a few bodies, at a pressure of 0·76 meters, are as follows:

Mercury = 350° C.
Linseed-oil = 316°.
Sulphuric Acid = 310°.
Sulphur = 299°.
Phosphorus = 290°.
Turpentine = 273°.
Water = 100°.
Alcohol (of specific gravity = 0·813) = 78·6°.
Sulphuric Ether = 37·8°.
Nitrous Acid = 28°.
Sulphurous Acid = — 10°.

Substances dissolved in water increase the temperature of the boiling point. For instance, water saturated with common salt (100 parts water + 41·2 common salt) boils, according to Legrand, at 108·4°; also, water saturated with carbonate of potash (100 parts water + 205 parts of carbonate of potash) at 133°, and water with chloride of lime (100 parts water + 325 parts of chloride of lime) at 179·5°.

The walls of vessels have also an influence upon the boiling point. Thus, for example, water in metallic vessels boils sooner than in glass vessels.

The expansion of bodies when transformed into steam is considerable. Thus, one cubic foot of water gives, at 100° temperature and 0·76 meters height of barometer, 1700 cubic feet steam, and its density is only $\frac{5}{8}$ of that of the air.

Vapors can be reduced to their liquid form by deprivation of heat or by increase of pressure, and therein consists their only difference from the gases proper, which, thus far, have not been reduced to the liquid form, either by the most intense cold or by the greatest pressure. Carbonic acid gas (vapor of liquid carbonic acid) can only be reduced to a liquid state at 0° and 36 atmospheres' pressure. At 30° of heat, this vapor has an expansive force of 73 atmospheres.

[Quite recently, all the so-called permanent gases have been liquefied, the discovery being announced almost simultaneously by MM. Cailletet and Pictet. (See *La Nature*, 1877, 1878; *Journal of the Franklin Institute*, cv., 187, 190, 319; *Scientific American*, xxxviii., 147; *Scientific American Supplement*, v., 1883; *Engineering*, xxv., 324.) To M. Cailletet, however, belongs the priority, and the apparatus employed by him is shown in the accompanying figures, which, together with the description that follows, was published in *Engineering* for April 26th, 1878:

"A tube *T*, Fig. 602*b*, of very fine bore towards its upper extremity, and capable of sustaining a pressure of 500 atmospheres, is partly filled with the gas to be liquefied. The lower portion of the tube expands to form a capacious bulb, which is filled with mercury, and inserted in a reservoir containing mercury and water. The walls of the reservoir are of steel, and sufficiently thick to allow pressures of 800 atmospheres to be used with perfect safety. The water is forced in by a strong pump; and, according as it enters, the mercury rises in the capillary tube, compressing the confined gas at the same time. A pressure of 200 atmospheres is attained by a few strokes of the pump, and is recorded by a metallic manometer. By means of a plunger *V*, the pressure may be very gradually increased up to 500 atmospheres. Meanwhile, the capillary tube may be surrounded by a freezing mixture, or by water at any desired temperature.

"In the case of most gases, the manometer remains stationary when a certain pressure has been reached. This phenomenon, which is at variance with Boyle's law (a law absolutely true only for a perfect gas), occurs as soon as liquefaction begins. When an appreciable quantity of the gas has been liquefied, upon gradually diminishing the pressure, the liquid begins to boil and returns to the gaseous condition. But if the valve V^1 be suddenly opened, the gas will as suddenly expand; and, undergoing a very considerable reduction of temperature, a portion of it will be liquefied, filling the

upper part of the tube with a sort of cloud or mist. These effects are readily seen with nitrous oxide and acetylene, and may be easily exhibited to an audience by projection on a screen. In the case of the other gases, the liquefaction has not yet proceeded further than its incipient stage. Thus, oxygen under a pressure of 270 atmospheres, and a temperature of 29° C.

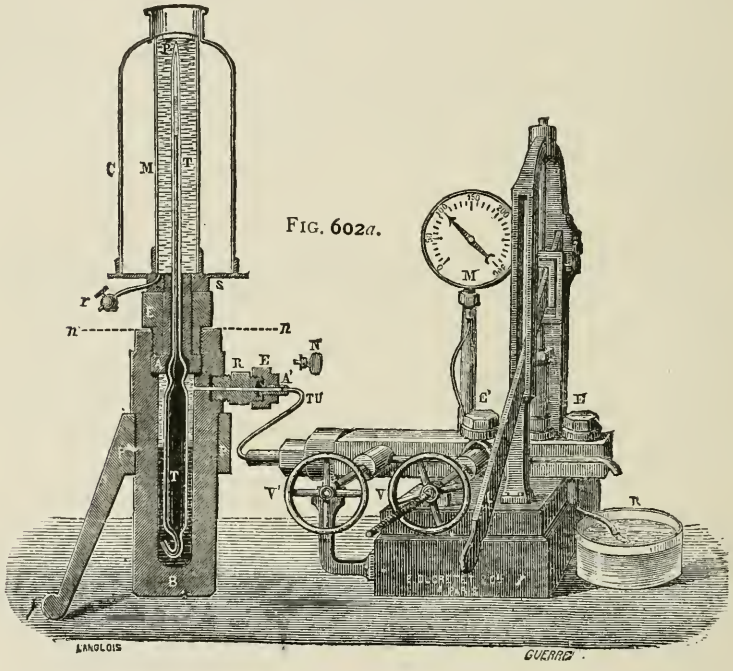


FIG. 602*a*.

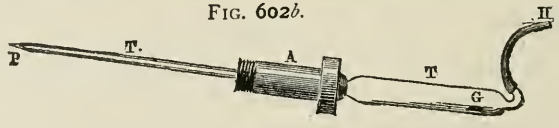


FIG. 602*b*.

was still gaseous; but, on suddenly opening the valve V', the tube was filled with a cloud, which denoted a commencement of liquefaction.

"In the experiments which were made in the laboratory of the Ecole Normale, Paris, on December 30th, 1877, in presence of several of the leading men of science, nitrogen was subjected to a pressure of 200 atmospheres

and the temperature of liquid sulphurous acid (—29° C.), without any signs of giving way. On opening the valve, tiny drops of an appreciable volume were visible for about three seconds. Hydrogen, as anticipated from its close resemblance to a perfect gas, showed itself the most refractory of all the gases; still, when compressed to 280 atmospheres and suddenly released, it assumed, though only for an instant, the form of a very fine, extremely tenuous mist. To leave no doubt on the matter, and to satisfy the physicists present, the experiment was repeated several times, and invariably with the same result.

"On January 14th, 1878, M. Cailletet experimented with atmospheric air. Having removed all traces of moisture and carbonic acid, and introduced it into the capillary tube, he surrounded it with liquefied nitrous oxide. When the manometer recorded 200 atmospheres, streams of liquid air flowed down the sides of the tube."]

§ **373.—Capacity for Heat.**—The quantity of heat in a body is, at any rate, proportional to the temperature and to the mass of the body, and may therefore be measured by the product of both. But it is also very different for different bodies. Some bodies require, in order to attain a certain temperature, more heat than others, and therefore possess a greater capacity for heat than the others. This property of bodies is measured by the *specific heat*, if we hereby understand by this that quantity of heat which is necessary to raise the temperature of a body of one pound in weight one degree. It is, moreover, not possible to indicate the quantity of heat itself, and we can only institute a comparison between the specific heats of different bodies. For this purpose, we assume that quantity of heat which *one pound of water* requires to raise its temperature one degree as a *heat-unit*, and call the same a calorie (Fr. calorie; Ger. Calorie). Accordingly, the quantity of heat which is necessary to raise a quantity of water of Q lbs. t degrees in temperature is

$$W = Q\,t,$$

and, on the other hand, for another body, the specific heat of which with respect to water is $= \omega$,

$$W_1 = \omega\,Q\,t.$$

In the following table, the specific heat of mercury is given as $= 0{\cdot}033$, and from this can be concluded that, for equal weights and equal increase of temperature, water requires $\frac{1}{0{\cdot}033} = \frac{1000}{33} = 30$ times as much heat as mercury.

In order to determine the specific heat of different bodies, various methods have been employed, chiefly the methods by mixture, melting, and cooling.

In the method by mixture, the previously heated body, the specific heat of which is to be determined, is brought into a water bath, and the increase of temperature of the water observed. If Q is the weight of the cooled body, as also Q_1 that of the cooling water, t the decrease of temperature of the body, and t_1 the increase of temperature of the water, then we have the loss of heat of the body $\omega Q t =$ to the gain of heat $Q_1 t_1$ of the water, and therefore the desired specific heat is

$$\omega = \frac{Q_1 t_1}{Q t}.$$

The method by melting consists in surrounding the body by ice, and measuring the amount of water resulting from the melting of the ice. Care having been taken that the ice and the water retain the temperature 0°, then we can put

$$\omega Q t = 79\ Q_1,$$

and therefore

$$\omega = \frac{79\ Q_1}{Q t},$$

because we know, from experiment, that, by the conversion of ice into water of 0°, 79 heat-units are consumed. (See § 380.)

Finally, in the method by cooling, the heated body is surrounded by a metallic envelope, and is suspended in a vessel exhausted of air, which is surrounded by water of a constant temperature, and the time is observed in which

the body sinks to a certain temperature, which is indicated by an inserted thermometer. If for two bodies of the weight Q and Q_1, having equal cooling surfaces, the periods of cooling are z and z_1, and the specific heats $= \omega$ and ω_1, then we have

$$\frac{z}{z_1} = \frac{\omega Q}{\omega_1 Q_1},$$

and therefore the proportion

$$\frac{\omega_1}{\omega} = \frac{Q z_1}{Q_1 z}.$$

EXAMPLE.—What quantity of heat is necessary to heat an iron boiler of 2500 lbs. weight, which is filled with 15000 lbs. of water, from 10° to 100°? The quantity of water requires the quantity of heat

$$W = Q t = 15000 . (100 - 10) = 15000 \times 90 = 1350000$$

calories or heat-units.

Since the specific heat of iron is only 0·11, the iron mass requires the quantity of heat

$$W_1 = \omega Q_1 t = 0{\cdot}11 \times 2500 \times 90 = 24750$$

calories; both together require therefore

$$1350000 + 24750 = 1374750$$

calories or heat-units.

REMARK.—By the aid of the specific heat, we may also inversely determine the temperature of a hot body by cooling it in water, and applying the above formula. Thus,

$$t = \frac{Q_1 t_1}{Q \omega}.$$

If, for instance, a hot brass body of 15 lbs. weight is brought into 80 lbs. of water of 10° heat, and thereby the temperature of the latter is raised from 10° to the common temperature 16°, then we have for the original temperature of the brass (since its specific heat is = 0·0939),

$$16° + \frac{Q_1 t_1}{Q \omega} = 16° + \frac{80 \times 6°}{0{\cdot}0939 \times 15} = 16° + \frac{480°}{1{\cdot}4085} = 357°.$$

Pouillet found in this way the temperature of melting iron = 1500° to 1600°

§ **374.—Specific Heat.**—Laplace and Lavoisier have, for the purpose of ascertaining the specific heat of different bodies, employed the method by melting; Dulong and Petit, the cooling method; Pouillet, and recently also Regnault, the method by mixture. (The latter seems to be the most accurate method.) The specific heats of some of the most important bodies in practice are given in the following table:

Iron	0·11379	acc. to	Regnault,	0·1100	acc. to	Dulong and	Petit.
Zinc	0·09555	"	"	0·0927	"	"	"
Copper	0·09515	"	"	0·0949	"	"	"
Brass	0·09391	"	"	.....	"	"	"
Silver	0·05701	"	"	0·0557	"	"	"
Lead	0·03140	"	"	0·0293	"	"	"
Bismuth	0·03084	"	"	0·0288	"	"	"
Antimony	0·05077	"	"	0·0507	"	"	"
Tin	0·05623	"	"	0·0514	"	"	"
Platinum	0·03243	"	"	0·0314	"	"	"
Gold	0·03244	"	"	0·0298	"	"	"
Sulphur	0·20259	"	"	0·1880	"	"	"
Coal	0·24111	"	"				
Coke	0·20307	"	"				
Graphite	0·20187	"	"				
Marble	0·20989	"	"				
Unslacked Lime	0·2169	"	Lavoisier and Laplace.				
Alcohol	0·700	(of 0·81 specific gravity) according to Dalton.					
Oak-wood	0·570	according to Mayer.					
Glass	0·19768	"	" Regnault.				
Mercury	0·03332	"	" "				
Oil of Turpentine	0·42593	"	" "				

Moreover, the specific heat of the same material is not perfectly constant, but increases as the density of the body decreases, and increases also somewhat when the temperature of the body becomes very high and approaches the boiling point. Thus, according to Dulong and Petit, the mean specific heat for

Iron,	between	0°	and	100°,	=	0·1098,	between	0°	and	300°	=	0·1218.
Mercury,	"	"	"		=	0·0330,	"	"	"		=	0·0350.
Zinc,	"	"	"		=	0·0927,	"	"	"		=	0·1015.
Copper,	"	"	"		=	0·0947,	"	"	"		=	0·1013.
Platinum,	"	"	"		=	0·0335,	"	"	"		=	0·0355.
Glass,	"	"	"		=	0·1770,	"	"	"		=	0·190.

REMARK.—The relation between the specific heat and the atomic weight which was originally established by Dulong and Petit, and has been recently confirmed by Regnault, is very remarkable. The product of the numbers, by which we express the specific heat and the atomic weight, is for all bodies nearly constant, and varies only from 38 to 42.

Thus is, for instance,	the specific heat,	and the atomic weight,	therefore, the product of both,
For Iron.....	= 0·11379	= 339·21	= 38·597
" Silver.....	= 0·05701	= 675·80	= 38·527
" Platinum.....	= 0·03243	= 1233·5	= 39·993
" Sulphur.....	= 0·20259	= 201·17	= 40·754

§ **375.**—The specific heat of *gases* is determined by the water calorimeter, through which the gases, whose temperature and expansive force have been accurately determined, are allowed to flow. We thus observe the increase of temperature of the accurately weighed cooling water due to the cooling of the gas, or the experiment is continued until the cooling water has assumed a constant temperature, so that just as much heat passes out as is conveyed to the water by the gas, and observe the excess of temperature of the water above that of surrounding objects. If equal volumes of gas flow through the calorimeter in the same time, then the specific heats of the different gases can be placed proportional to the observed differences of temperature.

According to the determinations of Regnault, the values of the specific heat of gases are as follows:

NAMES OF GASES AND VAPORS.	SPECIFIC HEAT.		DENSITY.
	By Weight.	By Volume.	
Atmospheric Air.....	0·2375	0·2375	1·0000
Oxygen.....	0·2175	0·2405	1·1056
Nitrogen.....	0·2440	0·2370	0·9713
Hydrogen.....	3·4090	0·2359	0·0692
Carbonic Acid (from 10° to 100°)....	0·2025	0·3096	1·5290
Carbonic Oxide.....	0·2470	0·2389	0·9673
Steam.....	0·4776	0·2966	0·6210

We must also distinguish with gases and vapors the specific heat at constant pressure from the specific heat at

constant volume. This is necessary by reason of the heating and cooling which bodies undergo when they are compressed or extended. This change of temperature is especially noticeable with gases, because these are found of very different densities.

If a quantity of air under constant pressure assumes a greater volume, by reason of a slight rise of temperature $\tau°$, and then if by compression it is brought back to its original volume, it undergoes a second slight increase of temperature of $\tau_1°$, without any more heat having been added. Therefore for *constant volume* the same mass shows the temperature $\tau + \tau_1$, while under *constant pressure* it expands and shows only the temperature τ. The specific heat ω for constant pressure, therefore, is greater than the specific heat ω_1 for constant volume, and we have

$$\omega\,\tau = \omega_1\,(\tau + \tau_1),$$

hence

$$\kappa = \frac{\omega}{\omega_1} = \frac{\tau + \tau_1}{\tau}$$

is the ratio of the specific heat for constant pressure to the specific heat for constant volume.

§ **376.**—If the temperature of a mass of air of the density γ, under a constant pressure p, increases to t, then the density becomes γ_1, and is given by the equation,

$$\gamma_1 = \frac{\gamma}{1 + \delta t}$$

If this mass of air is brought back to its original volume by an increase of pressure, then by this compression there is heat developed or an increase of temperature,

$$t_1 = \psi\left(1 - \frac{\gamma_1}{\gamma}\right) = \frac{\psi\,\delta\,t}{1 + \delta\,t},$$

assuming that, on a sudden change of the density γ_1 into γ, the temperature increases proportionally to the change of

density. Assuming this, the total increase of temperature is finally

$$t + t_1 = t + \frac{\psi\,\delta\,t}{1 + \delta\,t} = t\left(1 + \frac{\psi\,\delta}{1 + \delta\,t}\right),$$

and therefore the proportion of the heat at constant pressure to that at constant volume is

$$\kappa = 1 + \frac{\delta\,\psi}{1 + \delta\,t}.$$

The pressure of air of the density γ and temperature t (see Vol. I., § 393) can be put,

$$p = \mu\,\gamma\,(1 + \delta\,t),$$

where μ is an empirical constant.

Differentiating this expression with regard to p, γ, and t, we obtain,

$$dp = \mu\,(d\gamma + \delta t\,.\,d\gamma + \gamma\delta\,.\,dt),$$

or, as we can put,

$$t = \psi\left(1 - \frac{\gamma_1}{\gamma}\right)$$

and

$$dt = \psi\,\frac{d\gamma}{\gamma},$$

$$dp = \mu\,(1 + \delta\,t + \delta\,\psi)\,d\gamma.$$

Dividing by $p = \mu\,\gamma\,(1 + \delta\,t)$, we have the differential equation,

$$\frac{dp}{p} = \left(1 + \frac{\delta\,\psi}{1 + \delta\,t}\right)\frac{d\gamma}{\gamma} = \kappa\,\frac{d\gamma}{\gamma}.$$

As

$$\int\frac{dp}{p} = Ln\,.\,p$$

and

$$\int \frac{d\gamma}{\gamma} = Ln \,.\, \gamma$$

(see Vol. I., § 22, Introduction to the Calculus), we have

$$Ln \,.\, p = \kappa \, Ln \,.\, \gamma + \text{Const.},$$

as also

$$Ln \,.\, p_1 = \kappa \, Ln \,.\, \gamma_1 + \text{Const.},$$

and therefore

$$Ln \,.\, p_1 - Ln \,.\, p = \kappa \, (Ln \,.\, \gamma_1 - Ln \,.\, \gamma),$$

or

$$Ln \,.\, \left(\frac{p_1}{p}\right) = \kappa \, Ln \left(\frac{\gamma_1}{\gamma}\right),$$

Ln denoting the natural logarithm. Hence

$$\frac{p_1}{p} = \left(\frac{\gamma_1}{\gamma}\right)^{\kappa},$$

and

$$\frac{1 + \delta t_1}{1 + \delta t} = \frac{p_1}{p} \cdot \frac{\gamma}{\gamma_1} = \left(\frac{\gamma_1}{\gamma}\right)^{\kappa - 1} = \left(\frac{p_1}{p}\right)^{\frac{\kappa - 1}{\kappa}},$$

as we shall assume in the following investigations.

The formula

$$\frac{p_1}{p} = \left(\frac{\gamma_1}{\gamma}\right)^{\kappa}$$

expresses the so-called law of Poisson.

§ **377.**—The value of the ratio $\kappa = \frac{\omega}{\omega_1}$ can be ascertained by the following experiment: A vessel is filled with rarefied air, and then, by means of a cock, the orifice of discharge is opened for a short time, whereby the external air will naturally penetrate into the vessel, and the air previously contained by the vessel is condensed. At the same time, by means of a manometer which is connected with the

air reservoir, not only the manometric height ($-h$) of the enclosed air previous to the opening of the cock is observed, but also the manometric height ($-h_1$) directly after closure, as well as the manometric height ($-h_2$) after the complete cooling of the condensed air. If, now, b is the height of barometer outside, t the temperature of the air before and after the experiment, and t_1 the temperature of the air directly after ingress, we have, according to the previous article,

$$\frac{1+\delta t_1}{1+\delta t} = \left(\frac{\gamma_1}{\gamma}\right)^{\kappa-1} = \left(\frac{p_1}{p}\right)^{\frac{\kappa-1}{\kappa}} = \left(\frac{b-h_1}{b-h}\right)^{\frac{\kappa-1}{\kappa}},$$

and as the volume and consequently the density of the air does not change during the cooling of the condensed air,

$$\frac{b-h_1}{1+\delta t_1} = \frac{b-h_2}{1+\delta t} \quad \text{or} \quad \frac{1+\delta t_1}{1+\delta t} = \frac{b-h_1}{b-h_2},$$

so that by the elimination of $\frac{1+\delta t_1}{1+\delta t}$, we have

$$\left(\frac{b-h_1}{b-h}\right)^{\frac{\kappa-1}{\kappa}} = \frac{b-h_1}{b-h_2},$$

or

$$\frac{\kappa-1}{\kappa} \cdot Log.\left(\frac{b-h_1}{b-h}\right) = Log.\left(\frac{b-h_1}{b-h_2}\right),$$

and consequently the ratio of the specific heat of the air at equal pressure to that of equal volume,

$$\kappa = \frac{\omega}{\omega_1} = \frac{Log.(b-h_1) - Log.(b-h)}{Log.(b-h_2) - Log.(b-h)}.$$

If the differences $h-h_1$ and $h-h_2$ of the manometer are small, then we can put

$$Log.\left(\frac{b-h_1}{b-h}\right) = Log.\left(1+\frac{h-h_1}{b-h}\right) = \frac{h-h_1}{b-h},$$

and

$$Log.\left(\frac{b-h_2}{b-h}\right) = Log.\left(1+\frac{h-h_2}{b-h}\right) = \frac{h-h_2}{b-h}.$$

Hence we obtain for κ the simpler expression,

$$\kappa = \frac{\omega}{\omega_1} = \frac{h-h_1}{h-h_2}.$$

Clemens and Désormes in this way have found,

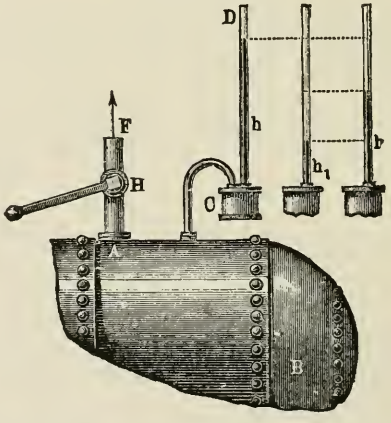
$$\kappa = \frac{\omega}{\omega_1} = 1{\cdot}348,$$

and Gay-Lussac,

$$= 1{\cdot}375.$$

The author has pursued an opposite course in order to determine κ. He first filled a steam boiler $A\,B$, Fig. 604,

Fig. 604.



with compressed air, then by means of a cock H opened the orifice F for a few moments, whereby a discharge of air, as also a rarefication and cooling of that retained in the boiler, took place. If, now, h is the original height of the manometer $C\,D$ h_1 that shortly after closing the orifice F, and h_2

that after heating up to its original temperature, observed about ten minutes later, then the desired proportion can be calculated by the formula,

$$\frac{\omega}{\omega_1} = \frac{Log.\ (b + h) - Log.\ (b + h_1)}{Log.\ (b + h) - Log.\ (b + h_2)}.$$

In such an experiment, the height of the barometer was

$$b = 0{\cdot}7342 \text{ meters},$$

and the manometer heights were observed to be,

$$h = 0{\cdot}7180,$$
$$h_1 = 0{\cdot}5890,$$
$$h_2 = 0{\cdot}6250 \text{ meters}.$$

Hence

$$\frac{\omega}{\omega_1} = \frac{Log.\ 1{\cdot}4522 - Log.\ 1{\cdot}3232}{Log.\ 1{\cdot}4522 - Log.\ 1{\cdot}3592} = \frac{0{\cdot}16203 - 0{\cdot}12162}{0{\cdot}16203 - 0{\cdot}13328}$$

$$= \frac{4041}{2875} = 1{\cdot}405.$$

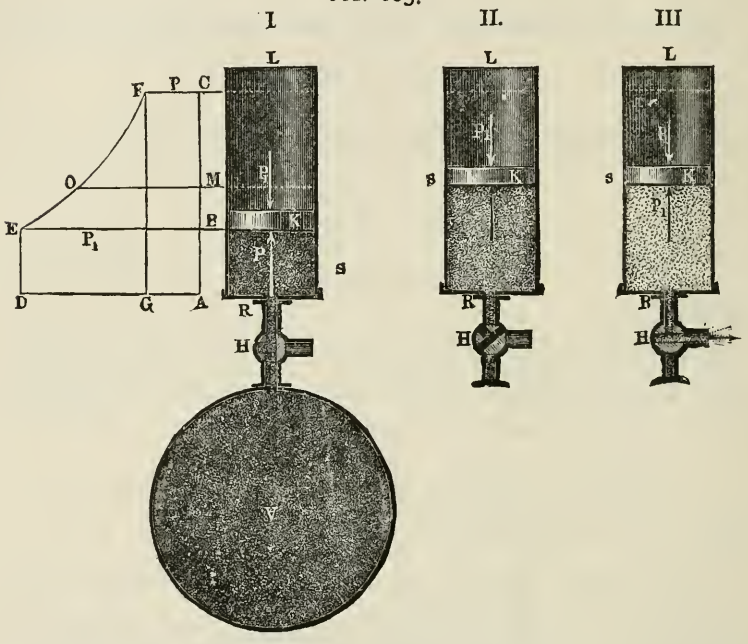
(See "Der Civilingenieur," Band 5, 1859.)

According to Masson, $\frac{\omega}{\omega_1} = 1{\cdot}419$. According to the experiments of Moll and van Beck upon the velocity of sound, $\kappa = 1{\cdot}41$. Since, during the very small time of efflux in the experiment above, a small quantity of heat is lost, we shall assume $\kappa = 1{\cdot}41$.

§ **378.—Work of Heat.**—If the piston K, Fig. 605, I., II., and III., the area of which may be taken as one square unit, moves within the cylinder $L\ R$, and is pressed upon by the air coming from the large reservoir V with the force p, represented by the straight line $A\ D = B\ E$, and has passed over a certain distance $\overline{A\ B} = s$, then the mechanical work performed is $p\ s$, and is graphically represented by the rectangle $A\ B\ E\ D$.

If the communication between the cylinder $L\,R$ and the reservoir V is closed by turning the cock H, as is represented in Fig. II., then the force p is no longer constant, but becomes less and less the more the air expands and presses the piston K forward. If, now, the temperature of

Fig. 605.



the air remains constant during this motion of the piston, then, according to the law of Mariotte, the pressure upon the piston would decrease, and at the end of a certain distance $\overline{A\,M} = x$, this pressure represented by $\overline{M\,O}$ would be

$$y = \frac{s\,p}{x}.$$

As the air, like all other bodies, absorbs heat while it expands, and consequently loses sensible heat, the above state of things could only take place if the enclosed air is supplied from without with just as much heat as it absorbs during its expansion.

If, then, we suppose that such a communication of heat from without does not take place, we cannot put

$$y = \frac{s\,p}{x},$$

but must put the steam pressure

$$y' = \frac{1+\delta\,t}{1+\delta\,t_1} \cdot \frac{s\,p}{x}$$

(see Vol. I., § 392), where t_1 indicates the original temperature corresponding to the pressure p_1, and t the varying temperature pertaining to the pressure y. On account of the cooling during the expansion, we have also

$$\frac{1+\delta\,t}{1+\delta\,t_1} = \left(\frac{s}{x}\right)^{\kappa-1},$$

where κ indicates the known ratio $\frac{\omega}{\omega_1}$. Therefore,

$$y = \left(\frac{s}{x}\right)^{\kappa} \cdot p = \left(\frac{1+\delta\,t}{1+\delta\,t_1}\right)^{\frac{\kappa}{\kappa-1}} \cdot p.$$

If, now, the piston κ passes through the very small distance dx, it performs the work

$$y\,dx = \left(\frac{s}{x}\right)^{\kappa} \cdot p\,dx,$$

and consequently the work which during the passage of the distance $\overline{BC} = s_1 - s$ is performed by the confined air, is represented by the area $BEFC$, and is equal to

$$\int_s^{s_1} y\,dx = s^{\kappa}\,p \int_s^{s_1} x^{-\kappa}\,dx = s\;p\left(\frac{s_1^{-\kappa+1}}{-\kappa+1} - \frac{s^{-\kappa+1}}{-\kappa+1}\right)$$

$$= \frac{s^{\kappa}\,p}{\kappa-1}\left(\frac{1}{s^{\kappa-1}} - \frac{1}{s_1^{\kappa-1}}\right) = \frac{p\,s}{\kappa-1}\left[1 - \left(\frac{s}{s_1}\right)^{\kappa-1}\right].$$

If during the entire motion of the piston through the distance $\overline{AC} = s_1$, the exterior air opposes it with the

force p_1, then the work $p_1 s_1$ represented by the rectangle $A C F G$ is lost, and the resulting mechanical work of the confined air or the work transferred to the piston is graphically represented by the area

$$G D E F = A B E D + B C F E - A C F G,$$

or the work performed is

$$L = p s + \frac{p s}{\kappa - 1}\left[1 - \left(\frac{s}{s_1}\right)^{\kappa - 1}\right] - p_1 s_1.$$

Presupposing that the tension of the enclosed air at the end of the stroke of the piston $\overline{A C} = s_1$ has become equal to the tension p of the exterior air, and that consequently the total capacity for work of the enclosed air has been transferred to the piston, we have

$$p_1 = \left(\frac{s}{s_1}\right)^{\kappa} p,$$

and, therefore,

$$L = \frac{\kappa}{\kappa - 1}\left[1 - \left(\frac{s}{s_1}\right)^{\kappa - 1}\right] p s.$$

By placing the cock H in the position shown in Fig. 605, III., so that the communication of the cylinder with the exterior air is established, and then pushing the piston slowly back to the bottom of the cylinder, work is hereby neither lost nor gained, because the air coming out of H exerts the same pressure p upon the one side of the piston, as the exterior air does upon the other side of the same.

Denoting the initial distance passed through by the piston $A B = s =$ unity, we obtain accordingly the mechanical work performed by one unit of volume, as, for instance, by one cubic meter of compressed air during expansion from s to s_1,

$$L = \frac{\kappa}{\kappa - 1}\left[1 - \left(\frac{s}{s_1}\right)^{\kappa - 1}\right] p$$

$$= \frac{\kappa}{\kappa - 1}\left[1 - \left(\frac{p_1}{p}\right)^{\frac{\kappa - 1}{\kappa}}\right] p,$$

and therefore the work which is performed by the expansion of a volume of air V of the tension p is,

$$\text{I. } L = \frac{\kappa}{\kappa - 1}\left[1 - \left(\frac{p_1}{p}\right)^{\frac{\kappa-1}{\kappa}}\right] V p,$$

or

$$= \frac{\kappa}{\kappa - 1}\left[1 - \left(\frac{s}{s_1}\right)^{\kappa-1}\right] V p.$$

If, on the contrary, the quantity of air V_1 is by the pressure p_1 compressed to p, then the work expended will be,

$$\text{II. } L = \frac{\kappa}{\kappa - 1}\left[\left(\frac{p}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right] V_1 p_1$$

$$= \frac{\kappa}{\kappa - 1}\left[\left(\frac{s_1}{s}\right)^{\kappa-1} - 1\right] V_1 p_1.$$

§ **379.—Mechanical Equivalent of Heat.**—The expressions I. and II. found in the preceding paragraph give the quantity of work which a certain mass of air performs when, in expanding, it passes from a greater to a lesser pressure, and which it requires in order to force it to pass from a lesser tension to a greater one. But as every change of density and tension is connected with a certain change of temperature, we can express the quantity of work by the temperatures of the air before and after performance of the work, and thus find a still more simple formula. We have then to put in the formulæ mentioned above instead of

$$\frac{p_1}{p}, \left(\frac{1 + \delta t_1}{1 + \delta t}\right)^{\frac{\kappa}{\kappa-1}},$$

and obtain consequently

$$\left(\frac{p_1}{p}\right)^{\frac{\kappa-1}{\kappa}} = \frac{1 + \delta t_1}{1 + \delta t},$$

and therefore for the mechanical work which is performed

by the cooling of the mass of air V from the temperature t_1 to t,

$$L = \frac{\kappa}{\kappa - 1}\left(1 - \frac{1 + \delta t}{1 + \delta t_1}\right) V p$$

$$= \frac{\kappa}{\kappa - 1} \cdot \frac{\delta (t_1 - t)\, V p}{1 + \delta t_1}.$$

But the density or the weight of a cubic meter of atmospheric air is

$$\gamma_1 = \frac{1 \cdot 2514\, p_1}{1 + \delta t_1},$$

if p indicates the pressure per square centimeter (see Vol. I., § 393). Therefore, we have here, when for p_1 the pressure upon the square meter is inserted,

$$L = \frac{\kappa}{\kappa - 1}\, \delta (t_1 - t) \cdot \frac{10000}{1 \cdot 2514} \cdot V \gamma.$$

Introducing $\delta = 0 \cdot 003665$ and $\kappa = 1 \cdot 410$, we obtain

$$L = 100 \cdot 72\, (t_1 - t)\, V_1\, \gamma_1.$$

Finally, the expenditure of heat of the quantity of air V, corresponding to the change of temperature from t_1 to t, is

$$W = \omega (t_1 - t)\, V \gamma = 0 \cdot 2375\, (t_1 - t)\, V \gamma;$$

therefore the corresponding quantity of work is

$$L = \frac{100 \cdot 72}{\omega} W = \frac{100 \cdot 72}{0 \cdot 2375} W = 424 \cdot 1\, . W \text{ meter kilogrammes.}$$

The work L which the air performs in cooling stands, therefore, in a fixed relation

$$A = \frac{L}{W} = 424 \cdot 1$$

to the lost quantity of heat W.

We call this relation A the *mechanical equivalent of heat*, and the same relates not only to the quantity of work gained by the cooling of the air, but also to the quantity of heat produced by the performance of work upon the air.

Understanding by the unit of heat that quantity of heat which is necessary to raise the temperature of one pound of water one degree Centigrade, we have, since one meter = 3·28 English feet,

$$L = 424\cdot1 \times 3\cdot28\ W = 1390\ W \text{ ft. lbs.},$$

and, therefore, the mechanical equivalent of the heat is = 1390 ft. lbs. (If we take Fahrenheit's scale, we have $1390 \times \frac{5}{9} = 772$ ft. lbs. for the mechanical equivalent of one heat-unit.)

While thus, by the quantity of heat which raises the temperature of one kilogramme of water one degree Centigrade, a work of 424·1 meter kilogrammes can be performed, a work of 1390 ft. lbs. can be performed by the quantity of heat which raises the temperature of one pound of water one degree Centigrade, and of 772 ft. lbs. by that which raises one pound of water one degree Fahrenheit.

Several physicists have endeavored to prove, that the above value $A = 424\cdot1$ of the mechanical equivalent of heat, is applicable not only to the development of heat by the expansion and compression of atmospheric air, but also to all other methods of producing heat, such as friction, concussion, electro-magnetism, etc., and for any other liquid or solid body. Joule has by various experiments proved that this relation of mechanical work to heat is nearly one and the same for different bodies and for different methods of heat generation. For this purpose, he placed a horizontal paddle-wheel in a vessel filled with water, and, by means of mechanism similar to that shown in Fig. 264, caused this wheel to revolve, and observed the increase of temperature of the water after the wheel had made a certain number of revolutions, and had performed a corresponding mechanical work measured by the fall of the weights which imparted motion. The ratio of this work

to the product of the weight of the water multiplied by the increase of temperature of the same gave the desired equivalent of work A of the heat. In this manner, Joule found as a mean, if the temperature is expressed in Fahrenheit degrees,

$$A = 773 \cdot 64 \text{ English ft. lbs.}$$

From which we have, for the Centigrade scale,

$$A = 425 \text{ meter kilogrammes} = 1394 \text{ ft. lbs.}$$

From the friction of an iron paddle-wheel in mercury, he found, in the same manner,

$$A = 776 \cdot 3 \text{ ft. lbs.} = 426 \text{ meter kilogrammes.}$$

Joule found also, by the friction of two cast-iron plates upon each other, that a work of $774 \cdot 88$ ft. lbs. $= 425$ meter kilogrammes, is necessary for each unit of heat generated. A somewhat greater value for A—viz., 460 meter kilogrammes—was found by him, by comparing the expenditure of work necessary for the revolution of an electro-magnetic rotation apparatus, with the quantity of heat liberated. Hirn found in his friction experiments (mentioned in Vol. I., § 173) the mechanical equivalent A between 315 and 425 meter kilogrammes. As a mean, by indirect friction with application of oils, he found

$$A = 365 \text{ meter kilogrammes.}$$

On the other hand, Person found for air

$$A = 424 \text{ meter kilogrammes.}$$

(See "Comptes Rendus de l'Académie des Sciences," Tome 39, Paris, 1854.)

REMARK.—The earliest acceptation and determination of the mechanical equivalent of heat is due to the German physicist Mayer. (See "Annalen der Chemie und Pharmacie," Band 42, 1842.) He found by the agitation of water $A = 365$ meter kilogrammes. In the development of this theory of work, based upon this constant relation or equivalence of heat and work, Clap-

peyron, Clausius, Helmholz, Hoppe, Kirchhoff, Rankine, Thomsom, Zeuner, and others, have been engaged. See also the later volumes of Poggendorff's "Physik und Chemie," as also the later "Philosophical Magazines." See also Zeuner's "Grundzüge der mechanischen Wärmetheorie," Leipzig, 1866, and the "Abhandlungen über die mechanische Wärmetheorie" by Clausius, Braunschweig, 1864.

§ **380.—Latent Heat.**—During the conversion of a solid body into the liquid condition, as also during the conversion of a liquid into steam, a certain quantity of heat is absorbed; and, on the contrary, during the solidification of a fluid body, as also during the condensation of steam or vapor, a certain quantity of heat is liberated. There is, therefore, in liquids, more heat contained than is indicated by the senses or by the thermometer, and this heat, which is therefore called the *latent heat*, is to be considered as the cause of the fluid condition of the body.

Different bodies absorb in this manner different quantities of heat, and the same body contains, while in the condition of vapor, more latent heat than when in the fluid condition, and in the latter more than when it is solid. If 1 lb. of water at 79° is mixed with 1 lb. of ice at 0°, the result will be 2 lbs. of water at 0°.

We can, therefore, conclude that the 1 lb. of ice at 0° required, for its conversion into water of 0°, 79 degrees or heat-units. Furthermore, by condensing 1 lb. of steam at 100° by $5\frac{1}{2}$ lbs. of water at 0° heat, the result will be $6\frac{1}{2}$ lbs. of water at 100° or $6{\cdot}5 \times 100$ heat-units; but only 100° of these being sensible, the latent heat of steam at 100° temperature is, therefore, 550° or heat-units (or $550 \times \frac{9}{5} = 990°$, according to Fahrenheit's scale).

The latest experiments of Provostaye and Desains, as also those of Regnault (see "Annal. de chimie et de physique," Sect. III., Tome VIII.), give the latent heat of water $= 79{\cdot}0$; the results as to the latent heat of metals are very uncertain. Hassenfratz gives for mercury $= 86\frac{2}{3}$; Irvine for lead $= 90$; Rudberg, on the contrary, $5{\cdot}838$, and so on. The absorption of heat during the conversion of a solid body into a liquid is practically applied in the use of

freezing mixtures. Thus, for instance, 1 part of common salt mixed with 5 parts of snow at 0° gives a fluid saline solution of $-17{\cdot}7°$ C., or the zero point of the scale of Fahrenheit (see § 351). A mixture of 3 parts of chloride of calcium and 2 parts of snow changes its temperature from 0° to $-28°$. Recent more accurate experiments upon latent heat of vapors have been made by Brix. (See Poggendorff's "Annalen," Band LV., 1842.) According to these, the latent heat is

For steam	540,
" alcohol vapor	219,
" oil of turpentine vapor	74.

Despretz had previously found values which deviate but slightly from the above.

Comparing the latent heat of different vapors with their densities, we find that they are almost inversely proportional to the latter. While, for instance, the density of the vapor of alcohol is $2{\cdot}58$ times as great as that of steam, we have the latent heat of the former only $\frac{219}{540} = \frac{1}{2{\cdot}47}$ of that of steam. Therefore we can assume that equal volumes of all vapors contain at the boiling point nearly the same amount of latent heat.

According to the latest experiments of Regnault, the *total heat* of steam at t degrees temperature is

$$W = 606{\cdot}5 + 0{\cdot}305\ t°.$$

Therefore, the specific heat of water is not perfectly constant, but is given by the formula

$$\omega = 1 + 0{\cdot}00004\ t + 0{\cdot}0000009\ t^2.$$

We have the "heat of the liquid" (Flüssigkeitswärme) or the *sensible heat* (*i.e.*, the heat required to raise the temperature before evaporation from a certain fixed temperature up to that at which evaporation begins) of the steam for the temperature t,

$$n = \text{mean of } \omega \text{ times } t$$

$$= \left(\frac{\omega_1 + \omega_2 + \ldots + \omega_n}{n}\right) t$$

$$= (1 + 0{\cdot}00004 \text{ mean of } t + 0{\cdot}0000009 \text{ mean of } t^2)\, t$$

$$= \left(1 + 0{\cdot}00004\,\frac{t}{2} + 0{\cdot}0000009\,\frac{t^2}{3}\right) t$$

$$= t + 0{\cdot}00002\, t^2 + 0{\cdot}0000003\, t^3,$$

and, finally, for the latent heat, or heat of evaporation,

$$w = W - n$$

$$= 606{\cdot}5 + 0{\cdot}305\, t - (1 + 0{\cdot}00002\, t + 0{\cdot}0000003\, t^2)\, t$$

$$= 606{\cdot}5 - 0{\cdot}695\, t - 0{\cdot}00002\, t^2 - 0{\cdot}0000003\, t^3,$$

from which follows:

Temperature t of Water.	Total Heat of Steam.	Sensible Heat of Steam.	Latent Heat.
0°	606·5°	0°	606·5°
25	614·1	25·0	589·0
50	621·7	50·1	571·7
75	629·4	57·2	554·7
100	637·0	100·5	536·5
125	644·6	125·8	518·6
150	652·2	151·5	500·8
175	659·9	177·2	482·7
200	667·5	203·2	464·3
225	675·1	229·4	445·5

For other vapors, we have other values of W, ω, and w_1. Thus, for ether,

$$W = 94{\cdot}00 + 0{\cdot}45000\, t - 0{\cdot}0005555\, t^2$$

$$\omega\, t = 0{\cdot}52901\, t + 0{\cdot}0002959\, t^2$$

and

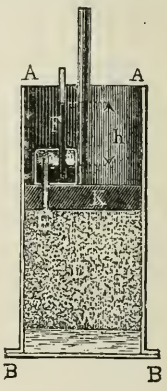
$$w_1 = W - \omega\, t = 94{\cdot}00 - 0{\cdot}07901\, t - 0{\cdot}0007514\, t^2.$$

CHAPTER II.

STEAM.

§ **381.—Steam.**—If we create a vacuum over a liquid—for instance, over a mass of water *W*, Fig. 606, by drawing upwards the piston *K*, which fits closely the vessel *A B*, and at first touches the surface of *W*, a portion of the liquid will be changed into steam, and, indeed, the more readily, the more empty space is furnished or the further the piston *K* is drawn up. If the quantity of water is not very great, we can, by the enlargement of the space *K W*, or by further withdrawal of the piston *K*, convert it wholly into vapor. If the temperature does not change during this process, the force of expansion of this vapor, indicated by the height *h* of a manometer, remains unchanged, though there may have been offered to the steam a greater or a lesser space for its development. But if, after the complete transformation of the water into steam, we pull the piston *K* further upwards, then the manometer will sink, and the force of expansion will become less. This decrease of the force of expansion is wholly in accordance with the law of Mariotte (see Vol. I., § 387)—*i. e.*, from that point where all the water is converted into steam, the force of expansion is directly proportional to the density of the steam, and consequently inversely as the volume. If from this point, for instance, we double the volume of steam by a further withdrawal

Fig. 606.



of the piston, then the mercurial column h supported by the steam will only be half as great as at first. By gradually lessening the space filled by the steam, by depressing the piston, the manometer will rise up to the height which it had at the moment all the water was converted into steam. From this point on, the manometer will remain, during the further depression of the piston, at the same height, and a portion of the steam changes again into water, and, indeed, more so the less space there remains, until finally, when the piston has again taken its original position, all the vapor has again passed into water.

§ 382.—**Maximum Tension of Steam.**—If we perform the operation described in the preceding paragraph at a higher or lower temperature of the liquid and its surroundings, then the phenomena indeed remain the same, only the height of the manometer, and hence also the force of expansion of the vapor, will be greater or less, and, on the other hand, the distance which the piston must be raised before the water has entirely passed into steam will be less or greater than in the former case. If, furthermore, for a constant position of the piston, such that there still remains water to evaporate, we heat the water, then still more water will pass into steam, denser steam will therefore be formed, and there will be a greater force of expansion. By a further elevation of temperature, the whole quantity of water can thus be converted into vapor, and if, when this has occurred, we continue to add heat, then, it is true, the force of expansion of the steam will be further increased, but there is no further increase of density connected with it, and the law of increase of tension is also a different one—viz., that of Gay-Lussac (see Vol. I., § 392). If, now, we gradually diminish the temperature, the opposite conditions take place; first, the force of expansion of the vapor decreases according to the law of Gay-Lussac, and then, upon the attainment of a certain temperature, there occurs a condensation of the vapor into water, and so

more and more vapor changes into water the more the temperature is lowered, and the density and force of expansion of the steam become less and less. This decrease of temperature may even run below 0° without the whole of the steam disappearing, for even at —20° the manometer still indicates a measurable force of expansion.

From this, we see that the condition of the steam, so long as it is still in contact with water, is very different from that when it fills a limited space by itself. In the first case, its density and force of expansion are dependent only on the temperature, but in the latter case, on the other hand, the density, force of expansion, and temperature of the vapor are, as expressed by the laws of Gay-Lussac and Mariotte, dependent on each other. If for the production of steam there is no lack of water, then steam of a certain density and expansive force is generated at every temperature, and as it is not possible to condense this or increase its tension by further diminution of volume, we can say that in this state it possesses its maximum density and tension. Commonly, such vapor is called *saturated steam* (Fr. vapeur saturée; Ger. gesättigter Dampf). The non-saturated steam is also called *superheated steam* (Fr. vapeur surchauffée; Ger. überhitzter Dampf).

§ 383.—**Experiments upon the Tension of Vapors.**— The important question is now to be considered, in what connection do the force of expansion and the temperature of steam at its maximum tension stand to each other? Experiments for the purpose of determining these relations have already been made in great number, especially by the German investigators, Schmidt, Arzberger, Kämtz, etc.; by the English, Watt, Robison, Dalton, Ure, etc., and by the French, Arago and Dulong, Regnault, etc.; but the extent and exactitude of these experiments are all very different, and the results do not yield the desired conformity. This is not the place to describe the different

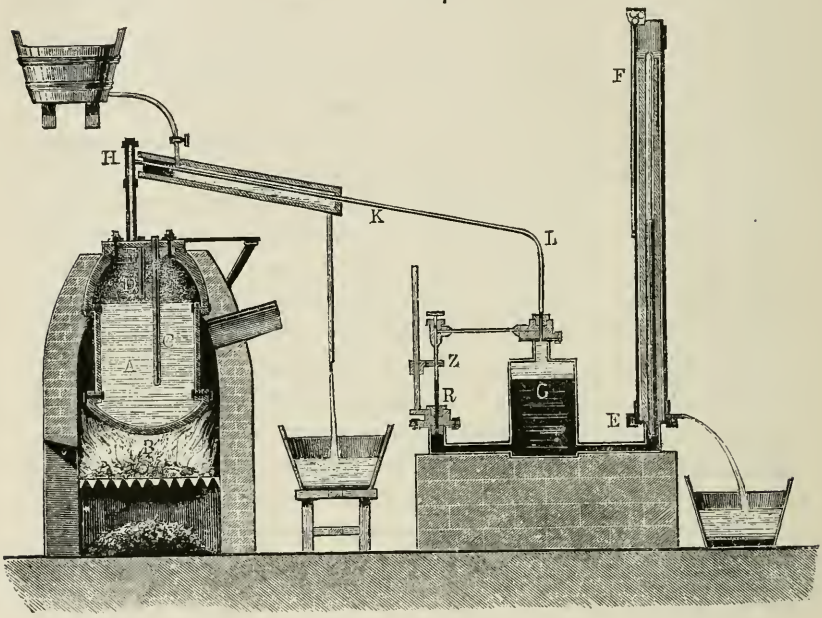
apparatus which has been employed in experiments upon the force of expansion of steam, and it is only possible for us to make the following general remarks. It is required essentially only to gradually heat the steam more and more, and to measure its temperature and its corresponding force of expansion. For the ascertainment of the temperature, thermometers are used; but these must not be brought into direct contact with the steam, but must be cased in iron tubes, so that the tube may not be crushed by the vapor. To find the force of expansion, a mercurial column is generally employed, which forms, as it were, a very long barometer, or also an air manometer may be used (see Vol. I., § 386). Arzberger as well as Southern made use of the latter instruments. But as appears by comparing the results of experiments, these give, as might be expected, somewhat too small values for the force of expansion. Very detailed experiments have been instituted by the Franklin Institute of Philadelphia and the Academy of Sciences at Paris. The latter are the most extensive, and, in point of accuracy, are possibly only surpassed by the latest experiments of Magnus and Regnault. The experiments which the first-named Institute has made, extend, as do also those of Arzberger, as high as 10 atmospheres, while those of the last-named Academy run up to 24 atmospheres. Moreover, at tensions of from 2 to 10 atmospheres, the former give greater forces of expansion than the latter, and at 10 atmospheres the deviations amount to $\frac{1}{9}$ of an atmosphere.

REMARK.—A condensed presentation of the experiments upon the force of expansion of steam is found in the "Mechanics' Pocket Dictionary," by W. Grier—article, Steam; also in the second volume of Robison's "System of Mechanical Philosophy," in P. Barlow's "Treatise on the Manufactures and Machinery of Great Britain," and in Tredgold's "Dampfmaschinenlehre."

§ 384.—Experiments of the Paris Academy.—On account of the importance of the subject, we give a cut (Fig. 607) and a short description of the apparatus which the French academicians, Arago, Dulong, etc., have em-

ployed for determining the force of expansion of steam. The generation of the steam took place in a strong sheet-

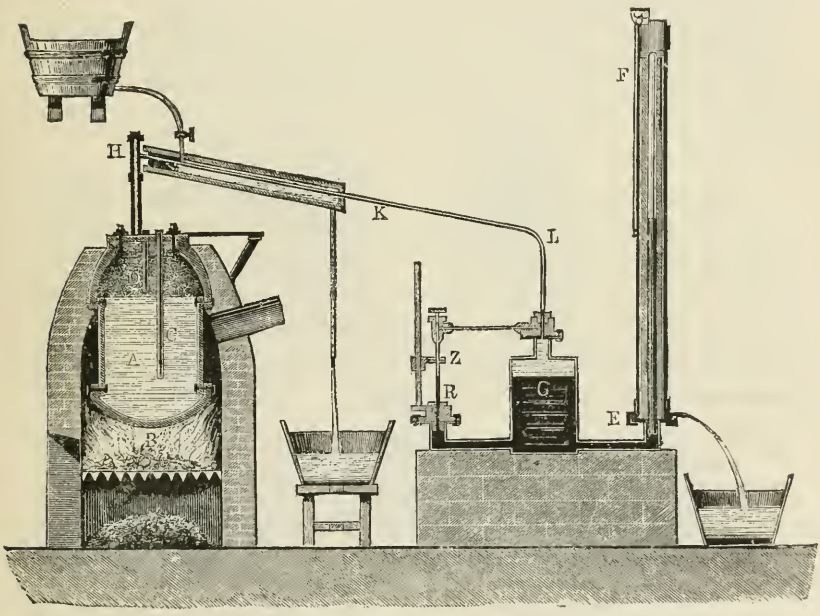
FIG. 607.



iron boiler *A* of 80 liters capacity, which for this purpose was set into the stove *B*. Into this boiler, two gun-barrels *C D* protruded, of which one reached under the water, but the other only into the steam space. In both, mercurial thermometers were placed, which were bent at the top and carried forward horizontally, and at this point were retained at a constant temperature by a stream of water. To measure the force of expansion of the steam, the air manometer *E F* was used, which was surrounded by a column of water constantly renewed so as to maintain a constant temperature. The iron vessel *G* of this manometer was nearly filled with mercury, and the upper portion, as well as the communicating tube *K L*, was filled with water, and to attain an invariable temperature, this tube was externally washed by water. The glass tube *R* and the

pointer *Z* served to determine the height of the mercury in the vessel *G*. The experiments were conducted in the following manner: At first the tube *H* and safety-valve being open, the water was allowed to boil for fifteen or twenty minutes in order to expel all the air from *A*; then both were closed, and by the addition of fuel a higher temperature was produced. As soon as the thermometer and manometer attained their maximum, one observer read

FIG. 608.



the one and another observer the other. In this manner, thirty observations were made at from 123° to 224·15° of temperature, or 2·14 to 23·994 atmospheres.

As the application of the air manometer is based upon the law of Mariotte, the French academicians considered it necessary to have other experiments testing the law of Mariotte for very high tensions, precede the experiment just described. For this purpose, they made use of the same apparatus, only upon the side at *R* they ap-

plied a vertical glass or barometric tube open above, and consisting of thirteen pieces, having a length of 26 meters and width of 5 millimeters, and at *L* they applied a force-pump. By means of this, a pressure was produced which was conveyed by the water to the mercury in *G*, and drove that into the manometer *E F*, as also at *R* into the barometer. By comparison of the height of the remaining column of air with the height of the column of mercury in the long tube, the correctness of the law of Mariotte could be proven.

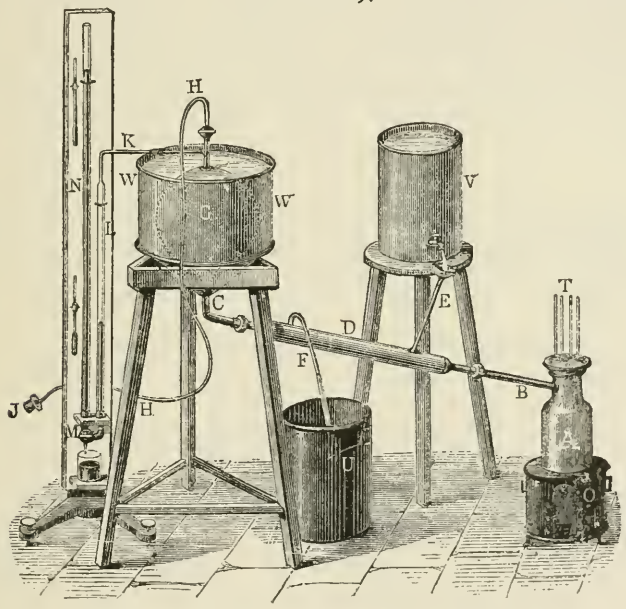
REMARK.—For a detailed discussion of these experiments, see the "Exposé des recherches faites par ordre de l'Académie Royal des Sciences pour déterminer les forces élastiques de la vapeur d'eau à hautes températures, Paris, chez Firmin Didot, 1830." See also Poggendorff's "Annalen," Vol. XVIII.

§ 385.—**The Experiments of Regnault.**—As the difference of the expansion of different kinds of glass, and consequently the influence of the same upon the mercurial thermometer, were not known at the time when Dulong and Arago instituted the experiments described in the preceding paragraph, Regnault considered it necessary to institute new experiments upon the force of expansion of steam.

The method of procedure described in what follows, can be applied as well to the determination of steam above 100° as to steam below 100°. The apparatus used for this purpose has the following arrangement (see Fig. 609): The hermetically sealed copper vessel *A* is about $\frac{1}{3}$ filled with water, and also contains four thermometers *T*, of which two are sunk to just beneath and two to just above the surface of the enclosed water. From this vessel a tube *B C* leads to a glass globe *G* of twenty-four liters capacity. This glass globe is connected with an air pump by a lead tube *H H I*, so that the air enclosed in it can be rarefied or condensed at will, and another tube *K* leads from this globe to an open manometer *L M N* (see Vol. I., § 386), which indicates the expansive force of the air in *G*. Moreover, the globe *G* is placed in a water bath *W W* in

order to maintain a constant temperature, and the tube *B C* is also surrounded by a cover *D*, in which water of a constant temperature circulates. This water is brought to the cover by the tube *E* from a vessel *V*, and discharged by means of the tube *F* into the vessel *U*. Now, if we heat the vessel *A* by means of the stove *O*, then a portion of the enclosed water is converted into steam, and the force of expansion of the latter is held in equilibrium by

FIG. 609.



the pressure of the air in *G* and *B C*. Lastly, we observe the height of the manometer *L M N* as soon as it has become constant, as also the height of the thermometers *T*. Now, by means of the air pump, we give a higher pressure to the air in *G*, and also apply to the vessel a greater heat, and again observe the manometer, as well as the corresponding temperature of the steam; and, continuing in this manner, we obtain a series of manometer readings and the corresponding temperatures. (See "Mémoires de l'Institut de France," Tome 21, 1847, and Tome 26, 1862.)

The experimental apparatus with which Regnault determined the force of expansion of steam beneath the boiling point is somewhat more simple. Here a globe of glass is filled with distilled water, brought into an exhausted glass globe which communicates above, and on one side by an elbow with an air pump, as well as on the other side with a barometer tube, and is surrounded by a tin vessel filled with water and furnished with a transparent glass slide. A thermometer immersed in the water indicates its temperature. The steam experimented upon is obtained from the water in the glass globe by exploding it by heating the apparatus. The apparatus which Magnus has employed for the same purpose is somewhat peculiar.

§ **386.**—The results of the experiments of Arago, Dulong, etc., upon the force of expansion of steam are contained in the following table:

Number of the Observation.	Temperature (C) according to the		Elasticity of the Steam.	
	Long Thermometer.	Short Thermometer.	Measured by the Height of a Mercurial Column.	Expressed in Atmospheres.
	Degrees.	Degrees.	Meters.	Atmospheres.
1	122·97	123·70	1·6292	2·14
2	132·58	132·82	2·1767	2·87
3	132·64	133·30	2·1816	2·88
4	137·70	138·30	2·5386	3·35
5	149·54	149·70	3·4759	4·58
6	151·87	151·90	3·6868	4·86
7	153·64	153·75	3·8810	5·12
8	163·00	163·40	4·9384	6·51
9	168·40	168·50	5·6054	7·39
10	169·57	169·40	5·7737	7·61
11	171·88	172·34	6·1510	8·11
12	180·71	180·70	7·5001	9·89
13	183·70	183·70	8·0352	10·60
14	186·80	187·10	8·6995	11·48
15	188·30	188·50	8·8400	11·66
16	193·70	193·70	9·9989	13·19
17	198·55	198·50	11·0190	14·53
18	202·00	201·75	11·8620	15·67
19	203·40	204·17	12·2903	16·21
20	206·17	206·10	12·9872	17·13

Number of the Observation.	Temperature according to the		Elasticity of the Steam.	
	Long Thermometer.	Short Thermometer.	Measured by the Height of a Mercurial Column.	Expressed in Atmospheres.
	Degrees.	Degrees.	Meters.	Atmospheres.
21	206·40	206·80	13·0610	17·23
22	207·00	207·40	13·1276	17·30
23	208·45	208·90	13·6843	18·05
24	209·10	209·13	13·7690	18·16
25	210·47	210·50	14·0634	18·55
26	215·07	215·30	15·4995	20·44
27	217·23	217·50	16·1528	21·31
28	218·30	218·40	16·3816	21·60
29	220·40	220·80	17·1826	21·66
30	223·88	224·15	18·1894	23·99

The following table gives the results of the experiments of Regnault upon the tensions of steam at from 1 to 4 atmospheres:

Number of the Observation.	Temperature		Force of Expansion.	
	Of the Water.	Of the Steam.	In Meters.	In Atmospheres.
	In Centigrade Degrees.			
1	99·83	99·82	0·75161	0·99
2	100·00	100·00	0·76000	1·00
3	100·71	100·71	0·77603	1·02
4	105·10	105·06	0·90460	1·19
5	111·78	111·70	1·13147	1·49
6	116·04	116·04	1·30237	1·71
7	121·16	121·13	1·53027	2·01
8	122·70	122·53	1·60125	2·11
9	123·94	123·91	1·67041	2·20
10	128·40	128·47	1·91512	2·52
11	128·54	128·47	1·92520	2·53
12	128·66	128·57	1·93114	2·54
13	130·12	130·18	2·01251	2·65
14	131·38	131·30	2·09469	2·75
15	131·51	131·63	2·09828	2·76
16	133·20	133·28	2·20908	2·91
17	135·70	135·65	2·37303	3·04
18	135·83	136·00	2·38681	3·14
19	137·75	137·52	2·51479	3·31
20	138·86	138·24	2·56173	3·37
21	140·90	141·01	2·75617	3·63
22	141·57	141·54	2·79968	3·68
23	143·85	143·83	2·99279	3·94
24	144·12	144·17	3·01008	3·96
25	145·70	145·64	3·14941	4·14
26	147·50	147·50	3·30695	4·35
27	148·20	148·30	3·36135	4·42

Comparing the very nearly corresponding values of both tables, we find a very satisfactory agreement. For instance, the first table gives for the mean temperature of 138° the tension of the steam 3·35 atmospheres, but the second for the mean temperature of 138·3° gives 3·37 atmospheres. We see from these tables that the readings of both thermometers, of which the one stood in the water and the other in the vapor, vary very little from each other.

REMARK.—Regnault has also made a series of experiments upon the elasticity of steam from 32° to 100° of temperature. By Magnus also a number of experiments upon the force of expansion of steam from — 20° to + 10° have been instituted. (See Poggendorff's "Annalen," Vol. 61.) In the memoirs already cited, Vol. 26, § 383, Regnault discusses his experiments upon the force of expansion of different vapors.

§ **387.—Formulæ for Elasticity.**—The relation between the temperature and the force of expansion of steam has not yet been embodied in a law, and we have therefore to content ourselves with empiric formulæ which more or less closely conform to experimental results. The method which is used for finding such formulæ consists in plotting upon paper the observed temperatures and the corresponding tensions as co-ordinates, so that the corresponding points are fixed, and then ascertaining which of the known curves, or which of the curves corresponding to known functions, most accurately conform to the curved line thus obtained. After having decided upon a certain curve, we have only to determine the constants for the same from the results of the experiments, and here the method of least squares,* treated of in the "Ingenieur," page 76, etc., finds special application. Up to the present time, more than forty-five such formulæ have been established. (See "Die Fortschritte der Physik im Jahre 1845," Jahrgang I., Berlin, 1847.)

For practical use, the most convenient formula is that first introduced by Young,

* See also "Elements of the Method of Least Squares," by Mansfield Merriman, Macmillan & Co., 1877.

$$p = (a + b\,t)^n,$$

in which t represents the temperature, p the corresponding force of expansion, and a, b, and n empirical numbers. But this does not satisfactorily agree with the results of experiments for all temperatures, and therefore in its application we have been compelled to determine special values for the constants for low, medium, and high temperatures.

For high temperatures, and particularly for tensions of more than 4 atmospheres, we have, according to Dulong and Arago,

$$p = (0{\cdot}2847 + 0{\cdot}007153\,t)^5 \text{ atmospheres},$$

and inversely,

$$t = 139{\cdot}8\,\sqrt[5]{p} - 39{\cdot}80°.$$

Expressing the force of expansion by the pressure per square inch, and taking the English system of weights and measures, we have, since, according to Vol. I., § 385, the pressure of one atmosphere is $= 14{\cdot}7$,

$$p = (0{\cdot}2847 + 0{\cdot}007153\,t)^5 \times 14{\cdot}7 = (0{\cdot}4873 + 0{\cdot}012244\,t)^5 \text{ lbs.},$$

and inversely,

$$t = 81{\cdot}66\,\sqrt[5]{p} - 39{\cdot}80.$$

For tensions from 1 to 4 atmospheres, Mellet, the French translator of Tredgold's "Theory of the Steam Engine," gives

$$p = \left(\frac{75 + t}{174}\right)^6 \text{ kilogrammes}$$

per square centimeter, and hence, since the pressure of one atmosphere per square centimeter $= 1{\cdot}0336$ kilogrammes,

$$p = \left(\frac{75 + t}{174}\right)^6 \times \frac{1}{1{\cdot}0336} = \left(\frac{75 + t}{175}\right)^6 \text{ atmospheres}$$

$$= \left(\frac{75 + t}{174}\right)^6 \times \frac{14{\cdot}7}{1{\cdot}0336} = \left(\frac{75 + t}{174}\right)^6 \times 14{\cdot}22 = \left(\frac{75 + t}{111{\cdot}78}\right)^6 \text{ lbs.}$$

to the square inch. Inversely, if p is given in atmospheres,

$$t = 175\sqrt[6]{p} - 75°,$$

and if p is given in pounds,

$$t = 111{\cdot}78\sqrt[6]{p} - 75°.$$

Pambour (see his "Théorie des machines à vapeur") takes for tensions of from 1 to 4 atmospheres,

$$p = \left(\frac{72{\cdot}67 + t}{171{\cdot}72}\right)^6 \text{ kilogrammes.}$$

Therefore inversely,

$$t = 171{\cdot}72\sqrt[6]{p} - 72{\cdot}67°.$$

Hence, if p is expressed in atmospheres,

$$p = \left(\frac{72{\cdot}67 + t}{172{\cdot}67}\right)^6 \text{ atmospheres}$$

and

$$t = 172{\cdot}67\sqrt[6]{p} - 72{\cdot}67°;$$

or, for English measures,

$$p = \left(\frac{72{\cdot}67 + t}{110{\cdot}93}\right)^6 \text{ lbs.,}$$

and

$$t = 110{\cdot}93\sqrt[6]{p} - 72{\cdot}67°.$$

The English Artisan Club, in its theory of the steam engine, gives the following formula:

For temperatures of over 100°,

$$p = \left(\frac{85 + t}{185}\right)^{6{\cdot}42} \text{ atmospheres;}$$

hence

$$t = 185\sqrt[6{\cdot}42]{p} - 85° = 185p^{0{\cdot}15576} - 85°;$$

for temperatures below 100°,

$$p = \left(\frac{115 + t}{215}\right)^{7.71507} \text{ atmospheres,}$$

and

$$t = 215 \sqrt[7.71507]{p} - 115^\circ = 215\, p^{0.12962} - 115^\circ.$$

Example 1.—What is the tension of saturated steam at 145° temperature? The formula of Mellet gives,

$$p = \left(\frac{75 + 145}{175}\right)^6 = \left(\frac{44}{35}\right)^6 = 3{\cdot}947 \text{ atmospheres,}$$

and Pambour's formula,

$$p = \left(\frac{72{\cdot}67 + 145}{172{\cdot}67}\right)^6 = \left(\frac{217{\cdot}67}{172{\cdot}67}\right)^6 = 4{\cdot}013 \text{ atmospheres.}$$

The formula of the academicians,

$$p = (0{\cdot}2847 + 145 \times 0{\cdot}007153)^5 = 1{\cdot}3219^5 = 4{\cdot}036 \text{ atmospheres;}$$

and, finally, that of the Artisan Club,

$$p = \left(\frac{85 + 145}{185}\right)^{6{\cdot}42} = \left(\frac{46}{37}\right)^{6{\cdot}42} = 4{\cdot}046 \text{ atmospheres.}$$

The mean of all these four values is 4·01 atmospheres.

(2) What is the pressure of steam at a temperature of 145° against a piston 3 ft. in diameter? The area of the surface of the piston is

$$F = \frac{9\,\pi}{4} \text{ sq. ft.} = 9 \times 36\,\pi = 1017{\cdot}9 \text{ sq. inches,}$$

and pressure on every square inch at 4 atmospheres is

$$p = 4 \times 14{\cdot}7 = 58{\cdot}8 \text{ lbs.;}$$

hence the pressure on the whole surface is

$$P = Fp = 1017{\cdot}9 \times 58{\cdot}8 = 59852 \text{ lbs.}$$

(3) What temperature corresponds to a tension of $\frac{1}{4}$ of an atmosphere? According to the second formula of the Artisan Club,

$$t = 215 \times (\tfrac{1}{4})^{0.12962} - 115 = 179{\cdot}64 - 115 = 64{\cdot}64^\circ.$$

§ **388.—More accurate Formulæ for Elasticity.**—Exponential or logarithmic formulæ coincide more closely with experiment than algebraic expressions. A rather simple exponential formula for the expansive force of steam was first given by Roche. (See Poggendorff's "Annalen," Vols. 18 and 27.) It has the form,

$$p = a\, b^{\frac{t}{m+nt}}.$$

Even if, as Regnault shows, this formula does not express the general law of the force of expansion of steam, it still gives, according to the calculations of Magnus, August, etc., quite a good result within the limits of observations and for temperatures of ordinary occurrence. According to the later calculations of Magnus,

$$p = 4{\cdot}525 \times 10^{\frac{7{\cdot}4475\, t}{234{\cdot}69 + t}},$$

and according to those of Holtzmann,

$$p = 4{\cdot}529 \times 10^{\frac{7{\cdot}2804\, t}{236{\cdot}22 + t}} \text{ millimeters.}$$

If we take the first formula, we obtain, since one atmosphere corresponds to a height of 760 millimeters,

$$p = \frac{4{\cdot}525}{760} \times 10^{\frac{7{\cdot}4475\, t}{234{\cdot}69 + t}} = 0{\cdot}005954 \times 10^{\frac{7{\cdot}4475\, t}{234{\cdot}69 + t}} \text{ atmospheres,}$$

or

$$log.\, p = 0{\cdot}77481 - 3 + \frac{7{\cdot}4475\, t}{234{\cdot}69 + t} = \frac{5{\cdot}2223\,(t - 100)}{234{\cdot}69 + t}.$$

Inversely,

$$t = \frac{234{\cdot}69\ log.\, p + 522{\cdot}23}{522{\cdot}23 - log.\, p}.$$

The following formula of August is also quite accurate:

$$p = \left(\frac{6415\,(1028\cdot4 + t)}{1000000000}\right)^{\frac{100-t}{100+\frac{3}{8}t}} \text{ atmospheres.}$$

Finally, Regnault has used, in his experiments upon the expansive force of steam, the following formulæ:

(1) For steam from $-32°$ to $0°$:

$$\log. p = a + b\,\alpha^t \text{ millimeters,}$$
where $a = -0\cdot08038$
$\log. b = 0\cdot6024724 - 1$
$\log. \alpha = 0\cdot0333980$
and $t = 32° + t_1$,

where t_1 is the temperature (negative) of the water as given by the air thermometer.

(2) For steam from $0°$ to $100°$:

$$\log. p = a + b\,\alpha^t - c\,\beta^t \text{ millimeters,}$$
where $a = 4\cdot7393707$
$\log. b = 0\cdot1340339 - 2$
$\log. c = 0\cdot6116485$
$\log. \alpha = 0\cdot006865036$
$\log. \beta = 0\cdot9967249 - 1$,

and t expresses the temperature above zero.

(3) For steam from $-20°$ to $220°$:

$$\log. p = a - b\,\alpha^t - c\,\beta^t,$$
where $a = 6\cdot2640348$
$\log. b = 0\cdot1397743$
$\log. c = 0\cdot6924351$
$\log. \alpha = 0\cdot9940493 - 1$
$\log. \beta = 0\cdot9983438 - 1$,
and $t = 20° + t_1$,

where t_1 indicates the temperature above $0°$ (the freezing point).

(4) The formula,

$$log.\, p = a - b\, \alpha^t$$

is quite accurate if we put

$$a = 5 \cdot 4233177$$
$$b = -\, 5 \cdot 4642763$$
$$\text{and} \quad log.\, \alpha = 0 \cdot 9972311 - 1.$$

Introducing $t = 20^\circ + t_1$ in the formula (3),

$$log.\, p = a - b\, \alpha^t - c\, \beta^t,$$

we have

$$log.\, (b\, \alpha^t) = log.\, b + 20\, log.\, \alpha + t_1\, log.\, \alpha$$
$$= 0 \cdot 0207601 - 0 \cdot 00595071\, t_1,$$

and

$$log.\, (c\, \beta^t) = log.\, c + 20\, log.\, \beta + t_1\, log.\, \beta$$
$$= 0 \cdot 659312 = 0 \cdot 00165614\, t_1.$$

Similar formulæ have, moreover, been already established by Prony and Biot.

The following two tables have been calculated from the above. The first of these tables gives the tension for temperatures expressed in degrees, while the second table shows the temperature corresponding to the tension expressed in atmospheres. In this, the pressure of an atmosphere is taken as equal to a column of mercury of the height of 76 centimeters (30 inches). According to the first table, for instance, we have for the temperature $t = 116^\circ$, the force of expansion $= 131 \cdot 147$ centimeters $= 1 \cdot 726$ atmospheres, and according to the second table a temperature of $152 \cdot 2^\circ$ corresponds to a tension of 5 atmospheres.

REMARK 1.—Dalton's assumption that the expansive force of steam increases in a geometrical ratio, while the temperature of the same increases in an arithmetical ratio, leads only to an approximate formula for the elasticity.

According to this, the force of expansion of steam in atmospheres is $p = a^{t-100°}$ where a indicates a constant to be determined by experiments. But, according to the results of experiments, we have for $t = 144°$ C. the expansive force $p = 4$ atmospheres, hence $4 = a^{44}$, and inversely

$$a = \sqrt[44]{4} = 1{\cdot}0320,$$

and therefore

$$p = (1{\cdot}0320)^{t-100} \text{ atmospheres,}$$

and

$$t = 100 + \frac{log.\ p}{log.\ 1{\cdot}0320},$$

i. e.,

$$t = 100 + 73{\cdot}10 + log.\ p \text{ in degrees Centigrade.}$$

According to this latter formula, we have, for instance,

for $p = 2$	atmospheres,	$t = 122{\cdot}0°$,
for $p = 3$	"	$t = 134{\cdot}9°$,
for $p = 4$	"	$t = 144{\cdot}0°$,
for $p = 5$	"	$t = 151{\cdot}1°$,
and for $p = 6$	"	$t = 156{\cdot}7°$,

while, according to experiment, we have, for $p = 2$, $t = 120°.6$; for $p = 3$, $t = 133°.9$; for $p = 4$, $t = 144°.0$; for $p = 5$, $t = 152°.2$; and for $p = 6$, $t = 159°.2$.

We see from this comparison that for moderate tensions of from 1 to 5 atmospheres, the simple formula $p = (1{\cdot}032)^{t-100}$ atmospheres accords tolerably well with experiment.

Remark 2.—The density also of steam (§ 389) can be determined *à priori* with tolerable accuracy. If, under equal pressure, from 1 volume of oxygen and 2 volumes of hydrogen, 2 volumes of steam are produced eudiometrically, and the density of the oxygen at 0°, and one atmosphere is 1·4298 kilogrammes, but, on the other hand, that of hydrogen is 0·0896 kilogrammes, then the density or the weight of a cubic meter of steam can be put

$$\frac{1{\cdot}6099}{2} = 0{\cdot}8045 \text{ kilogrammes.}$$

The weight of a cubic meter of atmospheric air is for equal temperature and pressure 1·2935 kilogrammes, consequently the specific gravity of steam with reference to atmospheric air is

$$\varepsilon = \frac{0{\cdot}8045}{1{\cdot}2935} = 0{\cdot}622,$$

or nearly $\frac{5}{8}$, which coincides very well with the experiments of Gay-Lussac and others.

Table I.

EXPANSIVE FORCE OF STEAM FOR TEMPERATURES FROM — 32° UP TO + 230° C. ACCORDING TO REGNAULT.

Temperature.	TENSION OF STEAM.		Temperature.	TENSION OF STEAM.	
	In Centimeters.	In Atmospheres.		In Centimeters.	In Atmospheres.
— 32°	0·0320	0·0004	+ 8°	0·8017	0·0107
31	0·0352	0·0005	9	0·8574	0·011
30	0·0386	0·0005	10	0·9165	0·012
29	0·0424	0·0006	11	0·9792	0·013
28	0·0464	0·0006	12	1·0457	0·014
27	0·0508	0·0007	13	1·1162	0·015
26	0·0555	0·0007	14	1·1908	0·016
25	0·0605	0·0008	15	1·2699	0·017
24	0·0660	0·0009	16	1·3536	0·018
23	0·0719	0·0009	17	1·4421	0·019
22	0·0783	0·0010	18	1·5357	0·020
21	0·0853	0·0011	19	1·6346	0·022
20	0·0927	0·0012	20	1·7391	0·023
19	0·1008	0·0013	21	1·8495	0·024
18	0·1095	0·0014	22	1·9659	0·026
17	0·1189	0·0015	23	2·0888	0·028
16	0·1290	0·0017	24	2·2184	0·029
15	0·1400	0·0018	25	2·3550	0·031
14	0·1518	0·0020	26	2·4988	0·033
13	0·1646	0·0022	27	2·5505	0·034
12	0·1783	0·0024	28	2·8101	0·037
11	0·1933	0·0025	29	2·9782	0·039
10	0·2093	0·0027	30	3·1548	0·042
9	0·2267	0·0030	31	3·3406	0·044
8	0·2455	0·0032	32	3·5359	0·047
7	0·2658	0·0035	33	3·7411	0·049
6	0·2876	0·0038	34	3·9565	0·052
5	0·3113	0·0041	35	4·1827	0·055
4	0·3368	0·0044	36	4·4201	0·058
3	0·3644	0·0048	37	4·6691	0·061
2	0·3941	0·0052	38	4·9302	0·065
1	0·4263	0·0056	39	5·2039	0·068
0	0·4600	0·0061	40	5·4906	0·072
+ 1	0·4940	0·0065	41	5·7910	0·076
2	0·5302	0·0070	42	6·1055	0·080
3	0·5687	0·0073	43	6·4346	0·085
4	0·6097	0·0080	44	6·7790	0·089
5	0·6534	0·0086	45	7·1391	0·094
6	0·6998	0·0092	46	7·5158	0·099
7	0·7492	0·0199	47	7·9093	0·104

Temperature.	TENSION OF STEAM.		Temperature.	TENSION OF STEAM.	
	In Centimeters.	In Atmospheres.		In Centimeters.	In Atmospheres.
+ 48°	8·3204	0·109	+ 98°	70·7280	0·931
49	8·7499	0·115	99	73·3305	0·965
50	9·1982	0·121	100	76·000	1·000
51	9·6661	0·127	101	78·7590	1·036
52	10·1543	0·134	102	81·6010	1·074
53	10·6636	0·140	103	84·5280	1·112
54	11·1945	0·147	104	87·5410	1·152
55	11·7478	0·155	105	90·6410	1·193
56	12·3244	0·163	106	93·8310	1·235
57	12·9251	0·170	107	97·1140	1·278
58	13·5505	0·178	108	100·4910	1·322
59	14·2015	0·187	109	103·965	1·368
60	14·8791	0·196	110	107·537	1·415
61	15·5839	0·205	111	111·209	1·463
62	16·3170	0·215	112	114·983	1·513
63	17·0791	0·225	113	118·861	1·564
64	17·8714	0·235	114	122·847	1·616
65	18·6945	0·246	115	126·941	1·670
66	19·5496	0·257	116	131·147	1·726
67	20·4376	0·267	117	135·466	1·782
68	21·3596	0·281	118	139·902	1·841
69	22·3165	0·294	119	144·455	1·901
70	23·3093	0·306	120	149·128	1·962
71	24·3393	0·320	121	153·925	2·025
72	25·4073	0·334	122	158·847	2·091
73	26·5147	0·349	123	163·896	2·157
74	27·6624	0·364	124	169·076	2·225
75	28·8517	0·380	125	174·388	2·295
76	30·0838	0·396	126	179·835	2·366
77	31·3600	0·414	127	185·420	2·430
78	32·6811	0·430	128	191·147	2·515
79	34·0488	0·448	129	197·015	2·592
80	35·4643	0·466	130	203·028	2·671
81	36·9287	0·486	131	209·194	2·753
82	38·4435	0·506	132	215·503	2·836
83	40·0101	0·526	133	221·969	2·921
84	41·6298	0·548	134	228·592	3·008
85	43·3041	0·570	135	235·373	3·097
86	45·0344	0·593	136	242·316	3·188
87	46·8221	0·616	137	249·423	3·282
88	48·6687	0·640	138	256·700	3·378
89	50·5759	0·665	139	264·144	3·476
90	52·5450	0·691	140	271·763	3·576
91	54·5778	0·719	141	279·557	3·678
92	56·6757	0·746	142	287·530	3·783
93	58·8406	0·774	143	295·686	3·890
94	61·0740	0·804	144	304·026	4·000
95	63·3778	0·834	145	312·555	4·113
96	65·7535	0·865	146	321·274	4·227
97	68·2029	0·897	147	330·187	4·344

Temperature.	TENSION OF STEAM.		Temperature.	TENSION OF STEAM.	
	In Centimeters.	In Atmospheres.		In Centimeters.	In Atmospheres.
+ 148°	339·298	4·464	+ 190°	944·270	12·425
149	348·609	4·587	191	965·093	12·699
150	358·123	4·712	192	986·271	12·977
151	367·843	4·840	193	1007·804	13·261
152	377·774	4·971	194	1029·701	13·549
153	387·918	5·104	195	1051·963	13·842
154	398·277	5·240	196	1074·595	14·139
155	408·856	5·380	197	1097·500	14·441
156	419·659	5·522	198	1120·982	14·749
157	430·688	5·667	199	1144·746	15·062
158	441·945	5·815	200	1168·896	15·380
159	453·436	5·965	201	1193·437	15·703
160	465·162	6·120	202	1218·369	16·031
161	477·128	6·278	203	1243·700	16·364
162	489·336	6·439	204	1269·430	16·703
163	501·791	6·603	205	1295·566	17·047
164	514·497	6·770	206	1322·112	17·396
165	527·454	6·940	207	1349·075	17·751
166	540·669	7·114	208	1376·453	18·111
167	554·143	7·291	209	1404·252	18·477
168	567·882	7·472	210	1432·480	18·848
169	581·890	7·656	211	1461·132	19·226
170	596·166	7·844	212	1490·222	19·608
171	610·719	8·036	213	1519·748	19·997
172	625·548	8·231	214	1549·717	20·391
173	640·660	8·430	215	1580·133	20·791
174	656·055	8·632	216	1610·994	21·197
175	671·743	8·839	217	1642·315	21·690
176	687·722	9·049	218	1674·090	22·027
177	703·997	9·263	219	1706·329	22·452
178	720·572	9·481	220	1739·036	22·882
179	737·452	9·703	221	1772·213	23·319
180	754·639	9·929	222	1805·864	23·761
181	772·137	10·150	223	1839·994	24·210
182	789·952	10·394	224	1874·607	24·666
183	808·084	10·633	225	1909·704	25·128
184	826·540	10·876	226	1945·292	25·596
185	845·323	11·123	227	1981·376	26·071
186	864·435	11·374	228	2017·961	26·552
187	883·882	11·630	229	2055·048	27·040
188	903·668	11·885	230	2092·640	27·535
189	923·795	12·155			

Table II.

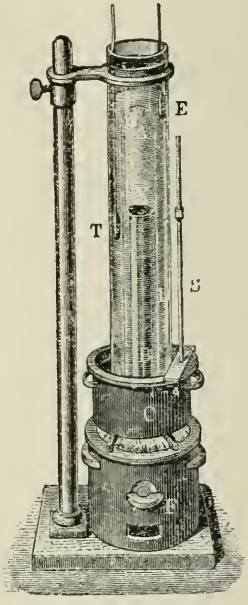
TEMPERATURE OF STEAM FOR TENSIONS FROM 1 ATMOSPHERE UP TO 28 ATMOSPHERES, ACCORDING TO REGNAULT.

EXPANSIVE FORCE.		Temperature in Degrees.	EXPANSIVE FORCE.		Temperature in Degrees.
In Atmosp'rs.	In Meters.		In Atmosp'rs.	In Meters.	
1	0·76	100·0	15	11·40	198·8
2	1·52	120·6	16	12·16	201·9
3	2·28	133·9	17	12·92	204·9
4	3·04	144·0	18	13·68	207·7
5	3·80	152·2	19	14·44	210·4
6	4·56	159·2	20	15·20	213·0
7	5·32	165·3	21	15·96	215·5
8	6·08	170·8	22	16·72	217·9
9	6·84	175·8	23	17·38	220·3
10	7·60	180·3	24	18·14	222·5
11	8·36	184·5	25	19·00	224·7
12	9·12	188·4	26	19·76	226·8
13	9·88	192·1	27	20·52	228·9
14	10·64	195·5	28	21·28	230·9

§ **389.—Density of Steam.**—The density of steam, as well as that of every kind of gas, depends equally upon the temperature and upon the force of expansion. (See Vol. I., § 392 and § 393.) But as the force of expansion of saturated steam is determined by the temperature, it follows that for this the density is dependent only upon the temperature or upon the force of expansion. But to be able to indicate the density of vapor at every temperature and force of expansion, it was necessary to ascertain the same by experiments at a fixed temperature and force of expansion; and with this object in view, Gay-Lussac pursued the following method: he filled a thin glass bulb with water and sealed its neck in the alcohol flame. By accurately weighing the empty and the filled bulb, the weight of the contained water was obtained.

Then this bulb was brought into the glass tube *A B*, Fig. 610, which was graduated and filled with mercury and stood in a vessel *C*, also filled with mercury, which could be heated by the furnace *F*. The tube *A B* was furthermore surrounded by a glass cylinder *D E*, and the interval between the two was filled with water. By applying heat below, the water in the globule *K* burst its case and became steam, and after all the water had been converted into steam by the maintenance of a constant temperature, the temperature was read off from a thermometer *T*, and the volume and the force of expansion of the steam from the graduated bar *S*.

Fig. 610.



In this way, Gay-Lussac found that one liter of steam at 100° and 0·76 meters height of barometer weighed $\frac{1}{1.6964} = 0.5895$ grammes. But, according to the same authority, the weight of one liter of atmospheric air under the same conditions is 0·9454 grammes, from which we have for the ratio of the density of steam to that of atmospheric air at equal tension and equal temperature,

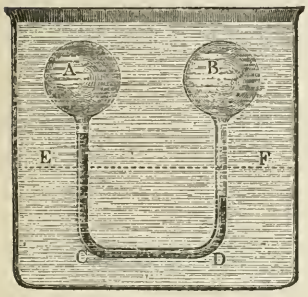
$$\frac{5895}{9454} = \frac{1000}{1603},$$

or quite closely five eighths.

Another method for the determination of the specific gravity of vapors has been employed by Dumas. Fairbairn and Tate have also instituted experiments upon the density of saturated and of superheated steam. (See "Useful Information for Engineers," by William Fairbairn. Second Series, London, 1860; also "Polytechn. Central-

blatt," Jahrgang 1860.) The apparatus employed consisted essentially of two communicating tubes *A C*, *B D*, Fig. 611, half filled with mercury, which terminated above in the bulbs *A* and *B*, which had previously to the experiment been deprived of air. If now unequal quantities of water were brought into these bulbs, then, as is well known, they at once became filled with saturated steam, the density of which could, by raising the temperature of the surrounding oil bath *E F*, be so increased that finally all the water in the one vessel would change into steam, and, with further heating, the latter would attain the superheated condition. The instant in which this occurs is indicated by the rising of the mercury in the one and the sinking of the mercury in the other leg of the communicating tubes. The difference in the level of the surface of both mercurial columns gives also the difference of pressure between the saturated steam in the one and the superheated steam in the other bulb. By a thermometer reaching into one bulb, the temperature, and, by a manometer communicating with the other bulb, the force of expansion of the saturated steam, were determined.

Fig. 611.



§ 390.—**Specific Volume of Steam.**—With the aid of the density ratio given in the preceding paragraph, the density of steam for every temperature and tension can be calculated according to the laws of Mariotte and Gay-Lussac, and the formula developed in Vol. I., § 393. According to that formula, we have the density of steam corresponding to the temperature t and tension p in French measure,

$$\gamma = \frac{\frac{5}{8} \times 1 \cdot 2935\, p}{1 + 0 \cdot 00367\, t} = \frac{0 \cdot 8084\, p}{1 + 0 \cdot 00367\, t} \text{ kilogrammes.}$$

For steam at its maximum tension, we can still express by means of one of the formulæ of paragraphs 387 and 388, the force of the tension p in terms of the temperature t, or, inversely, the temperature t in terms of the force of tension p, and can therefore directly determine γ from t or p. Using, for instance, the formula of Mellet-Tredgold, we can put

$$\gamma = \frac{0\cdot 8084}{1 + 0\cdot 00367\, t}\left(\frac{75 + t}{175}\right)^6,$$

or

$$\gamma = \frac{0\cdot 8084\, p}{1 + 0\cdot 00367\,(175\sqrt[6]{p} - 75)} \text{ kilogrammes.}$$

The density γ_0 of water is 1000 kilogrammes, hence the ratio of the densities of steam and water is

$$\frac{\gamma}{\gamma_0} = \frac{\gamma}{1000} = \frac{0\cdot 0008084\, p}{1 + 0\cdot 00367\,(175\sqrt[6]{p} - 75)},$$

and, inversely, the ratio between the volume of the vapor and that of the water for equal weights, or the so-called *specific volume* of the steam at its maximum tension, is

$$\mu = \frac{V}{V_0} = \frac{\gamma_0}{\gamma} = \frac{1 + 0\cdot 00367\,(175\sqrt[6]{p} - 75)}{0\cdot 0008084\, p}.$$

This ratio can, according to Navier, be approximately expressed by

$$\mu = \frac{\alpha}{\beta + p},$$

or in numbers, if p expresses the pressure of the steam in atmospheres,

$$\mu = \frac{1000}{0\cdot 09 + 0\cdot 50026\, p} = \frac{2000}{0\cdot 1800 + p}.$$

But, according to Pambour, this formula is sufficiently correct only for high temperatures, and gives too great results for tensions below one atmosphere. He takes therefore for steam at low pressures,

$$\mu = \frac{1935}{0\cdot1161 + p},$$

and for steam of high pressure,

$$\mu = \frac{2054}{0\cdot2922 + p},$$

and upon these bases his theory of the steam engine.

According to the experiments of Fairbairn and Tate, the specific volume of steam is

$$\mu = 25\cdot62 + \frac{1659\cdot2}{0\cdot02413 + p}.$$

REMARK.—Very simple expressions for the force of expansion and densities of steam are given by Watterson (see "Philosoph. Transactions," 1852; also Poggendorff's "Annalen," Ergänzungsband 4, 1853).

§ **391.**—By the mechanical theory of heat, the specific volume of steam can be calculated, according to Zeuner, by the formula,

$$\mu = 1 + \frac{13186400 + 40704\,t - 8\cdot48\,t^2 - 0\cdot1272\,t^3}{p},$$

where p indicates the pressure of the steam per square meter, or by the formula,

$$\mu = 1 + \frac{1275\cdot9 + 3\cdot9385\,t - 0\cdot00082051\,t^2 - 0\cdot000012308\,t^3}{p},$$

when p indicates the pressure of the steam in atmospheres at 10335 kilogrammes to the square meter (14·7 lbs. per square inch).

According to this formula, the values in the following table have been calculated:

TABLE OF THE SPECIFIC VOLUME OF STEAM AT FROM 0·1 TO 10·9 ATMOSPHERES.

	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
0	indeter.	14556	7542	5141	3917	3172	2671	2309	2037	1822
1	1650	1509	1390	1289	1202	1127	1060	1002	949	902
2	859·8	821·2	786·1	753·9	724·4	697·1	671·9	648·5	626·7	606·4
3	587·4	569·6	552·9	537·1	522·3	508·2	499·4	482·4	470·5	495·1
4	448·4	438·1	428·3	418·9	410·0	401·4	393·2	385·4	377·8	370·6
5	363·6	356·9	350·5	344·3	338·3	332·5	326·9	321·6	316·4	311·3
6	306·4	301·7	297·2	292·7	288·4	284·3	280·2	276·3	272·5	268·7
7	265·2	261·7	258·3	254·9	251·7	248·5	245·3	242·5	239·6	236·7
8	233·9	231·2	228·6	226·0	223·3	220·8	218·5	216·2	213·9	211·7
9	209·5	207·3	205·2	203·1	201·0	199·1	197·2	195·3	193·4	191·5
10	189·7	188·0	186·3	184·6	182·9	181·2	179·7	178·1	176·6	175·1

In this table the first vertical column indicates the whole atmospheres and the first horizontal row tenths of an atmosphere, and the number which is in the same horizontal row with the whole and in the same vertical column with the tenths gives the specific volume of steam corresponding to this tension. Accordingly, the specific volume of steam at 1·3 atmospheres is = 1289, because the latter number is found at the intersection of the horizontal line beginning with 1 and the vertical column under 0·3. Again, a cubic foot of water at 4·2 atmospheres gives 428·3 cubic feet of steam, because this number stands at the place where the horizontal line beginning with 4 and the vertical column beginning with 0·2 intersect each other.

From the foregoing, we see that the density of saturated steam increases as the temperature or the force of expansion increases, and even approaches nearer and nearer to that of water. According to the precise formula, the density of steam at the melting point of zinc would be $\frac{1}{4}$ that of water, and at the red heat of iron would equal that of water.

According to a more recent determination by Zeuner (see "Zeitschrift des Vereins deutscher Ingenieure," Vol. XI.), when p indicates the tension in atmospheres, and τ the absolute temperature beginning with $-273°$, then we may put with great accuracy for saturated and also for unsaturated steam,

$$p\,v' = 0{\cdot}0049287\,\tau - 0{\cdot}18781\,\sqrt[4]{p},$$

whence the specific volume of steam is

$$\mu = 1000\,v' = \frac{4{\cdot}9287\,\tau - 187{\cdot}81\,\sqrt[4]{p}}{p}.$$

Thus, for instance, for $p = 1$ atmosphere and $\tau = 373°$,

$$\mu = 4{\cdot}9287 \times 373 - 187{\cdot}81 = 1838{\cdot}4 - 187{\cdot}8 = 1650{\cdot}6,$$

while the table gives $\mu = 1650$. If with the same pressure, the temperature $\tau = 500$, and hence the steam is superheated, then

$$\mu = 2464{\cdot}35 - 187{\cdot}81 = 2276{\cdot}54.$$

Furthermore, for saturated steam at $p = 4$ atmospheres, and $\tau = 273 + t = 273 + 144 = 417°$ of absolute heat,

$$\mu = \frac{4{\cdot}9287 \times 417 - 187{\cdot}81\,\sqrt[4]{4}}{4} = 447{\cdot}4,$$

while the table gives $\mu = 448{\cdot}4$, and the formula of Fairbairn gives

$$\mu = 25{\cdot}62 + \frac{1659{\cdot}2}{4{\cdot}02413} = 437{\cdot}9.$$

If the temperature of steam at 4 atmospheres is raised to 200°, so that $\tau = 473°$, then

$$\mu = \frac{4{\cdot}9287 \times 473 - 265{\cdot}6}{4} = 516{\cdot}4,$$

while, according to the experiments of Hirn, it would be

$$\mu = 522.$$

EXAMPLE 1.—What quantity of water is necessary for the production of a quantity of steam Q of 500 cubic feet under the pressure of 3 atmospheres? According to the formula of Fairbairn,

$$\mu = 25{\cdot}62 + \frac{1659{\cdot}2}{3{\cdot}024127} = 574{\cdot}3,$$

hence the desired quantity of water

$$Q_1 = \frac{Q}{\mu} = \frac{500}{574{\cdot}3} = 0{\cdot}871 \text{ cubic ft.} = 0{\cdot}871 \times 62{\cdot}55 = 54{\cdot}48 \text{ lbs.}$$

According to the table, we have

$$Q_1 = \frac{500}{587{\cdot}4} = 0{\cdot}851 \text{ cubic ft.} = 53{\cdot}19 \text{ lbs.}$$

2. What quantity of water corresponds to 500 cubic ft. of steam under the pressure of 3 atmospheres and temperature of 150°? As a tension of 4·712 atmospheres corresponds to the latter temperature, this steam is unsaturated, and, therefore, the specific volume of the same is to be calculated according to the formula,

$$\mu = \frac{4{\cdot}9287\ \tau - 187{\cdot}81\sqrt[4]{p}}{p}.$$

Here $p = 3$ and $\tau = 273 + 150 = 423°$, hence

$$\mu = \frac{2085{\cdot}0 - 247{\cdot}2}{3} = 612{\cdot}6,$$

and the corresponding quantity of water,

$$Q = \frac{500}{612{\cdot}6} = 0{\cdot}816 \text{ cubic ft.} = 51{\cdot}0 \text{ lbs.}$$

[Zeuner, in his treatise, *De la Chaleur*, has called attention to the fact that when water is converted into steam by the action of heat, the heat that becomes *latent* during the process is composed of two parts—one, which is employed in changing the molecular condition of the body, the other being required to perform the work of expansion in the medium in which the steam is formed. The manner of calculating these separate portions of the latent heat of evaporation is shown in the appended table of formulas. This table gives, in a concise form, some of the most reliable formulas yet established for calculating the properties of saturated steam, and the tables that follow illustrate the application of the formulas. A few remarks in relation to

these tables and formulas may be appropriate. Regnault's experiments, already referred to, give relations between the temperature and pressure of saturated steam, between 25·6° and 446° Fahrenheit. These experiments also give the latent and total heat of evaporation for different temperatures, together with the laws by which these quantities vary. Professor Rankine published a single formula (contained in the accompanying table) in 1849, which expresses the relations between temperature and pressure, nearly in accordance with Regnault's experiments. This formula has been used for making the computations beyond the limits of the experiments. The indirect formula for calculating the relation between the volume and temperature of steam is given by Professor Rankine in his "Treatise on the Steam Engine," this formula expressing the results of Regnault's experiments on the quantity of heat in a given weight of steam. The results given by this formula show a close agreement with those obtained by the direct experiments of Sir William Fairbairn and Mr. T. Tate ("Philosophical Transactions," 1860, 1862). Professor Rankine remarks, in reference to tables computed by this formula: "It is certain that whatsoever may prove to be the law connecting the density, pressure, and temperature of steam under other circumstances, the densities and volumes in these tables cannot err, to an extent appreciable in practice, for steam obtained under circumstances similar to those of Regnault's experiments, which circumstances are, in all important points, similar to those under which steam is obtained in ordinary steam engines" ("Treatise on the Steam Engine," 1866, p. 231). The formulas for the weight and volume of distilled water at different temperatures are taken from Watt's "Dictionary of Chemistry," vols. iii., v., and vi., and are deduced from the experiments of Kopp, Matthiessen, Sorby, and Rosetti. These experiments have not been carried beyond a temperature of 412° Fahrenheit, so that the results given in the table for higher temperatures have not been verified. It is not probable, however, that they are greatly in error. But few explanations as to the use of the tables will be required, since the contents of the different columns are fully detailed in the tables themselves.]

FORMULAS RELATING TO THE PROPERTIES OF SATURATED STEAM.

QUANTITY.			SYMBOL.	FORMULA.
Pressure.	Above a Vacuum.	Pounds per square inch.	P	$P = \frac{p}{144}$, Log. $P = 6{\cdot}1007 \quad \frac{2731{\cdot}62}{t} - \frac{396944}{t^2}$
		Pounds per square foot.	p	$p = P \times 144$, Log. $p = 8{\cdot}2591 - \frac{2731{\cdot}62}{t} - \frac{396944}{t^2}$
		Inches of mercury, at 32° Fah.	M	$M = P \times 2{\cdot}03759$
		Feet of distilled water, at temperature of maximum density.	F	$F = P \times 2{\cdot}306768$
		Atmospheres.	A	$A = P \times 0{\cdot}0680967$
	Above the atmosphere, in pounds per square inch.		G	$G = P - 14{\cdot}685$
Temperature.	Fahrenheit's scale.		T	$T = t - 461°{\cdot}2$
	Absolute scale, Fahrenheit degrees.		t	$t = 1 + \left(\sqrt{\frac{8{\cdot}2591 - \text{Log. } p}{396944} + 0{\cdot}00001184} - 0{\cdot}003441\right)$
Quantity of heat.	Per pound of steam in British thermal units.	Required to raise the temperature of the water from 32° to T°.	S	$S = T - 32 + 0{\cdot}000000103\,(T - 39{\cdot}1)^3$
		Required to change the water into steam. (Internal latent heat.)	I	$I = L - E$
		Required to overcome the pressure of the surrounding medium. (External latent heat.)	E	$E = p \times \frac{C - v}{772}$
		Latent heat of evaporation, under constant pressure, P.	L	$L = 1091{\cdot}7 - 0{\cdot}695\,(T - 32) - 0{\cdot}000000103\,(T - 39{\cdot}1)^3$
		Total heat of evaporation above 32°.	H	$H = 1091{\cdot}7 + 0{\cdot}305\,(T - 32)$
	Total heat of evaporation per pound of steam, above 32°, in units of evaporation.		U	$U = \frac{H}{966{\cdot}1}$
Foot-pounds of energy, in latent heat of evaporation, per cubic foot of steam.			l	$l = 2{\cdot}3026 \times p \times \left(\frac{2731{\cdot}62}{t} + \frac{793888}{t^2}\right)$
Weight.	Of a cubic foot of steam, in pounds.		W	$W = \frac{l}{772 \times L}$
	Of a cubic foot of distilled water, in pounds, at temperature T.		w	$w = \frac{62{\cdot}425}{v}$
Volume.	Of a pound of steam, in cubic feet.		C	$C = \frac{1}{W}$
	Ratio of volume of steam to volume of equal weight of distilled water at temperature of maximum density.		V	$V = C \times 62{\cdot}425$
	Ratio of volume of distilled water, at temperature T, to volume of equal weight at temperature of maximum density.		v	For temperatures from 32° to 70°, $v = 1{\cdot}00012 - 0{\cdot}000033914\,(T - 32) + 0{\cdot}00000023822\,(T - 32)^2 - 0{\cdot}000000006403\,(T - 32)^3$ For temperatures above 70°, $v = 0{\cdot}99781 + 0{\cdot}00006117\,(T - 32) + 0{\cdot}00000001059\,(T - 32)^2$

Table of the Properties of Saturated Steam.

Pressure above a vacuum, in pounds per square inch.	Temperature, Fahrenheit degrees.	Quantities of Heat. In British thermal units. Required to raise the temperature of the water from 32° to T°.	Internal latent heat.	External latent heat.	Latent heat of evaporation at pressure P, = I + E.	Total heat of evaporation above 32° = S + L.	Total heat of evaporation above 32°, in units of evaporation.	Weight of a cubic foot of steam, in pounds.	Volume. Of a pound of steam in cubic feet.	Ratio of volume of steam to volume of equal weight of distilled water at temperature of maximum density.	Pressure above a vacuum, in pounds per square inch.
P.	T.	S.	I.	E.	L.	H.	U.	W.	C.	V.	P.
1	102·018	70·040	981·396	61·619	1043·015	1113·055	1·1522	·003027	330·4	20623	1
2	126·302	94·368	961·980	64·114	1026·094	1120·462	1·1599	·005818	171·9	10730	2
3	141 654	109·764	949·725	65·655	1015·380	1125·144	1·1647	·008522	117·3	7325	3
4	153·122	121·271	940·597	66·773	1007·370	1128·641	1·1683	·011172	89·51	5588	4
5	162·370	130·563	933·239	67·660	1000·899	1131·462	1·1712	·013781	72·56	4530	5
6	170·173	138·401	927·038	68·403	995·441	1133 842	1·1737	·016357	61·14	3816	6
7	176·945	145·213	921·654	69·041	990·695	1135·908	1·1758	·018908	52·89	3302	7
8	182·952	151·255	916·883	69·602	986·485	1137·740	1·1777	·021436	46·65	2912	8
9	188·357	156·699	912·584	70·106	982·690	1139·389	1·1794	·023944	41·77	2607	9
10	193·284	161·660	908·672	70·560	979·232	1140·892	1·1810	·026437	37·83	2361	10
11	197·814	166 225	905·083	70·967	976·050	1142·275	1·1824	·028911	34·59	2159	11
12	202·012	170·457	901·766	71·332	973·098	1143·555	1·1837	·031376	31·87	1990	12
13	205·929	174·402	898·683	71·663	970·346	1144·748	1·1849	·033828	29·56	1845	13
14	209·604	178·112	895·784	71·973	967·757	1145·869	1·1861	·036265	27·58	1721	14
14·69	212 000	180 531	893·894	72·175	966·069	1146·600	1·1869	·037928	26·37	1646	14·69
15	213·067	181·608	893·044	72 274	965·318	1146·926	1·1872	·038688	25·85	1614	15

TABLE OF THE PROPERTIES OF SATURATED STEAM—(*Continued*).

P.	T.	S.	I.	E.	L.	H.	U.	W.	C.	V.	P.
16	216·347	184·919	890·458	72·549	963·007	1147·926	1·1882	·041107	24·33	1519	16
17	219·452	188·056	888·007	72·811	960·818	1148·874	1·1892	·043519	22·98	1434	17
18	222·424	191 058	885·661	73·060	958·721	1149·779	1·1901	·045920	21·78	1359	18
19	225·255	193·918	883·427	73·298	956·725	1150·643	1·1910	·048312	20·70	1292	19
20	227·964	196·655	881·289	73·525	954·814	1151·469	1·1919	·050696	19·73	1231	20
21	230·565	199·285	879·239	73·739	952·978	1152·263	1·1927	·053074	18·84	1176	21
22	233 069	201·817	877·267	73·942	951·209	1153·026	1·1935	·055446	18·04	1126	22
23	235·479	204·258	875·368	74·136	949·504	1154·762	1·1943	·057812	17·30	1080	23
24	237·803	206·610	873·538	74·323	947·861	1155·471	1·1950	·060171	16·62	1038	24
25	240·053	208·887	871·767	74·503	946·270	1155·157	1·1957	·062524	16·00	998·4	25
26	242·225	211·089	870·052	74·678	944·730	1155·819	1·1964	·064870	15·42	962·3	26
27	244·333	213·223	868·391	74·847	943·238	1156·461	1·1971	·067210	14·88	928·8	27
28	246·376	215·293	866·780	75·011	941·791	1157·084	1·1978	·069545.	14·38	897·6	28
29	248 363	217·308	865·215	75·168	940·383	1157·691	1·1984	·071875	13·91	868·5	29
30	250·293	219·261	863·700	75·319	939·019	1158·280	1·1990	·074201	13·48	841·3	30
31	252·171	221 165	862·221	75·466	937·687	1158·852	1·1996	·076522	13·07	815·8	31
32	254 002	223 021	860·781	75·608	936·389	1159·410	1·2002	·078839	12·68	791·8	32
33	255·782	224·827	859·382	75·745	935·127	1159·954	1·2008	·081152.	12·32	769·2	33
34	257·523	226·594	858·013	75·878	933·891	1160·485	1·2013	·083461	11·98	748·0	34
35	259 221	228·316	856·680	76·007	932·687	1161·003	1·2018	·085766.	11·66	727·9	35
36	260·883	230·001	855·375	76·133	931·508	1161·509	1·2023	·088067	11·36	708·8	36
37	262·505	231·650	854·097	76·255	930·354	1162·004	1·2028	·090364	11·07	690·8	37
38	264·093	233·261	852·852	76·375	929·227	1162·488	1·2033	·092657	10·79	673·7	38
39	265·647	234·840	851·629	76·493	928·122	1162·962	1·2038	·094946	10·53	657·5	39
40	267·168	236·386	850·432	76·608	927·040	1163·426	1·2043	·097231	10·28	642·0	40
41	268·660	237·902	849·261	76·719	925·980	1163·882	1·2048	·099514	10·05	627·3	41
42	270·122	239·389	848·113	76·827	924·940	1164·329	1·2053	·101794	9·826	613·3	42
43	271·557	240·846	846·988	76·932	923·920	1164·766	1·2058	·104071	9·609	599·9	43
44	272 965	242·275	845·884	77·035	922·919	1165·194	1·2062	·106345	9·403	587·0	44
45	274·347	243·680	841·799	77·136	921·935	1165·615	1·2066	·108616	9·207	574·7	45
46	275·704	245·061	843·733	77·235	920·968	1166·029	1·2070	·110884	9·018	563·0	46
47	277·036	246·418	842·687	77·331	920·018	1166·436	1·2074	·113149	8·838	551·7	47
48	278·348	247·752	841·659	77·425	919·084	1166·836	1·2078	·115411	8·665	540·9	48
49	279·637	249·064	840·647	77·517	918·164	1167·228	1·2082	·117670	8·498	530·5	49
50	280·904	250·355	839·653	77·607	917·260	1167·615	1·2086	·119927	8·338	520·5	50
51	282·151	251·624	838·675	77·696	916·371	1167·995	1.2090	·122181	8·185	510·9	51
52	283·381	252·875	837·710	77·784	915·494	1168·369	1·2094	·124433	8·037	501·7	52
53	284·589	254·106	836·762	77·870	914·632	1168 738	1·2098	·126682	7·894	492·8	53
54	285·781	255·321	835·827	77·954	913·781	1169·102	1·2102	·128928	7·756	484·2	54
55	286·955	256·518	834·906	78·036	912·942	1169·460	1·2106	·131172	7·624	475·9	55

Table of the Properties of Saturated Steam—(*Continued*).

P.	T.	S.	I.	E.	L.	H.	U.	W.	C.	V.	P.
56	288·111	257·695	834·001	78·117	912·118	1169·813	1·2110	·133414	7·496	467·9	56
57	289·251	258·857	833·108	78·196	911·304	1170·161	1·2114	·135654	7·372	460·2	57
58	290·374	260·002	832·228	78·273	910·501	1170·503	1·2117	·137892	7·252	452·7	58
59	291·483	261·132	831·361	78·348	909·709	1170·841	1·2120	·140128	7·136	445·5	59
60	292·575	262·248	830·507	78·421	908·928	1171·176	1·2123	·142362	7·024	438·5	60
61	293·653	263·348	829·663	78·494	908·157	1171·505	1·2127	·144594	6·916	431·7	61
62	294·717	264·433	828·830	78·566	907·396	1171·829	1·2130	·146824	6·811	425·2	62
63	295·768	265·506	828·005	78·638	906·643	1172·149	1·2133	·149052	6·709	418·8	63
64	296·805	266·566	827·191	78·709	905·900	1172·466	1·2136	·151277	6·610	412·6	64
65	297·830	267·612	826·388	78·779	905·167	1172·779	1·2140	·153500	6·515	406·6	65
66	298·842	268·644	825·596	78·847	904·443	1173·087	1·2143	·155721	6·422	400·8	66
67	299·843	269·666	824·814	78·913	903 727	1173·393	1·2146	·157940	6·332	395·2	67
68	300·831	270·674	824·042	78·978	903·020	1173·694	1·2149	·160157	6·244	380·8	68
69	301·807	271·669	823·280	79·042	902·322	1173·991	1·2152	·162372	6·159	384·5	69
70	302·774	272·657	822·524	79·105	901·629	1174·286	1·2155	·164584	6·076	379·3	70
71	303·728	273·633	821·778	79·167	900·943	1174·578	1·2158	·166794	5·995	374·3	71
72	304·669	274·597	821·041	79·228	900·269	1174·866	1·2161	·169003	5·917	369·4	72
73	305·603	275·550	820·312	79·288	899·600	1175·150	1·2164	·171210	5·841	364·6	73
74	306·526	276·493	819·589	79·349	898·938	1175·431	1·2167	·173417	5·767	360·0	74
75	307·440	277·427	818·873	79·410	898·283	1175·710	1·2170	·175622	5·694	355·5	75
76	308·344	278·350	818·166	79·469	897·635	1175·985	1·2173	·177825	5·624	351·1	76
77	309·239	279·265	817·468	79·526	896·994	1176·259	1·2176	·180027	5·555	346·8	77
78	310·123	280·170	816·777	79·582	896·359	1176·529	1·2179	·182229	5·488	342·6	78
79	311·000	281·066	816·090	79·633	895·729	1176·795	1·2181	·184429	5·422	338·5	79
80	311·866	281·952	815·413	79·695	895·108	1177·060	1·2184	·186627	5·358	334·5	80
81	312·725	282·830	814·742	79·749	894·491	1177·321	1·2187	·188823	5·296	330·6	81
82	313·576	283·701	814·077	79·802	893·879	1177·580	1·2190	·191017	5·235	326·8	82
83	314·417	284·562	813·419	79·856	893·275	1177·837	1·2193	·193210	5·176	323·1	83
84	315·250	285·414	812·768	79·909	892·677	1178·091	1·2195	·195401	5·118	319·5	84
85	316·076	286·260	812·122	79·961	892·083	1178 343	1·2198	·197591	5·061	315·9	85
86	316 893	287·096	811·484	80·012	891·496	1178·592	1·2200	·199781	5·006	312·5	86
87	317·705	287·927	810·850	80·063	890·913	1178·840	1·2203	·201969	4·951	309·1	87
88	318·510	288·750	810·222	80·113	890·335	1179·085	1·2205	·204155	4·898	305 8	88
89	319·306	289·565	809·601	80·162	889·763	1179·328	1·2208	·206340	4·846	302·5	89
90	320·094	290·373	808·986	80·210	889·196	1179·569	1·2210	·208525	4·796	299·4	90
91	320·877	291·176	808·375	80·258	888·633	1179·809	1·2212	·210709	4·746	296·3	91
92	321·653	291·970	807·770	80·305	888·075	1180·045	1·2215	·212892	4·697	293.2	92
93	322·422	292·758	807·170	80·351	887·521	1180·279	1·2217	·215074	4·650	290·2	93
94	323·183	293·539	806·575	80·397	886·972	1180·511	1·2220	·217253	4·603	287·3	94
95	323·939	294·314	805·985	80·442	886·427	1180·741	1·2222	·219430	4·557	284·5	95

TABLE OF THE PROPERTIES OF SATURATED STEAM—(*Continued*).

P.	T.	S.	I	E.	L.	H.	U.	W.	C.	V.	P.
96	324·688	295·083	805·400	80·487	885·887	1180·970	1·2224	·221604	4·513	281·7	96
97	325·431	295·845	804·821	80·531	885·352	1181·197	1·2227	·223778	4·469	279·0	97
98	326·169	296·601	804·245	80·576	884·821	1181·422	1·2229	·225950	4·426	276·3	98
99	326·900	297·350	803·675	80·620	884·295	1181·645	1·2232	·228122	4·384	273·7	99
100	327·625	298·093	803·108	80·665	883·773	1181·866	1·2234	·230293	4·342	271·1	100
101	328·345	298·832	802·544	80·709	883·253	1182·085	1·2236	·232464	4·302	268·5	101
102	329·060	299·566	801·985	80·752	882·737	1182·303	1·2238	·234634	4·262	266·0	102
103	329·769	300·293	801·432	80·794	882·226	1182·519	1·2240	·236803	4·223	263·6	103
104	330·471	301·014	800·884	80·835	881·719	1182·733	1·2242	·238972	4·185	261·2	104
105	331·169	301·731	800·339	80·875	881·214	1182·945	1·2245	·241139	4·147	258·9	105
106	331·862	302·444	799·796	80·916	880·712	1183·156	1·2247	·243304	4·110	256·6	106
107	332·550	303·152	799·258	80·956	880·214	1183·366	1·2249	·245467	4·074	254·3	107
108	333·232	303·854	798·725	80·995	879·720	1183·574	1·2251	·247629	4·038	252·1	108
109	333·911	304·551	798·196	81·034	879·230	1183·781	1·2254	·249789	4·003	249·9	109
110	334·582	305·242	797·672	81·072	878·744	1183·986	1·2256	·251947	3·969	247·8	110
111	335·250	305·927	797·153	81·110	878·263	1184·190	1·2258	·254105	3·935	245·7	111
112	335·914	306·609	796·637	81·147	877·784	1184·393	1·2260	·256263	3·902	243·6	112
113	336·573	307·285	796·125	81·184	877·309	1184·594	1·2262	·258420	3·870	241·6	113
114	337·226	307·956	795·617	81·221	876·838	1184·794	1·2264	·260576	3·838	239·6	114
115	337·874	308·621	795·114	81·257	876·371	1184·992	1·2266	·262732	3·806	237·6	115
116	338·518	309·281	794·614	81·293	875·907	1185·188	1·2268	·264887	3·775	235·7	116
117	339·159	309·939	794·114	81·330	875·444	1185·383	1·2270	·267041	3·745	233·8	117
118	339·796	310·592	793·619	81·366	874·985	1185·577	1·2272	·269195	3·715	231·9	118
119	340·430	311·241	793·126	81·403	874·529	1185·770	1·2274	·271348	3·685	230·1	119
120	341·058	311·885	792·637	81·439	874·076	1185·961	1·2276	·273500	3·655	228·3	120
121	341·681	312·524	792·152	81·474	873·626	1186·150	1·2278	·275651	3·628	226·5	121
122	342·300	313·161	791·669	81·509	873·178	1186·339	1·2280	·277801	3·600	224·7	122
123	342·916	313·795	791·189	81·543	872·732	1186·527	1·2282	·279949	3·572	223·0	123
124	343·528	314·423	790·711	81·578	872·289	1186·714	1·2284	·282097	3·545	221·3	124
125	344·136	315·051	790·236	81·612	871·848	1186·899	1·2286	·284243	3·518	219·6	125
126	344·741	315·672	789·765	81·646	871·411	1187·083	1·2288	·286389	3·492	218·0	126
127	345·340	316·289	789·298	81·679	870·977	1187·266	1·2290	·288533	3·466	216·4	127
128	345·936	316·903	788·831	81·711	870·545	1187·448	1·2292	·290677	3·440	214·8	128
129	346·530	317·513	788·374	81·742	870·116	1187·629	1·2293	·292820	3·415	213·2	129
130	347·121	318·121	787·914	81·774	869·688	1187·809	1·2295	·294961	3·390	211·6	130
131	347·706	318·725	787·458	81·805	869·263	1187·988	1·2296	·297102	3·366	210·1	131
132	348·287	319·325	787·004	81·837	868·841	1188·166	1·2298	·299242	3·342	208·6	132
133	348·867	319·922	786·554	81·868	868·422	1188·344	1·2300	·301382	3·318	207·1	133
134	349·443	320·515	786·105	81·900	868·005	1188·520	1·2302	·303521	3·295	205·7	134
135	350·015	321·105	785·659	81·931	867·590	1188·695	1·2304	·305659	3·272	204·2	135
136	350·584	321·692	785·215	81·962	867·177	1188·869	1·2306	·307797	3·249	202·8	136
137	351·149	322·274	784·775	81·992	866·767	1189·041	1·2308	·309934	3·227	201·4	137

Table of the Properties of Saturated Steam—(*Continued*).

P.	T.	S.	I.	E.	L.	H.	U.	W.	C.	V.	P.
138	351·71	322·853	784·339	82·021	866·360	1189·213	1·2309	·312070	3·204	200·0	138
139	352·27	323·429	783·905	82·050	865·955	1189·384	1·2311	·314205	3·182	198·7	139
140	352·827	324·003	783·472	82·080	865·552	1189·555	1·2313	·316338	3·161	197·3	140
141	353·380	324·573	783·042	82·109	865·151	1189·724	1·2315	·318471	3·140	196·0	141
142	353·931	325·141	782·613	82·133	864·751	1189·892	1·2316	·320603	3·119	194·7	142
143	354·478	325·705	782·188	82·166	864·354	1190·059	1·2318	·322735	3·099	193·4	143
144	355·022	326·265	781·766	82·194	863·960	1190·225	1·2320	·324867	3·078	192·2	144
145	355·562	326·823	781·346	82·221	863·567	1190·390	1·2321	·326998	3·058	190·9	145
146	356·100	327·378	780·927	82·249	863·176	1190·554	1·2323	·329128	3·038	189·7	146
147	356·636	327·930	780·510	82·277	862·787	1190·717	1·2324	·331257	3·019	188·5	147
148	357·169	328·479	780·096	82·304	862·400	1190·879	1·2326	·333386	3·000	187·3	148
149	357·697	329·024	779·684	82·332	862·016	1191·040	1·2328	·335515	2·981	186·1	149
150	358·223	329·566	779·275	82·359	861·634	1191·200	1·2330	·337643	2·962	184·9	150
160	363·346	334·850	775·296	82·616	857·912	1192·762	1·2346	·358886	2·786	173·9	160
170	368·226	339·892	771·505	82·854	854·359	1194·251	1·2361	·380071	2·631	164·3	170
180	372·886	344·708	767·891	83·072	850·963	1195·671	1·2376	·401201	2·493	155·6	180
190	377·352	349·329	764·430	83·273	847·703	1197·032	1·2390	·422280	2·368	147·8	190
200	381·636	353·766	761·111	83·462	844·573	1198·339	1·2404	·443310	2·256	140·8	200
210	385·759	358·041	757·916	83·640	841·556	1199·597	1·2417	·464295	2·154	134·5	210
220	389·736	362·168	754·831	83·808	838·642	1200·810	1·2430	·485237	2·061	128 7	220
230	393·575	366·152	751·862	83·966	835·828	1201·980	1·2442	·506139	1·976	123·3	230
240	397·285	370·008	748·988	84·115	833·103	1203·111	1·2454	·527003	1·893	118·5	240
250	400·883	373·750	746·203	84·256	830·459	1204·209	1·2465	·547831	1·825	114·0	250
260	404·370	377·377	743·508	84·388	827·896	1205·273	1·2476	·568626	1·759	109·8	260
270	407·755	380·905	740·891	84·510	825·401	1206·306	1·2487	·589390	1·697	105·9	270
280	411·048	384·337	738·350	84 623	822·973	1207·310	1·2497	·610124	1·639	102·3	280
290	414·250	387·677	735·878	84·731	820·609	1208·286	1·2507	·630829	1·585	99·0	290
300	417·371	390·933	733·470	84·835	818·305	1209·238	1·2517	·651506	1·535	95·8	300
350	431·96	406·26	722·20	85·28	807·48	1213·74	1·256	·754534	1·325	82·7	350
400	444·92	419·76	712·34	85·60	797·94	1217·70	1·260	·857185	1·167	72·8	400
450	456·62	432·18	703·28	85·84	789·12	1221·30	1·264	·959536	1·042	65·1	450
500	467 42	443·52	695·01	86·01	781·02	1224·54	1·267	1·061700	·942	58·8	500
550	477·50	454·14	687·34	86·12	773·46	1227·60	1·270	1·16380	·859	53·6	550
600	486·86	464·22	680·08	86·18	766·26	1230·48	1·273	1·26586	·790	49·3	600
650	495·68	473·58	673·40	86·20	759·60	1233·18	1·276	1·36791	·731	45·6	650
700	504·14	482·40	667·11	86·19	753·30	1235·70	1·279	1·46995	·680	42·4	700
750	512·06	490·86	661·04	86·14	747·18	1238 04	1·282	1·57198	·636	39·6	750
800	519·62	498·88	665·31	86·08	741 42	1240·30	1·285	1·67401	·597	37·1	800
850	526·82	506·66	649·84	86·00	735·84	1242·50	1·287	1·77603	·563	34·9	850
900	533·66	514·03	644·71	85·91	730·62	1244·65	1·289	1·87804	·532	33·0	900
950	540·32	521·30	639·60	85·80	725·40	1246·70	1·291	1·98004	·505	31·4	950
1000	546·80	528·30	634·68	85·68	720·36	1248·66	1·293	2·08203	·480	30·0	1000

The column headed "U" in the table of the properties of saturated steam is useful for reducing the performance of different boilers to a common standard—this standard being that most generally accepted by engineers: the equivalent evaporation at atmospheric pressure and the temperature of boiling water, or, as it is frequently called, the evaporation from and at 212°. In the table it is assumed that the temperature of the feed water is 32°, and an auxiliary table is added, giving corrections for any temperature of feed from 32° to 212°.

CORRECTION FOR TOTAL HEAT IN UNITS OF EVAPORATION.

Temperature of feed, Fahrenheit degrees.	Correction.	Temperature of feed, Fahrenheit degrees.	Correction.	Temperature of feed, Fahrenheit degrees.	Correction.	Temperature of feed, Fahrenheit degrees.	Correction.	Temperature of feed, Fahrenheit degrees.	Correction.
33	·0010	69	·0383	105	·0756	141	·1129	177	·1504
34	·0021	70	·0393	106	·0766	142	·1140	178	·1514
35	·0031	71	·0404	107	·0777	143	·1150	179	·1525
36	·0041	72	·0414	108	·0787	144	·1160	180	·1535
37	·0052	73	·0424	109	·0797	145	·1171	181	·1545
38	·0062	74	·0435	110	·0808	146	·1181	182	·1556
39	·0073	75	·0445	111	·0818	147	·1192	183	·1566
40	·0083	76	·0456	112	·0829	148	·1202	184	·1577
41	·0093	77	·0466	113	·0839	149	·1213	185	·1587
42	·0104	78	·0476	114	·0849	150	·1223	186	·1598
43	·0114	79	·0487	115	·0860	151	·1233	187	·1608
44	·0124	80	·0497	116	·0870	152	·1244	188	·1618
45	·0135	81	·0507	117	·0880	153	·1254	189	·1629
46	·0145	82	·0518	118	·0891	154	·1264	190	·1639
47	·0155	83	·0528	119	·0901	155	·1275	191	·1650
48	·0166	84	·0538	120	·0911	156	·1285	192	·1660
49	·0176	85	·0549	121	·0922	157	·1296	193	·1670
50	·0186	86	·0559	122	·0932	158	·1306	194	·1681
51	·0197	87	·0569	123	·0943	159	·1316	195	·1691
52	·0207	88	·0580	124	·0953	160	·1327	196	·1702
53	·0217	89	·0590	125	·0963	161	·1337	197	·1712
54	·0228	90	·0601	126	·0974	162	·1348	198	·1723
55	·0238	91	·0611	127	·0984	163	·1358	199	·1733
56	·0248	92	·0621	128	·0994	164	·1368	200	·1743
57	·0259	93	·0632	129	·1005	165	·1379	201	·1754
58	·0269	94	·0642	130	·1015	166	·1389	202	·1764
59	·0279	95	·0652	131	·1025	167	·1400	203	·1775
60	·0290	96	·0663	132	·1036	168	·1410	204	·1785
61	·0300	97	·0673	133	·1046	169	·1420	205	·1796
62	·0311	98	·0683	134	·1057	170	·1431	206	·1806
63	·0321	99	·0694	135	·1067	171	·1441	207	·1817
64	·0331	100	·0704	136	·1077	172	·1452	208	·1827
65	·0342	101	·0714	137	·1088	173	·1462	209	·1837
66	·0352	102	·0725	138	·1098	174	·1473	210	·1848
67	·0362	103	·0735	139	·1109	175	·1483	211	·1858
68	·0372	104	·0746	140	·1119	176	·1493	212	·1869

The following example will illustrate the use of the table. Suppose a boiler evaporates 8·25 pounds of water per pound of coal, at a pressure of 80 pounds per square inch, the temperature of the feed being 120° : what is the equivalent evaporation from and at 212° ?

By column U in the table on page 6, the proper factor for the reduction if the temperature of the feed is 32°, is 1·2184 ; and, applying the correction from the table on page 9, the equivalent evaporation is found to be,

$$8{\cdot}25 \times (1{\cdot}2184 - 0{\cdot}0911) = 9{\cdot}3 \text{ pounds of water per pound of coal.}$$

VOLUME AND WEIGHT OF DISTILLED WATER.

Temperature, Fahrenheit degrees.	Ratio of volume to volume of equal weight at temperature of maximum density.	Weight of a cubic foot in pounds.	Temperature, Fahrenheit degrees.	Ratio of volume to volume of equal weight at temperature of maximum density.	Weight of a cubic foot in pounds.	Temperature, Fahrenheit degrees.	Ratio of volume to volume of equal weight at temperature of maximum density.	Weight of a cubic foot in pounds.
32·	1·000129	62 417	210	1·04226	59·894	390	1·15538	54·030
39·1	1·000000	62·425	212	1·04312	59·707	400	1·16366	53·635
40·	1·000004	62·423	220	1·04668	59·641	410	1·17218	53·255
50·	1·000253	62·409	230	1·05142	59·372	420	1·18090	52·862
60·	1·000929	62·367	240	1·05633	59·096	430	1·18982	52·466
70·	1·001981	62·302	250	1·06144	58·812	440	1·19898	52·065
80·	1·00332	62·218	260	1·06679	58·517	450	1·20833	51·662
90·	1·00492	62·119	270	1·07233	58·214	460	1·21790	51·256
100·	1·00686	62·000	280	1·07809	57·903	470	1·22767	50·848
110·	1·00902	61·867	290	1·08405	57·585	480	1·23766	50·438
120·	1 01143	61·720	300	1·09023	57·259	490	1·24785	50·026
130·	1·01411	61·556	310	1·09661	56·925	500	1·25828	49·611
140·	1 01690	61·388	320	1·10323	56·584	510	1·26892	49·193
150	1·01995	61·204	330	1·11005	56·236	520	1·27975	48·778
160·	1·02324	61·007	340	1·11706	55 883	530	1·29080	48·360
170·	1·02671	60·801	350	1·12431	55·523	540	1·30204	47·941
180·	1·03033	60·587	360	1·13175	55·158	550	1·31354	47·521
190·	1 03411	60·366	370	1·13942	54·787			
200	1·03807	60·136	380	1·14729	54·411			

As the table showing the relation between the temperature and volume of water is for pure water, when water contains foreign matter it will be necessary to multiply the tabular coefficient by the specific gravity of the water. For ordinary rain, spring, or river water, the correction is ordinarily so slight that it may be neglected in practice. Below are given the specific gravities of waters from different localities, the most of which have been taken from

Professor Chandler's lecture on "Water," published in the thirty-first annual report of the American Institute:

Atlantic Ocean	1·0275
Dead Sea	1·17205
Great Salt Lake	1·17
Mississippi River	1·00068
Croton (New York water supply)	1·00008
Ridgewood (Brooklyn water supply)	1·000067
Cochituate (Boston water supply)	1·000053
Schuylkill (Philadelphia water supply)	1·00006
Delaware River	1·000059
Lake Erie	1·000107
Lake Michigan	1·000113
Genesee River	1·000226
Passaic River	1·000127
Thames, at London	1·000279
Seine, above Paris	1·000151

It will be seen from these figures that, for most cases, it will be sufficiently accurate to use the weights given in the table. If the weight of a gallon of water at any temperature is desired, it may be obtained by dividing the weight of a gallon of water at the temperature of maximum density (8·3389 pounds for a U. S. gallon, and 10·001077 pounds for an imperial gallon) by the relative volume at the required temperature. It may also be obtained by multiplying the weight of a cubic foot of water, at the given temperature, by 0·1335631 to find the weight of a United States gallon, and by 0·1603412 to find the weight of an imperial gallon. When water contains foreign matter in solution, its rate of expansion by heat is not exactly the same as in the case of distilled water. There have not been enough experiments, however, to determine the law of the variation, and no great error will arise from the assumption that the expansion is the same as given in the table.]

§ 392. **Vapors in General.**—According to Dalton, *the expansive force of all vapors at the same number of degrees above or below the boiling point is the same.* Hence, by means of the boiling point the expansive force of different vapors can be calculated from that of steam. Thus, for instance, alcohol boils at 78° (see § 372), and therefore alcohol vapor at 113° or at 113° − 78° = 35° above its point of ebullition has the same tension as steam at 35° above the boiling point of water, or at 135°—viz., of 3 atmospheres.

But from the more recent experiments of Regnault (see Poggendorff's "Annalen," Vol. 93, 1854), it appears that this law is only approximately correct. According to these, for instance, for temperatures of from 0° to 130°, the forces of expansion of alcohol, sulphuric ether, and oil of turpentine vapors are as follows:

Temperature.	0	10	20	40	60	80	100	110	120	130 degrees.
										Centimeters.
Alcohol.....	1·273	2·408	4·40	13·41	35·00	81·28	168·5	235·2	320·8	433·1
Sulphuric Ether..	18·23	28·65	43·48	91·36	173·03	294·72	492·04	624·9	....	....
Oil of Turpentine.	0·210	0·230	0·430	1·120	2·69	6·12	13·49	18·73	25·70	34·70
Steam.....	0·460	0·9165	1·7391	5·491	14·879	35·164	76·00	107·54	149·13	203·03

According to the experiments of Rudberg, the force of expansion of steam developed from boiling saline solutions is the same, whatever may be the temperatures of their boiling points. Experiments upon the tension of vapors of saline mixtures have recently been made by Wüllner (see Poggendorff's "Annalen," Vol. 156).

To find the densities of different vapors, the above-mentioned procedure of Gay-Lussac and in part also the procedure of Dumas may be used. The latter consists in bringing a sufficient quantity of the fluid to be examined into a glass globe, which is drawn out to a fine point, and this is heated in a bath of water, oil, chloride of zinc, etc., until the discharge of steam through the point of the globe ceases and the liquid is therefore completely vaporized, and then the point is sealed in the blow-pipe flame. From the weight G_1 of this globe, when filled with the vapor to be examined, the density of the vapor can easily be calculated as soon as the capacity V of the globe and its weight G have been determined, when it is filled with atmospheric air. Thus the required density of the steam at the instant the tube is sealed is,

$$\gamma_1 = \frac{G_1 - G + V\gamma}{V},$$

where γ denotes the density of the atmospheric air at the

temperature and height of barometer at which the weighing took place.

The densities of some of the vapors, compared to that of the air when very near their boiling points, are as follows:

Atmospheric Air	= 1·000
Steam	= 0·6235
Alcohol Vapor	= 1·6138
Sulphuric Ether Vapor	= 2·5860
Vapor of the Oil of Turpentine	= 3·0130
Vapor of Mercury	= 6·976.

Moreover, the densities of vapors are nearly *inversely as their latent heats.*

Thus, according to Brix (see Poggendorff's "Annalen," Vol. 55), the latent heat of steam is = 540, and of the vapor of alcohol = 214, hence the ratio of these latent heats is $= \frac{540}{214} = 2{\cdot}52$; and, according to Gay-Lussac, the density of the vapor of alcohol = 1·6138, and that of steam is = 0·6235, and, therefore, the inverse ratio of the densities,

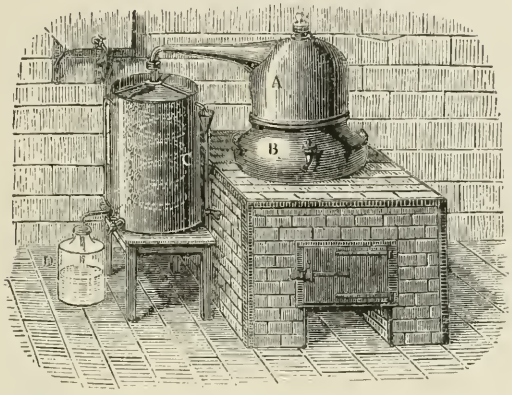
$$\frac{1{\cdot}6138}{0{\cdot}6235} = 2{\cdot}58.$$

§ **393.—Distillation.**—When two communicating vessels *A* and *B*, Fig. 612, which contain the same liquid are unequally heated, then the vapors formed do not assume a mean tension, but one corresponding to the lower temperature, because the vapor of the lower temperature cannot assume a higher tension without being partly condensed. For instance, if a vessel *A* contains steam of 0° of temperature, and a vessel *B* communicating with *A* contains steam of 100°, then the tension of the vapor in *A* and *B* is = 0·46 centimeter, the same as that of steam at 0°.

Upon this is based the application of the condenser to steam engines and the process of distillation. In distilling, the fluid contained in an alembic or retort *B*, Fig. 612,

is converted into vapor by means of heat, in order to liberate it from the less volatile substances that may be

Fig. 612.



held in solution. The vapors formed are intercepted by the cap-shaped end of a downward directed tube *A C*, and there again condensed by exterior cooling.

The condensed vapor then flows into a vessel *D*, while the less volatile substances remain in the retort. To accelerate the condensation of the vapors, the tube *C* which conducts them has a spiral shape and is immersed in a vessel filled with cold water. In order that this water shall not become too much heated by the condensing vapors; it must be continually renewed, and for this purpose we have two tubes in communication with the cooling vessel, of which the one continually conveys cold water to it from below, and the other removes an equal quantity of warm water from above.

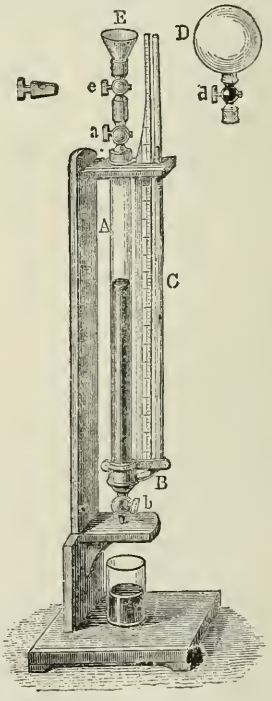
In this way, river and well water is distilled, in order to free it from any salts which may be held in solution, such as, for instance, carbonate of lime, sulphate of lime, etc.

As, in consequence of the above law, the tension of the vapor in the retort is the same as that which corresponds to the temperature of the cooling tube, the evaporation of the liquid in the retort must go on more actively than if the tension of the vapor were a higher one.

§ **394.—Gas and Steam Mixtures.**—If two gases which do not exert any chemical influence upon each other are enclosed in the same vessel, they do not, like liquids, arrange themselves in layers one upon another, according to their specific gravities, but they expand themselves equally throughout the whole space of the vessel, and hence *the force of expansion of the mixture is equal to the sum of the tensions which each single gas would have if it alone occupied the vessel.*

Besides this law established by Dalton, the following also holds good for vapors: *When a liquid is brought into a space filled with gas, as much of the liquid is converted into vapor as though that space were void.*

Fig. 613.



The accuracy of both these laws can be confirmed by the following experiments: The glass tube *A B*, Fig. 613, communicates below with a glass tube *B C*, and is furnished at both ends with cocks *a* and *b*. If we open the cock *a* and close the cock *b*, then we can fill the apparatus by pouring in mercury from above. This being done, *a* is closed and *b* is opened until so much mercury has drained off, that above the mercury remaining in the tube *A B* a vacant space is left. Now closing *b*, we can read from the scale between the two tubes the difference of level h_1 which measures the pressure of the outer air. Now screw on above the cock *a* a globe *D*, which is capable of being closed by a cock *d*, and which is filled with dry air, and then open all three cocks *a*, *b*, and *d*, so that the air enclosed in *D* can expand in the upper end of the tube *A B*. If when,

under these circumstances, the mercury in $A\,B$ has sunk a certain height, b is closed, and the difference of level h_2 is again read off between the mercurial columns $A\,B$ and $B\,C$. The tension of the air enclosed in D and A will be the difference $x = h_1 - h_2$ between the first and last difference of level.

After this, the cock a is closed, and, instead of the globe D, a funnel E, that can be closed by a narrow cock e, is screwed on, into which water or the liquid whose vapor is to be examined is poured, and allowed to drop into the tube $A\,B$ by sudden openings of the cock e. So long as the vapor formed by this liquid depresses the mercury in $A\,B$, so long is fresh liquid allowed to drop in; but when this sinking ceases, then the air has become thoroughly saturated with the vapor of the introduced liquid. We now add mercury through B until the surface of the mercury in $A\,B$ has reassumed its former position, and the difference of level h_3 of the two mercury columns is again, and for the third time, read off. The tension of the air saturated with vapor and enclosed in A is again the difference $y = h_1 - h_3$ between the first and last difference of level, and, therefore,

$$y = x + (h_2 - h_3)$$

greater by $h_2 - h_3$ than the tension x of the dry air. If, finally, it is found that $h_2 - h_3$ is nearly equal to the tension of the saturated vapor at the temperature of the experiment, the approximate correctness of the law of Dalton is thereby proved.

§ 395.—**Humid Air.**—Free air usually contains a greater or less amount of watery vapor, and the determination of this amount is the province of hygrometry. If the air is saturated with vapor, then from the temperature t and the tension p of the same its density γ may be determined as follows: By means of the temperature t, the tension p_1 of the vapor in the air may be found by the formulæ of § 387 and § 388, and then by subtraction the

tension $p_2 = p - p_1$ of dry air. But, now, the density of the vapor is

$$\gamma_1 = \tfrac{5}{8} \times \frac{1 \cdot 3\ p_1}{1 + \delta\ t},$$

and that of dry air is

$$\gamma_2 = \frac{1 \cdot 3\ p_2}{1 + \delta\ t} = \frac{1 \cdot 3\ (p - p_1)}{1 + o\ t} \text{ kilogrammes. (See Vol. I., § 393.)}$$

Hence, the density of the air when saturated with vapor is

$$\gamma = \gamma_1 + \gamma_2 = \frac{1 \cdot 3}{1 + \delta\ t}\ (p - p_1 + \tfrac{5}{8}\ p_1) = \frac{1 \cdot 3}{1 + \delta\ t}\ (p - \tfrac{3}{8} p_1),$$

or

$$\gamma = \frac{1 \cdot 3\ p}{1 + \delta\ t} \left(1 - \tfrac{3}{8} \frac{p_1}{p}\right) \text{ kilogrammes,}$$

where the tension p is given in atmospheres.

If, as is usual, the air is not saturated with vapor, the degree of humidity of the air must also be inserted in the formula. By this we understand the relation ψ between the real quantity of vapor in the air to that quantity of vapor which the air contains when in the saturated state. Consequently, if γ_1 is the density of saturated vapor, then the density of the unsaturated vapor can be put $= \psi\ \gamma_1$. Assuming this, we have, then, for the density of humid air at the degree of humidity ψ and the tension p,

$$\gamma = \gamma_1 + \gamma_2 = \tfrac{5}{8} \times \frac{1 \cdot 3\ \psi\ p_1}{1 + \delta\ t} + \frac{1 \cdot 3\ (p - \psi\ p_1)}{1 + \delta\ t}$$

$$= \frac{1 \cdot 3\ p}{1 + \delta\ t} \left(1 - \tfrac{3}{8} \frac{\psi\ p_1}{p}\right).$$

As $\frac{3}{8}\ \psi\ \frac{p_1}{p}$ is generally but a small fraction, we can also put

$$\gamma = \frac{1 \cdot 3\, p}{(1 + \delta\, t)\left(1 + \frac{3}{8}\, \psi\, \frac{p_1}{p}\right)} = \frac{1 \cdot 3\, p}{1 + \delta\, t + \frac{3}{8}\, \psi\, \frac{p_1}{p}}$$

$$= \frac{1 \cdot 3\, p}{1 + \left(\delta + \frac{3}{8}\, \psi\, \frac{p_1}{p\, t}\right) t}.$$

In the mean, the degree of humidity of free air is $\psi = \frac{1}{2}$; assuming the temperature of the same at $t = 10°$, and putting $\frac{p_1}{p} = 0 \cdot 012$, we obtain

$$\frac{3}{8}\, \psi\, \frac{p_1}{p\, t} = \frac{3}{8} \times \frac{1}{2} \times \frac{0 \cdot 012}{10} = 0 \cdot 00023\,;$$

consequently,

$$\delta + \frac{3}{8}\, \psi\, \frac{p_1}{p\, t} = 0 \cdot 00367 + 0 \cdot 00023 = 0 \cdot 0039\,;$$

for which we can put 0·004, so that the density of the free air in the mean condition of humidity is more simply

$$\gamma = \frac{1 \cdot 3\, p}{1 + 0 \cdot 004\, t} \text{ kilogrammes.}$$

If p is given in kilogrammes per square centimeter, we have

$$\gamma = \frac{0 \cdot 7821\, p}{1 + 0 \cdot 004\, t} \text{ kilogrammes;}$$

and if p is given in pounds per sqı are inch, then the density or the weight of one cubic foot of humid air is to be put

$$\gamma = \frac{0 \cdot 003434\, p}{1 + 0 \cdot 004\, t} \text{ lbs.} \quad \text{(See Vol. I., § 393.)}$$

§ **396.—Hygrometers.**—To measure the degree of humidity of the air, various instruments called hygrometers are employed. These are either chemical or absorption or condensing hygrometers.

If we allow the air, the degree of humidity of which is to be ascertained, to flow through a tube which contains a substance for which the vapor of water has a great affinity—as, for instance, chloride of calcium—this substance will absorb the watery vapor contained in the air, and the air emerges perfectly dry. If we weigh the absorbing body before and after its use, the difference of the weights will indicate the weight of the imbibed water, and if this is divided by the volume of the air conducted through the tube, we obtain the quantity of water per unit of space expressed in weight. An aspirator—*i. e.*, a discharge reservoir closed above—serves for the production of the current of air. If a tube loosely filled with calcic chloride is allowed to discharge into the aspirator which is filled with water, just so much air will flow through the tube into the aspirator as is necessary to completely fill the space which the water, in flowing out, leaves behind.

More simple, but far less satisfactory, are the absorption hygrometers, which are based upon the property that certain organic substances have of expanding when wet and contracting when dry. To this class belongs especially the hair hygrometer of Saussure. The hair employed is cleansed of its oil and fastened at one end, while the other is wound around a guide roller which is provided with a pointer and with opposing weights, so that the roller with the pointer moves from side to side according as the hair elongates or contracts—*i. e.*, as the degree of humidity of the air is greater or less.

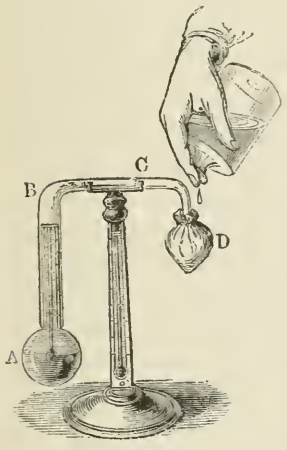
By means of the condensing hygrometer, the humidity of the air is determined by gradually cooling a body and observing at what temperature the vapor of the air condenses on the body as dew. As with the appearance of the dew the condition of saturation is attained, we can calculate, from the temperature of the body, by means of the known formulæ (see § 387 and § 388), the corresponding force of expansion as well as the density of the watery vapor in the air, and upon now comparing this with the tension which corresponds to the temperature of the air in the

saturated condition, we can obtain the degree of humidity of the air itself. If, for instance, the temperature of the air were $t = 20°$, and, on the other hand, that of the body on which the condensation of the water took place were $t_1 = 5°$, then, as there corresponds to the temperature $t = 20°$ the force of expansion $p = 1 \cdot 7391$ centimeters, and to the temperature $t_1 = 5°$ the force of expansion $p_1 = 0 \cdot 6534$ centimeters, the degree of humidity of the air would be

$$\psi = \frac{0 \cdot 6534}{1 \cdot 7391} = 0 \cdot 376.$$

In Daniell's hygrometer, *A B C D*, Fig. 614, the body *A* on which the vapor of the air condenses consists of a glass bulb *A*, covered with lustrous gold or platinum, which is two thirds filled with sulphuric ether and contains the bulb of a thermometer, by means of which the temperature at the moment of dew formation can be read off. This bulb is connected, by means of a bent tube *C B*, with another glass bulb *D*, and the whole apparatus is exhausted of air. To accomplish the necessary refrigeration of the first globe, it is only necessary to allow sulphuric ether to drop on the cloth-covered globe *D*. The evaporation of this ether produces cold in *D*, whereon a diminution of tension of the ethereal vapor in the whole apparatus occurs, and there is not only the condensation of this vapor in *D*, but also new vapor rises in *A* and the bulb *A* is cooled.

Fig. 614.



In the main, the hygrometer of Regnault as also the psychrometer of August are based upon the same principle.

§ 397.—Heating Power.—The heat which is necessary to change water into steam is generally gained by the combustion of bodies (Fr. combustion; Ger. Verbrennung),

Combustion consists in the rapid union of the fuel (Fr. combustible; Ger. Brennstoff) with oxygen. Carbonaceous bodies are chiefly used as fuel, but the oxygen is supplied by the atmospheric air, which, in its usual condition, contains about 23 per cent. The heating power (Fr. puissance calorifique; Ger. Erwärmungskraft) or the amount of heat which is evolved by this union differs greatly with different combustibles. For instance, it is greater with hydrogen gas than with carbon, and greater with carbon than with wood, etc. Rumford, Lavoisier, and Laplace, as also Despretz, and especially Dulong, have made experiments upon the heating power of different substances, measuring it by the heat imparted to a certain quantity of water, by the combustion of a given quantity of the fuel. In this way, Dulong found that 1 gramme of hydrogen by its combustion can heat 34600 grammes of water one degree, and requires to burn it 4·32 grammes of oxygen, while, on the other hand, 1 gramme of carbon raises only 7290 grammes of water one degree, and requires 2·73 grammes of oxygen, and one gramme of carbonic oxide gas only raises 2490 grammes of water one degree, and requires 4·36 grammes of oxygen. According to § 373, the heating power of hydrogen gas is consequently = 34600, that of carbon = 7290, and that of carbonic oxide gas = 2490 heat-units (calories).

As to the quantity of oxygen necessary for combustion, this can also be directly calculated from the products of combustion. By the complete combustion of coal, the product is carbonic acid (Fr. acide carbonique; Ger. Kohlensäure), and this consists of 27·36 parts of carbon and 72·64 parts of oxygen; hence, 1 gramme of carbon requires for its combustion $\frac{72\cdot64}{27\cdot36} = 2\cdot65$ grammes of oxygen, or $\frac{2\cdot65}{0\cdot23}$ = 11·52 grammes of atmospheric air, since, by weight, the latter contains 23 parts of oxygen to 77 parts of nitrogen.

§ 398.—**Heat of Combustion.**—More recent experiments upon the heat of combustion have been instituted by Andrews (Poggendorff's "Annalen," Vol. 75), as also

by Favre and Silbermann ("Annales de chim. et de phys.," Sér. III., Tome 34).

The calorimeter which the latter experimenters employed consisted chiefly of a metallic combustion chamber of about five centimeters width and ten centimeters height, which was dipped into a vessel filled with water and was exteriorly in connection with three tubes, whereby the oxygen necessary for the combustion and the gas to be burned were conveyed to the chamber, and the gaseous products of combustion were removed. To communicate completely the heat of the latter to the water, the third or drain pipe had a great length and was wound spirally about the combustion chamber. If, instead of a gas, a solid or fluid body was to be burned, it had, of course, to be brought into the chamber before the experiment, and the second or gas-conveying tube is, of course, closed. To observe the process of combustion from without, another additional tube, closed by a strong plate of glass, was applied to the middle of the chamber, and above it an inclined mirror was placed. Further, the cooling vessel was surrounded by a jacket, and, with this, was set in still another vessel filled with water so that no heat could be absorbed from without, and, finally, to diffuse the heat throughout the cooling water as equably as possible, this water was set in motion by an agitator consisting of two sheet-iron rings. From the weight of the cooling water Q and the observed increase of heat t of the same in consequence of the combustion, the corresponding quantity of heat $W = Q\,t$ (see § 373) could be calculated.

In this manner was found for the combustion of one kilogramme (2·20 lbs.) of

Charcoal	8080	heat-units,
Graphite	7797	"
Carbonic Oxide Gas	2403	"
Hydrogen Gas	34462	"

etc.

According to these experiments, the heat of combustion or the heating power of coal or pure carbon is greater

than as given by Dulong and others. But, according to Favre and Silbermann, the difference is due to the fact that usually the coal is not completely burnt and converted into carbonic acid, but partly also into carbonic oxide gas. These experimenters have, accordingly, determined the amount of the latter gas separately, and added the heat which is given off by its combustion in order to obtain the whole of the heat of combustion.

While carbonic acid gas consists by weight of 27·27 parts of carbon and 72·73 parts of oxygen, carbonic oxide gas consists by weight of 42·86 parts of carbon and 57·14 of oxygen, and, consequently, for the combustion of one gramme of coal to carbonic oxide gas there is necessary only $\frac{57 \cdot 14}{42 \cdot 86} = 1 \cdot 333$ grammes of oxygen, or $\frac{1 \cdot 333}{0 \cdot 23} = 5 \cdot 8$ grammes of air—*i.e.*, only half as much atmospheric air as for the combustion to carbonic acid. Therefore, the carbonic oxide gas is formed in greater quantity when there is a lack of a current of air or of the quantity of oxygen necessary for the formation of carbonic acid gas.

As, according to the experiments of Favre and Silbermann, the conversion of one kilogramme of carbon into carbonic acid gas gives 8080 heat-units, but, on the other hand, that of one kilogramme of carbonic oxide gas into carbonic acid gives 2403 heat-units, and as carbonic oxide gas contains 42·86 per cent of carbon, therefore the conversion of one kilogramme of carbon from carbonic oxide to carbonic acid gas is equivalent to $\frac{2403}{0 \cdot 4286} = 5607$ heat-units, and, consequently, the amount of heat which is developed by the conversion of the carbon into carbonic oxide gas is $8080 - 5607 = 2473$ heat-units, or about $\frac{3}{10}$ of that quantity of heat (8080 heat-units) which is produced by the complete combustion to carbonic acid gas.

The quantities of heat which are developed by the combustion of the hydrocarbons can be easily calculated from the quantities of heat of their constituents. Marsh gas (fire-damp) consists by weight of 25 per cent of hydro-

gen and 75 per cent of carbon, and, consequently, gives by its combustion

$\frac{1}{4} \times 34462 + \frac{3}{4} \times 8080 = 8615 \cdot 5 + 6060 = 14675 \cdot 5$ heat-units,

but olefiant gas consists of $\frac{1}{7}$ hydrogen and $\frac{6}{7}$ oxygen, and by its combustion furnishes, therefore, only

$$\frac{1}{7} \times 34462 + \frac{6}{7} \times 8080 = 4923 + 6926 = 11849 \text{ heat-units.}$$

Remark.—In regard to the development of heat from other chemical combinations, as well as upon sources of heat in general, see Müller's "Physik," Vol. II., as also Wühlner's "Experimentalphysik," Vol. II.

§ **399.—Combustibles.**—The materials of combustion which are used for the production of heat are particularly pit coal, brown coal, turf, wood, and coke. These are combinations of carbon (C), hydrogen (H), and oxygen (O), at times containing also some nitrogen (N), and most always various quantities of inorganic constituents which, after the combustion, remain behind as ashes. Besides this, they also contain a greater or less amount of free or hygroscopic water, which assumes the form of vapor during the combustion, and thus absorbs a certain quantity of heat, whereby the heating power of the fuel is considerably decreased. Therefore, the fuel should be dried as thoroughly as possible before it is employed. Newly hewn wood contains from 35 to 50 per cent of water, and thoroughly dry wood still contains 20 to 25 per cent. As one pound of water requires about 640 heat-units to transform it into steam, and one pound of thoroughly dried wood develops by its combustion 3600 heat-units, then one pound of wood with 25 per cent of water will by its combustion only furnish $3600 \times 0 \cdot 75 = 2700$ heat-units, and, moreover, will give off $640 \times 0 \cdot 25 = 160$ heat-units to the water for its conversion into steam, so that, consequently, only $2700 - 160 = 2540$ heat-units can be utilized.

The quantity of oxygen O found in fuels by chemical analysis is combined with a portion $H_1 = \frac{O}{8}$ of the hydro-

gen H to form water; consequently, only the quantity of hydrogen

$$H - H_1 = H - \frac{O}{8}$$

is available for combustion, and develops the quantity of heat

$$W_1 = 34462 \left(H - \frac{O}{8} \right).$$

Adding to this the quantity of heat,

$$W_2 = 8080\ C,$$

which proceeds from the combustion of the carbon C, we obtain the whole theoretical heating power of a combustible

$$W = W_1 + W_2 = 34462 \left(H - \frac{O}{8} \right) + 8080\ C.$$

Anthracite is that material of combustion most rich in carbon. In the mean, it consists of 91 per cent of carbon, 3 per cent of hydrogen, 3 per cent of oxygen, and 3 per cent of ash, and hence the theoretical heating power is

$$W = 34462 \times (0{\cdot}03 - \tfrac{1}{8} \times 0{\cdot}03) + 8080 \times 0{\cdot}91$$

$$= 905 + 7353 = 8258 \text{ heat-units.}$$

Pit coal (Fr. houille; Ger. Steinkohle) consists in the mean of 80 per cent of carbon, 5 per cent of hydrogen, 10 per cent of oxygen, and 5 per cent of ash; consequently, its theoretical heating power is

$$W = 34462 \times (0{\cdot}05 - \tfrac{1}{8} \times 0{\cdot}1) + 8080 \times 0{\cdot}80$$

$$= 1292 + 6464 = 7756 \text{ heat-units.}$$

Brown coal (Fr. lignite; Ger. Braunkohle), on the other hand, contains in the mean only 60 per cent of car-

bon, 5 per cent of hydrogen, 25 per cent of oxygen, and 10 per cent of ash. The theoretical heating power of this fuel is, therefore,

$$W = 34462 \times (0{\cdot}05 - \tfrac{1}{8} \times 0{\cdot}25) + 8080 \times 0{\cdot}60$$

$$= 646 + 4848 = 5494 \text{ heat-units.}$$

Peat (Fr. tourbe; Ger. Torf) contains in the mean 52 per cent of carbon, 5 per cent of hydrogen, 33 per cent of oxygen, and 10 per cent of ash. The theoretical heating power is, therefore,

$$W = 34462 \times (0{\cdot}05 - \tfrac{1}{8} \times 0{\cdot}33) + 8080 \times 0{\cdot}52$$

$$= 301 + 4202 = 4503 \text{ heat-units.}$$

As to wood, this consists, on the average, of 49 per cent of carbon, 6 per cent of hydrogen, 44 per cent of oxygen, and 1 per cent of ash, so that the theoretical heating power is

$$W = 34462 \times (0{\cdot}06 - \tfrac{1}{8} \times 0{\cdot}44) + 8080 \times 0{\cdot}49 = 172 + 3959$$

$$= 4131 \text{ heat-units.}$$

By carbonization (Fr. carbonisation; Ger. Verkohlung) of the material, not only is the hydrogen and oxygen removed, but a part of the carbon is also lost, as there takes place a simultaneous union of the hydrogen, carbon, and oxygen, which escapes in the form of gas. For this reason, one pound of air-dried wood with 20 per cent hygroscopic water and 40 per cent of carbon will only give from 0·18 to 0·25 pounds of charcoal (Fr. charbon de bois; Ger. Holzkohle), and, in like manner, one pound of pit.coal will give only from 0·45 to 0·6 pounds of coke. Yet neither charcoal nor coke are pure carbon, but both still contain, beside the solid constituents giving ashes, some hydrogen and oxygen, and their theoretical heating power is therefore from 7000 to 7500 heat-units.

Accordingly, the employment of decarbonized substances is connected with a great loss of heat, and only

where the production of intense heat is required, or for the removal of certain volatile constituents—as, for instance, sulphur—is their use permissible. The quantity of heat capable of being *utilized*, or that which is obtained from the combustion of fuel in furnaces, is always much less than the above indicated theoretical quantity, because a complete conversion into carbonic acid never takes place, because the products of combustion carry away a considerable quantity of the heating power, and because heat is also lost by waste and conducted away by the walls of the furnace. From numerous experiments, especially from the very thorough ones instituted by Dr. W. Brix (see his researches upon the heating power of the most important combustibles), it follows that the mean effective heating power of most furnaces is about two thirds of the theoretical heating power of the fuel.

§ **400.—Combustion.**—The quantity of air necessary for the complete combustion of a certain quantity of fuel, as also the amount of gas generated which must be discharged by the chimney, can be ascertained as follows:

The quantity of carbon C of the fuel requires for the formation of carbonic acid the amount of oxygen,

$$O_1 = \tfrac{8}{3}\, C = 2{\cdot}67\, C,$$

and the amount of carbonic acid formed is

$$C + O_1 = \tfrac{11}{3}\, C = 3{\cdot}67\, C.$$

Further, the conversion of the quantity of free hydrogen $H - \frac{O}{8}$ to water requires an amount of oxygen,

$$O_2 = 8\left(H - \frac{O}{8}\right) = 8\,H - O,$$

and gives the quantity of water,

$$9\left(H - \frac{O}{8}\right) = 9\,H - \tfrac{9}{8}\,O.$$

Hence the whole amount of oxygen required is

$$O = O_1 + O_2 = 2{\cdot}67\ C + 8\ H - O,$$

and, therefore, the necessary quantity of atmospheric air is

$$L = \frac{2{\cdot}67\ C + 8\ H - O}{0{\cdot}231} = 11{\cdot}56\ C + 34{\cdot}63\ H - 4{\cdot}33\ O,$$

or in cubic meters, if C, H and O are expressed in kilogrammes, and it is assumed that at a mean temperature of 10° and 0·76 meters height of barometer, 1 cubic meter of air weighs $\gamma = 1{\cdot}25$ kilogrammes,

$$\tfrac{4}{5}\ L = 9{\cdot}25\ C + 27{\cdot}70\ H - 3{\cdot}46\ O \text{ cubic meters.}$$

On the other hand, the necessary quantity of air for one pound of fuel is

$$V = \frac{35{\cdot}316}{2{\cdot}2046}\,(9{\cdot}25\ C + 27{\cdot}70\ H - 3{\cdot}46\ O)$$

$$= 148{\cdot}1\ C + 443{\cdot}5\ H - 55{\cdot}4\ O \text{ cubic ft.}$$

According to the above, for instance, for one pound of pit coal, $C = 0{\cdot}80$, $H = 0{\cdot}05$, and $O = 0{\cdot}10$ pounds, and hence the amount of atmospheric air necessary is

$$V = 148{\cdot}1 \times 0{\cdot}8 + 443{\cdot}5 \times 0{\cdot}05 - 55{\cdot}4 \times 0{\cdot}10$$

$$= 118{\cdot}5 + 22{\cdot}2 - 5{\cdot}5 = 135{\cdot}2 \text{ cubic ft.}$$

To obtain a rapid and complete combustion, it is necessary to convey a double quantity of air to the furnace.

As to the quantity of gas to be removed by the chimney, this consists of the nitrogen of the atmospheric air used, of the carbonic acid gas produced by the combustion, and also of the vapor of water formed at the same time.

The quantity of nitrogen resulting from decomposition of the atmospheric air is by weight,

$$Q_1 = \frac{0.769}{0.231}(2.67\,C + 8\,H - O) = 3.329 \times (2.67\,C + 8\,H - O)$$

$$= 8.88\,C + 26.63\,H - 3.33\,O \text{ kilogrammes,}$$

or as the density of nitrogen gas at 10°, and the mean height of barometer 0.76 meters is

$$= 1.25 \times 0.9713 = 1.2141 \text{ kilogrammes,}$$

$$Q_1 = (8.88\,C + 26.63\,H - 3.33\,O) \div 1.2141$$

$$= 7.315\,C + 21.93\,H - 2.74\,O \text{ cubic meters,}$$

and, consequently, the quantity of nitrogen gas per pound of fuel is

$$Q_1 = 117.1\,C + 351.1\,H - 43.8\,O \text{ cubic ft.}$$

Further, since the density of carbonic acid gas amounts to

$$\gamma = 1.25 \times 1.529 = 1.911 \text{ grammes,}$$

the quantity of this gas resulting from 1 kilogramme of fuel is

$$= \frac{3.67\,C}{1.911} = 1.919\,C \text{ cubic meters;}$$

hence, this quantity per pound of fuel,

$$Q_2 = 16.01 \times 1.919\,C = 30.7\,C \text{ cubic ft.}$$

Finally, from the hydrogen H proceeds the quantity of water $9\,H$, which, as one cubic meter of steam weighs $\frac{5}{8} \times 1.25$ grammes $= 0.78125$ kilogrammes, gives for the quantity of steam,

$$\frac{9\,H}{0.78125} = 11.52\,H \text{ cubic meters,}$$

so that, per pound of fuel, the steam furnishes

$$Q_3 = 16{\cdot}01 \times 11{\cdot}52\ H = 184{\cdot}4\ H \text{ cubic ft.}$$

Therefore, the volume of gas furnished by complete combustion is

$$Q = Q_1 + Q_2 + Q_3 = 117{\cdot}1\ C + 351{\cdot}1\ H - 43{\cdot}8\ O + 30{\cdot}7\ C$$

$$+ 184{\cdot}4\ H = 147{\cdot}8\ C + 535{\cdot}5\ H - 43{\cdot}8\ O \text{ cubic ft.}$$

The weight of this mixture is the weight G of the fuel plus the weight of the supplied air,

$$L = 11{\cdot}56\ C + 34{\cdot}63\ H - 4{\cdot}33\ O;$$

consequently, the weight of one cubic foot of the same is

$$\gamma = \frac{11{\cdot}56\ C + 34{\cdot}63\ H - 4{\cdot}33\ O}{147{\cdot}8\ C + 535{\cdot}5\ H - 43{\cdot}8\ O}.$$

For instance, for $C = 0{\cdot}8$, $H = 0{\cdot}05$, and $O = 0{\cdot}10$, as above,

$$Q = 147{\cdot}8 \times 0{\cdot}8 + 535{\cdot}5 \times 0{\cdot}05 - 43{\cdot}8 \times 0{\cdot}10 = 140{\cdot}6$$

cubic ft., and weighs

$$G = 11{\cdot}56 \times 0{\cdot}8 + 34{\cdot}63 \times 0{\cdot}05 - 4{\cdot}33 \times 0{\cdot}10 = 10{\cdot}54 \text{ lbs.},$$

so that the density of this mass of gas is

$$\gamma = \frac{10{\cdot}54}{140{\cdot}6} = 0{\cdot}07496 \text{ lbs.},$$

while the density of the atmospheric air is

$$= \frac{1{\cdot}25}{15{\cdot}13} = 0{\cdot}08262 \text{ lbs.}$$

If we supply double the quantity of air which is neces-

sary for a slow combustion, then the quantity of the mass of gas to be removed by the chimney is still greater by

$$V = 148{\cdot}1\ C + 443{\cdot}5\ H - 55{\cdot}40\ O;$$

consequently,

$$Q = 295{\cdot}9\ C + 979{\cdot}0\ H - 99{\cdot}2\ O \text{ cubic ft.},$$

and the density of the gaseous mixture is

$$\gamma = \frac{23{\cdot}12\ C + 69{\cdot}26\ H - 8{\cdot}66\ O}{295{\cdot}9\ C + 979\ H - 99{\cdot}2\ O}.$$

These values for Q and γ refer only to the mean temperature (10°) of the admitted air; but as the temperature t of the gases of combustion that are carried off is higher, we have to put the volume

$$\left(\frac{1 + \delta\ t}{1 + 10\ \delta}\right) Q = \left(\frac{1 + 0{\cdot}00367\ t}{1{\cdot}0367}\right) Q.$$

Usually we assume $t = 300°$, and, therefore, obtain the quantity of gas,

$$= \frac{2{\cdot}101}{1{\cdot}0367}\ Q = 2{\cdot}027\ Q,$$

and its density,

$$= \frac{\gamma}{2{\cdot}027} = 0{\cdot}4934\ \gamma.$$

Accordingly, for one pound of pit coal, if we assume $t = 10°$, there is, therefore,

$$Q = 295{\cdot}9 \times 0{\cdot}8 + 979{\cdot}0 \times 0{\cdot}05 - 99{\cdot}2 \times 0{\cdot}10 = 275{\cdot}75 \text{ cub. ft.}$$

and

$$\gamma = \frac{21{\cdot}08}{275{\cdot}75} = 0{\cdot}07644 \text{ lbs.}$$

On the other hand, if we put $t = 300°$,

$$Q = 2{\cdot}027 \times 275{\cdot}75 = 558{\cdot}9 \text{ cubic ft.},$$

and

$$\gamma = 0{\cdot}4934 \times 0{\cdot}07644 = 0{\cdot}03771 \text{ lbs.}$$

The following table gives the theoretical heating power of the best fuels, as well as the quantities of air necessary, for their combustion, and the quantities of gas generated:

Combustibles.	Quantities of Heat.	Cold air for the combustion of one pound of fuel.	Quantity of Gas proceeding from the Combustion, Reduced	
			to 0°	to 300°.
	Cal.	Cub. Feet.	Cub. Feet.	Cub. Feet.
Thoroughly dried wood	3600	102	111	233
Air-dried wood with 20 per cent of water.	2800	82	93	194
Charcoal	7000	248	248	519
Thoroughly dried peat	4800	171	178	371
Peat with 20 per cent of water	3600	137	146	305
Peat coal	5800	200	200	418
Average pit coal	7500	274	279	584
Coke with 15 per cent of ash	6000	227	227	475
Pure coke	7050	250	250	520

Example.—How much air-dried wood is necessary to heat 30 cubic ft. of water from 10° of temperature to 70°? The quantity of heat necessary is, if the cubic foot of water is assumed to weigh 62·5 lbs.,

$$62{\cdot}5 \times 30 \times (70 - 10) = 181875 \times 60 = 112500 \text{ heat-units or calories.}$$

But one pound of air-dried wood furnishes by its combustion 2800 cal., hence for the required heating,

$$\frac{112500}{2800} = 40{\cdot}18 \text{ lbs.},$$

or about one cubic foot of wood is necessary. Moreover, the amount of air necessary for this combustion is

$$= 82 \times 40{\cdot}18 = 3295 \text{ cubic ft.},$$

and the produced amount of gas at the temperature of 250°,

$$= (1 + 0{\cdot}00367 \times 250) \times 93 \times 40{\cdot}18 = 1{\cdot}9175 \times 93 \times 40{\cdot}18 = 7165 \text{ cubic ft.}$$

§ **401.—Quantity of Fuel.**—The quantity of fuel which is necessary for the production of a certain amount of steam can also be easily calculated. We have indicated above (§ 380) that the total heat of steam of the temperature $t°$ is (according to Regnault),

$$W = 606{\cdot}5 + 0{\cdot}305\ t,$$

and, therefore, we can put the amount of heat which is necessary to convert one pound of water of the temperature t_1 into steam of the temperature t,

$$W = 606{\cdot}5 + 0{\cdot}305\ t - t_1 \text{ calories},$$

or more precisely, as the temperature t_1 of the water corresponds to the amount of heat,

$$W_1 = t_1 + 0{\cdot}00002\ t_1^2 + 0{\cdot}0000003\ t_1^3,$$

$$W = 606{\cdot}5 + 0{\cdot}305\ t - (1 + 0{\cdot}00002\ t_1 + 0{\cdot}0000003\ t_1^2)\ t_1 \text{ cal.}$$

Previous to the experiments of Regnault, the amount of heat of the steam was calculated either by means of a hypothetical formula of Watt or by means of another one of Southern. According to Watt, Sharp, Clément-Désormes, and chiefly according to the more recent observations of Pambour, the total heat of steam is the same at all temperatures; hence if the latent heat diminishes, the sensible heat becomes greater. Assuming that for the formation of steam of 100°, a quantity of heat of 540 cal. is required, then we have the amount of heat which is necessary to convert water of the temperature $t_1°$ into steam of any temperature simply,

$$W = 540 + 100 - t_1 = 640 - t_1.$$

According to Southern, Poncelet, and others, the latent heat of steam would be constant (540 cal.), the total heat would hence increase with the temperature, and therefore the amount of heat would be

$$W = 540 + t - t_1.$$

Assuming the temperature of the water as = 0, and putting that of the steam $t = 100$, 125, 150 degrees, etc., the following comparison can be made:

Temperature of the Steam.		50°	75°	100°	125°	150°	175°	200°
Quantity of Heat according to	Watt.....	640	640	640	640	640	640	640
	Southern....	590	615	640	665	690	715	740
	Regnault....	611·7	629·4	637	644·6	652·2	659·9	667·5

From this we see that at temperatures of from 100 to 150 degrees, such as most frequently occur in steam engines, the law of Watt does not deviate considerably from the formula of Regnault, but that, on the other hand, the rule of Southern leads to marked deviations at temperatures of over 120°.

If, according to Regnault, we put

$$W = 606 \cdot 5 + 0 \cdot 305\, t - t_1,$$

we have for the amount of heat which is necessary for the conversion of the quantity of water $Q\gamma$ pounds into steam,

$$W = (606 \cdot 5 + 0 \cdot 305\, t - t_1)\, Q\gamma \text{ calories.}$$

Adopting mean values for t and t_1, putting $t = 125$ and $t_1 = 15°$, we obtain

$$W = 630 \text{ calories.}$$

If we use pure carbon for combustion, and suppose that $\frac{2}{3}$ of the heat developed thereby produces effect, then we can put the quantity of heat obtained by one pound of coal,

$$= \tfrac{2}{3} \times 7050 = 4700 \text{ calories,}$$

and as, according to the previously mentioned rule, the quantity of heat which is required to convert 1 pound of water of the temperature of 15° into steam amounts to 630 cal., we can quite accurately assume that every pound of carbon furnishes by its combustion $\frac{4700}{630} = 7\frac{1}{2}$ pounds of steam, or that 1 pound of steam requires for its production $\frac{2}{15} = 0 \cdot 133$ pounds of carbon. Experimentally, 1 pound of pit coal gives from 5 to 7 pounds of steam, 1 pound of coke from $4\frac{2}{3}$ to $5 \cdot 8$ pounds, 1 pound charcoal 6 pounds, 1 pound of wood from $2 \cdot 5$ to $2 \cdot 7$ pounds of steam. (See "Guide du chauffeur, par Grouvelle et Jaunez.")

For the pit coal used for the production of steam, the following mean values can be used:

Pit Coal.	Weight of raw Pit Coal per Cubic Foot.	Percentage of Water.	Percentage of Ash.	Effective Vaporizing Power. Amount of Steam per Pound of raw Coal.
North America.....	51·17 lbs.	1·39	10·3	9·11 lbs.
England.....	55·5	3·37	7·8	8·6
Prussia.....	49·5	3·00	4·8	9·1
Saxony.....	52·1	10·83	25·5	9·0

Example.—What quantity of pit coal does a steam engine which consumes per minute 500 cubic feet of steam of three atmospheres require hourly, if the feed water has a temperature of 30°?

According to Table II., § 388, the tension of three atmospheres corresponds to the temperature $t = 133{\cdot}9°$, and, according to the table in § 391, 500 cubic feet of steam at three atmospheres tension require

$$\frac{500}{587{\cdot}4} \text{ cubic feet} = \frac{500}{587{\cdot}4} \times 62{\cdot}5 = 53{\cdot}20 \text{ pounds of water,}$$

and this requires, according to the above formula, the quantity of heat,

$$W = (606{\cdot}5 + 0{\cdot}305\, t - t_1)\, Q\, \gamma_1 = (606{\cdot}5 + 0{\cdot}305 \times 134 - 30) \times 53{\cdot}20$$
$$= (606{\cdot}5 + 41{\cdot}2 - 30) \times 53{\cdot}20 = 617{\cdot}5 \times 53{\cdot}20 = 32851 \text{ cal.}$$

Assuming that one pound of pit coal furnishes 4000 cal. of effective heat, we obtain for the necessary quantity of coal per minute,

$$K = \frac{W}{4000} = \frac{32851}{4000} = 8{\cdot}213 \text{ pounds;}$$

hence, hourly, $60\,K = 493$ pounds.

Final Remark.—Besides the works on Physics of Müller, Ganot, Wüllner, and others, the following papers treat the subject of heat and fuel: "Traité de la chaleur consid. dans les applications, par E. Péclet, III. édition," Paris, 1860; the "Wärmemesskunst und deren Anwendung," by C. Schinz, Stuttgart, 1858; "Untersuchungen über die Heizkraft der wichtigeren Brennstoffe im preussischen Staate," by P. W. Brix, Berlin, 1853; "A Report to the Navy Department of the United States of America on American Coals, etc.," Philadelphia, 1844; the Transactions of the "Vereines zur Beförderung des Gewerbfleiszes in Preussen," 1846. See also "Formules, Tables, etc., ou Aide-Memoire des Ingenieurs, etc., par J. Claudel," Paris, 1854; also "Untersuchungen ber die Heizkraft der Steinkohlen Sachsens von Ernst Hartig," Leipzig, 1860; as also "Des Machines à vapeur, par Morin et Treska," Tome I., "Production de la vapeur," Paris, 1863.

[A very complete series of experiments on combustion, conducted by MM. Scheurer-Kestner and Meunier-Dollfus (see *Bulletin de la Société Industrielle de Mulhouse*, 1868, 1869), show that the method usually employed for determining the heating power of fuel, from its elementary analysis, is incorrect. No very satisfactory explanation of this fact has, however, yet been given. It seems probable that the heating effect depends upon the state in which the carbon exists in the coal; and that, while the effect of the combustion of carbon in the form of charcoal has been accurately determined, it may be much higher in the case of carbon that is not so much condensed. An interesting discussion of these experiments, by M. L. Gruner, was published in the *Engineering and Mining Journal*, xviii., and the two tables that follow give, in a concise form, some of the most important results obtained by the experimenters:

SUMMARY OF ANALYSES AND CALORIMETRIC TRIALS OF FUEL, BY MESSRS. SCHEURER-KESTNER AND MEUNIER-DOLLFUS.

		Ronchamp.				Saarbrück.				
		No. 1.	No. 2.	No. 3.	No. 4.	Dutweiler.	Altenwald.	Heinitz.	Friedrichsth'l	Louisenth'l
Elementary analysis of coal.	Carbon	76·45	68·65	76·23	73·1	71·25	69·3	70·33	67·81	64·69
	Hydrogen	4·39	3·97	4·06	3·75	4·1	4·26	4·3	4·19	3·94
	Solid Refuse	15·02	20·8	12·8	16·19	13·25	13·5	11·57	12·7	12·38
	Nitrogen	1·09	1·06	1·	1·	0·5	0·5	0·5	0·5	0·5
	Oxygen	3·05	4·75	5·91	4·87	9·15	9·9	11·51	13·8	15·02
	Water		0·77		1·09	1·75	2·54	1·79	1·	3·57
Combustible and volatile constituents.	Uncombined carbon	72·57	74·74	71·44	71·58	62·84	63·15	61·57	58·44	56·15
	Carbon in Hydrocarbon	17·39	12·75	15·09	16·8	20·98	19·59	18·92	20·53	20·72
	Hydrogen	5·09	5·1	4·56	4·42	4·6	4·73	4·71	4·67	4·68
	Nitrogen	1·28	1·35	1·14	1·2	0·71	0·66	0·68	0·59	0·6
	Oxygen	3·67	6·06	6·87	6·	10·87	11·85	14·12	15·77	17·85
Heat of combustion per lb. of combustible by experiment.	Thermal units	16493	16401	16346	16103	15703	15539	15277	15223	14787
	Lbs. of water from and at 212° Fah.	17·06	16·98	16·91	16·60	16·25	16·08	15·8	15 75	15·3
Excess of experimental value above that calculated by analysis, thermal units.		547	1186	1244	1013	1319	1195	1361	1548	1379
Difference between experimental and calculated values, in per cent of experimental value.		3·32	7·08	7·61	6·29	8·4	7·69	8·91	10·17	9·32

Name of Fuel.	Combustible and volatile constituents.			Heat of combustion per lb. of combustible, by experiment.	
	Carbon.	Hydrogen.	Nitrogen and Oxygen.	Thermal units.	Lbs. of water evaporated from and at 212° Fah.
Saarbrück, Sulzbach	83·05	4·95	12·	15485	16·02
" von der Heydt	81·56	4·93	13·46	15232	15·76
Creusot, caking (Chaptal Shaft)	88·48	4·41	7·11	17320	17·92
" anthracite (St. Pierre shaft)	92·36	3·66	3·98	17021	17·61
" semi-bituminous (St. Paul)	90·07	4·1	5·13	16965	17·55
" flaming (St. Paul)	90·79	4·24	4·97	16673	17·25
Elanzy, Montceau	78·58	5·23	16·19	14985	15·5
" anthracite	87·02	4·72	8·26	16400	16·97
Anzin	84·45	4·21	11·32	16663	17·24
Denain	83·94	4·43	11·63	16290	16·85
English, Bwlf	91·08	3·83	5·09	15804	16·35
" Powell Duffryn	92·49	4·04	3·47	16108	16·66
Russian, Grouchefski anthracite	96·66	1·35	1·99	14866	15·38
" Mionski caking	91·45	4·5	4·05	15651	16·19
" Goloubofski flaming	82·67	5·07	12·26	14438	14·94
Lignite, blue, from Rochet	72·98	4·04	22·98	11669	12·07
" flaming, from Manosque	70·57	5·44	23·99	13253	13·71
" lean, from Manosque	66·31	4·85	28·84	12584	13·02
" caking, from Bohemia	76·58	8·27	15·15	14263	14·76
" changing to fossil wood	66·51	4·72	28·77	11444	11·84
Fossil wood, changing to lignite	67·6	4·55	27·85	11360	11·75
Russian lignite from Toula	73·72	6·09	20·19	13837	14·31

The data contained in the above table have been taken from the *Bulletin de la Société Industrielle de Mulhouse*, 1868, 1869, 1871; *Ann. de Chimie*, 4th Series, xxx., 5th Series, 11, xxi., 6th Series, 11; and *Comptes Rendus de l'Academie*, 2d Series, 1869.]

CHAPTER III.

STEAM-GENERATING APPARATUS.

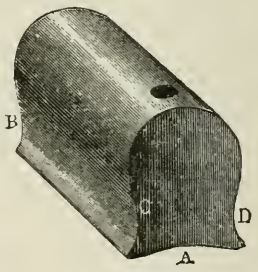
§ 402.—**Steam Boilers.**—The steam boiler (Fr. la chaudière à vapeur ; Ger. Dampfkessel) is a metallic vessel in which water is heated and converted into steam. A properly proportioned steam boiler must in a given time produce a certain quantity of steam of a certain pressure, with the least possible consumption of fuel and the greatest possible security against bursting. To satisfy these requirements, a proper material has to be selected, fixed forms and dimensions must be given to it, and the boiler must be fitted out with all the necessary auxiliary contrivances, etc. Generally strong sheet-iron plates are used for the construction of steam boilers; more rarely copper plates are used for this purpose, and only for narrow or tubular boilers is cast-iron or brass employed. The union of the plates with each other is accomplished by strong rivets (Fr. cloues ; Ger. Nietnägel) placed closely together. On account of its greater conducting power (see Vol. II., § 367), copper should be preferred to iron, but, on account of its cost, it is seldom used for steam boilers. More recently, cast-steel plates have been employed with advantage in the construction of boilers. As to the form of the boiler, it must be borne in mind that upon this depend both the durability and the capacity of evaporation. The durability or resistance of a boiler is greater the more regular and rounded its form ; on the other hand, the capacity of evaporation increases with the extent of surface of the boiler the more, therefore, it deviates from a regular and rounded

form. As, then, the requirements for the best form of boiler conflict with each other, we have always to pursue a middle course, and must make the form dependent on the pressure of the steam, selecting especially for the generation of steam of high pressure more rounded, and for the generation of steam of low pressure more angular, forms. An apparatus for the generation of steam, consisting of tubes or single boilers, is suitable in both respects; it offers a large heating surface to the fire, and gives also great security.

§ 403. **Forms of Steam Boilers.**—In regard to form, boilers can be classified as follows:

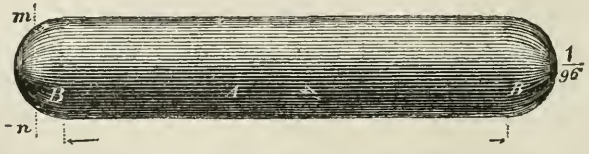
1. *Wagon or trunk boilers* (Fr. chaudière à tombeau; Ger. Wagenkessel), Fig. 615. These can be used only for low pressures (of 4 to 6 lb. gauge pressure upon the square inch), because their strength is not sufficient to stand a higher pressure. The fire passes along the bottom *A*, and then once more along the sides *B C*, *C D* . . . around the whole boiler, before entering the chimney. To prevent the bending out of the concave bottom and lateral surfaces, these parts of the boiler are stayed internally by iron rods.

Fig. 615.



2. *Cylindrical boilers with external furnace* (Fr. chaudières cylindriques à foyer extérieur; Ger. Walzenkessel mit äusserer Feuerung), Fig. 616. These are chiefly employed

Fig. 616.

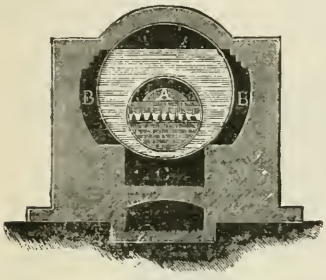


for the generation of steam of high pressure. The ends of these boilers are not plane, but are generally hemispher-

cal in shape, as shown in *B B*. The draught runs externally around the wall of the boiler the same as in the wagon boiler.

3. *Cylindrical boilers with internal furnace*, so-called "Cornish boiler" (Fr. chaudières à foyer intérieur; Ger. Walzenkessel mit innerer Feuerung), Fig. 617. Here the fireplace and grate are placed in a tube *A*, which extends through the whole boiler. These boilers, which for equal size have a greater heating surface than the other boilers, are known as Cornish boilers. The gases of combustion, after passing through the interior of the boiler, pass through side flues *B B* around the whole boiler, and also through a flue *C* beneath the boiler.

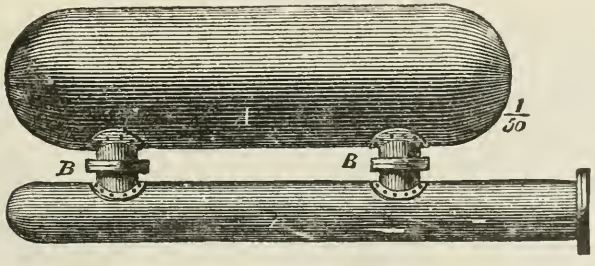
Fig. 617.



Large boilers are furnished with parallel fire tubes. In the so-called "butterfly boiler," the furnace is in front of the termination of the heating tubes, which pass through the main boiler.

4. *Boilers with boiler tubes* (Fr. bouilleurs; Ger. Kessel mit Siederöhren), Fig. 618. The boiler tube *C* lies beneath

Fig. 618.

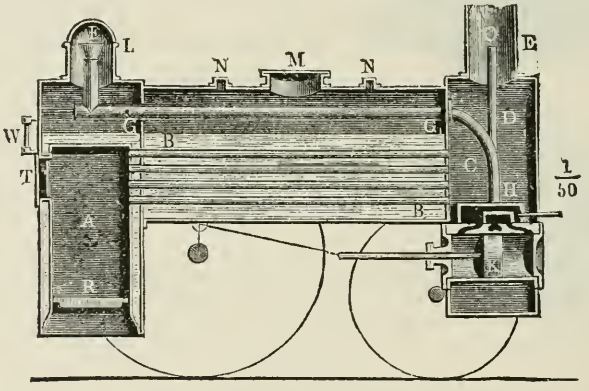


the main boiler *A*, and is connected with it by vertical tubes *B, B*. The advantage of these boilers is apparent; the main boiler does not come directly in contact with the fire, and

is therefore very well preserved; also the boiler tubes, on account of their small diameter, may have thinner walls. The use of two boiler tubes beneath the main boiler is very advantageous. Steam boilers with boiler tubes differ from those with heating tubes, in that the former have the furnace beneath the boiler tubes, but the latter beneath the main boiler; in the first, the flues lead from the boiler tubes to the main boiler, and from thence into the chimney, but in the latter the gases pass through the main boiler by means of the heating tubes, and then into the chimney.

5. *Tubular boilers* (Fr. chaudières tubulaires; Ger. Röhrenkessel), especially locomotive boilers, Fig. 619. These

Fig. 619.

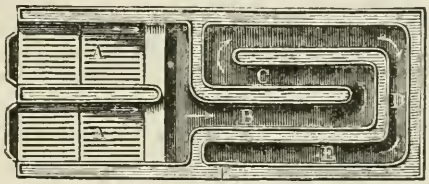


are employed when it is desired to economize space and to accelerate the generation of steam, wherefore they are specially applicable to locomotives and steamships. In the older tubular boilers of J. Barlow, the tubes were filled with water and surrounded by the hot air, but in the more recent tubular boiler of Seguin, the heating or fire tubes are conducted through the boiler. The heating tubes of locomotive boilers consist either of brass or wrought iron; they have a diameter of $1\frac{1}{2}$ to $2\frac{1}{2}$ inches, a length of 6 to 12 feet, and their number varies from 100 to 200 or more. In the figure, *A* is the fire box with grate *R* and furnace door

T; *B B* is the water space with the heating pipes, *C D* the smoke box, and *E* the chimney. The remaining parts of the figure are explained further on. In other tubular boilers—for instance, those of Zambeaux—the heating tubes are vertical.

6. *Boilers with chambers* (Fr. chaudières à galeries, Ger. Kessel mit Kammern), especially those with vertical chambers used in steamships, Fig. 620. Here the fire

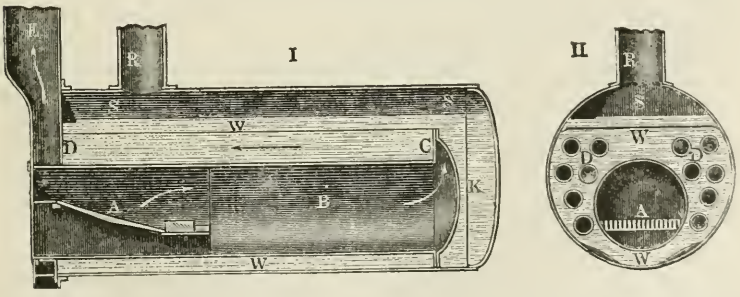
Fig. 620.



space is enclosed by water, and the products of combustion traverse a longer distance *A B C D E* before entering the chimney at *F*. These boilers are very economical, but they can only be employed for low pressures on account of their sharp corners and angles. These marine boilers have been recently much altered and improved.

7. *Compound boilers with returning flue* (Fr. chaudière composé à retour de flamme; Ger. zusammengesetzte Kessel mit rückströmender Flamme). Sections of such a

Fig. 621.



boiler are shown in Fig. 621, I, II. *A B S* is an ordinary boiler with internal fire space *A B*, and *C D* are twelve heat-

ing tubes, through which the heated air from the chamber K is conducted back to the chimney at E. The water WKW surrounds all the heating tubes, and the steam which collects in the steam space SS is conducted to the steam chamber by the tube R.

REMARK.—Besides these, there are still many other methods of constructing boilers. In Germany, the boilers of Alban and Henschel are the most noteworthy. Alban's apparatus is a steam boiler with a system of boiler tubes, which lie directly above the furnace (see Alban's paper, "Die Hochdruckmaschine"). Henschel's boiler has one or more oblique boiler tubes and a steam pipe situated horizontally above. To clean the heating tubes, it is necessary to provide tubular boilers with a removable fire-box and with a detachable cover. Such tubular boilers for stationary steam engines have been constructed by Farcot et Fils, as also by others. Steam boilers with water circulation have not yet met with general acceptance. These convert a continuous current of water almost immediately into steam, and require therefore but very little space, and have the advantage of being less dangerous and also of producing steam more rapidly. But, on the other hand, the development of steam is very irregular. To this class belong the steam boilers of Perkins, Belleville, Boutigny, etc. In the latter, the water is converted into steam during its passage through holes in plates placed upon each other. (See Morin, "Des Machines à Vapeur," Tome I.)

[Within the last few years, many engineers and inventors have turned their attention to the construction of water-tube or sectional boilers, and their efforts have been quite successful, so that, at the present time, the manufacture of sectional boilers is a large and important business.

The boiler invented by Joseph Harrison, Jr., was one of the first sectional boilers manufactured in this country. It was used in the works of William Sellers & Co., Philadelphia, in 1859, and has been regularly manufactured for the market since 1864. The construction of the boiler is thus described by a committee of the Franklin Institute:

"These boilers are of cast-iron, formed of a combination of hollow spheres, each eight inches diameter externally and three eighths of an inch thick, connected by curved necks three and one quarter inches diameter. These spheres are held together by wrought-iron bolts, and in one direction are cast in sets of two or four, with opposite lateral openings to each sphere, and are called by the inventor two- or four-ball units, as the case may be.

"He assumes that the boiler, in its smallest form, may be considered as one of these balls, with its opposite lateral openings closed by caps held in place by bolts. Two balls united by a neck, with caps over the four lateral openings, in the same manner would also make a boiler of a larger size. Four balls so united in one casting would be a still larger boiler, and that any number of these balls or spheres may be united by bolts passing through

them so as to form large boilers, and the strength of the boilers so made will be the strength of the weakest sphere or ball in the structure.

"In manufacturing the boiler for ordinary use, a number of these units are ordinarily so arranged as to form sections twelve and thirteen balls long, six balls wide. These sections are all tested by hydrostatic pressure, as high as 300 lbs. per square inch, before being delivered to purchasers. The committee saw one of these sections subjected to a bursting pressure of water, one sphere bursting when the pressure had reached 600 lbs. per square inch. A second one, tested in the same manner, burst at 625 lbs. They were shown a section in which one unit had burst at 900 lbs. per square inch, the damage having been repaired by the insertion of a new unit. The section then stood 1100 lbs. per square inch before bursting in a new place. The available strength of the section in all cases being the strength of the weakest unit in it, the inventor holds that the boiler is safer than any other in use; in fact, he considers it entirely free from any danger of disastrous explosion. To prove which, he had a section equal to six horse-power, similar to the one tested by hydrostatic pressure, and such as he is regularly selling, placed in an extemporary furnace built in a clay bank, and set in the usual manner for a boiler of this kind.

"The boiler was filled with water to the usual height, say about two thirds full, with no outlet or safety-valve of any kind, and sealed up tight, a small tube leading from the upper ball to a high-pressure gauge, placed at a safe distance, about two hundred feet, from the boiler. A fire was made under and around the boiler with the fuel of dry pine wood. The wind was very high at the time of the experiment, blowing from the west directly into the furnace, thus fanning the flames to an intense heat.

"The gauge soon gave indication of the formation of steam, the pressure steadily increasing up to four hundred and fifty pounds to the square inch.

"At this pressure there seemed to be a sudden discharge of steam, as from a small opening. The discharge did not continue for many seconds, and the committee are not certain that it proceeded from the boiler; there may have been some water discharged from the bank of wet earth into the fire. The pressure then increased at a uniform rate until it had reached the enormous strain of *eight hundred and seventy-five pounds per square inch*, when a sudden discharge of steam took place, seemingly no greater in volume than might issue from a safety-valve of two and a half inches diameter, or even less; after which the pressure fell to four hundred and fifty pounds, at which it stood when the fire was drawn for examination.

November 13th, 1866.—At four o'clock P.M. the committee met at the factory They examined the boilers tested at the former meeting. The boiler which had been subjected to its own steam-pressure of eight hundred and seventy-five pounds per square inch had been removed to the factory for examination. Mr. Harrison's foreman stated that when the boiler was first dragged from the fire, after its water had been forced out (as detailed in the account of the experiment), the three lower bolts were quite slack, but the next morning, when it had become cold, one of them was again tight.

The committee are confirmed in their belief that in this extreme test, the pressure at eight hundred and seventy-five pounds was enough to stretch some of the bolts, and thus relieved the strain on the boiler."

The above statement is taken from the "Report of the Committee on Science and the Arts, constituted by the Franklin Institute, on the Harrison Boiler," published in the *Journal of the Franklin Institute*, February, 1867. This report also contains accounts of other experiments with the Harrison boiler, quite as severe as that detailed above; and, as a result of these trials, together with the practical test afforded by the general use of the boiler, the Rumford medal was awarded to Mr. Harrison in 1871.

At present there are a large number of sectional boilers, varying considerably in detail, in successful use. Descriptions of a few representative types are appended.

The Root boiler, Fig. 621*a*, is composed of wrought-iron tubes, four inches in diameter, set in brick-work at an angle of between 20° and 30° from the horizontal. Cast-iron chambers are screwed upon the ends of the tubes. Each chamber has a small opening, and is provided with a square flange at the end. The several chambers are laid upon each other, so as to stagger the tubes, and the ends are then connected by hollow triangular caps, held in place by crow-feet, and packed with rubber grummets around the openings. There is a steam drum above the tubes connected to the upper row by short pieces of 2½-inch pipe. The water level is about two feet below the steam drum. The contraction of the passages, caused by the method of connecting the tubes, is designed to facilitate the disengagement of the steam from the water in a comparatively dry state.

FIG. 621*a*.

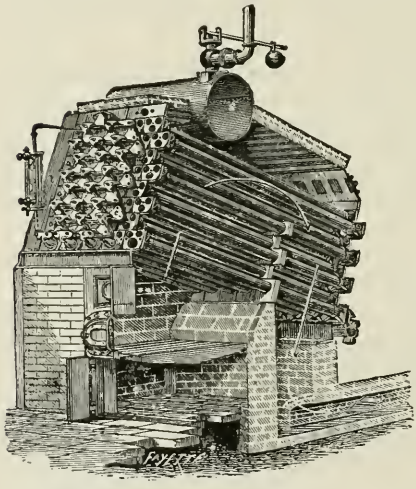
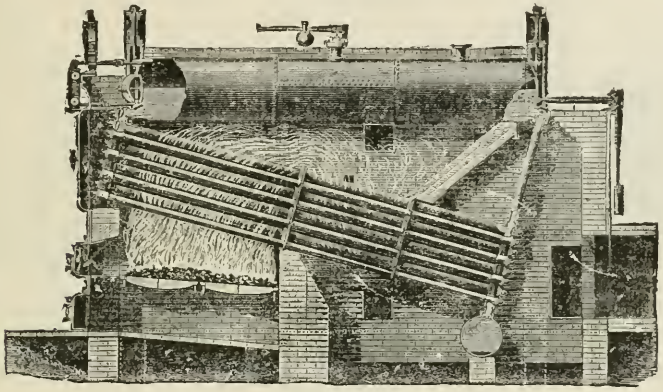


Fig. 621*b* represents the Babcock and Wilcox boiler, which is composed of inclined tubes, 3½ inches in diameter. These tubes are expanded at each end into cast-steel uptakes, this system of connection being used for all joints. There is a steam and water drum above the tubes, connected at each end, and a mud drum below the tubes, connected at their lower ends. The water level is maintained at about the middle of the upper drum. Hand-holes are provided, for purposes of cleaning, at each end of every tube, and there is a man-hole in upper drum, as well as an opening in mud drum, for the same purpose. The tubes are staggered, and there are deflecting plates

which cause the products of combustion to pass between and across the tubes three times before reaching the chimney.

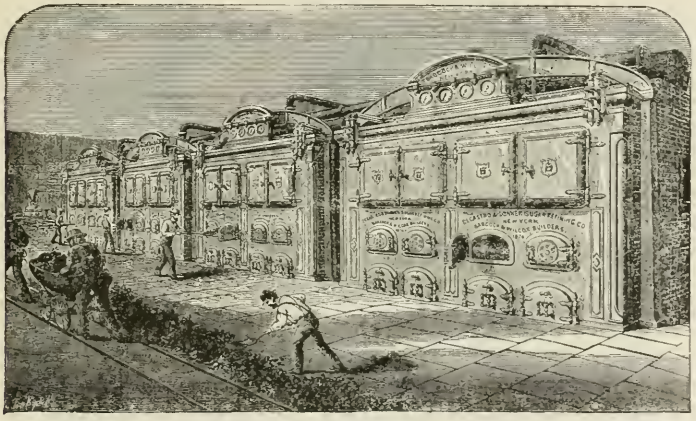
A view of a battery of these boilers, in use at a sugar refinery in Williamsburgh, N. Y., and rated at 1500 horse-power, is shown in Fig. 621*c*.

FIG. 621*b*.



The Howard boiler, Figs. 621*d* and 621*e*, an English invention, is composed of 9-inch tubes, connected together in groups, and placed at an angle with the horizontal. The tubes are connected together at both ends by caps

FIG. 621*c*.



cast upon them For an account of a trial of one of these boilers, containing many interesting details, see Van Nostrand's *Eclectic Engineering Magazine*, February and March, 1866.

In the Whittingham boiler, Fig. 621*f*, the water is contained in the narrow spaces between the two tubes, as shown in section, and the products of com-

bustion first pass around the outer tubes, and then through the inner ones, so that there is a large amount of heating surface, compactly arranged. The outer tubes are fitted at the ends in castings of a zigzag form. The inner

FIG. 621*d*.

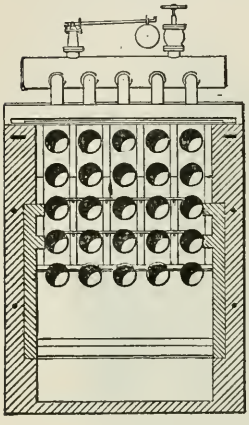


FIG. 621*e*.

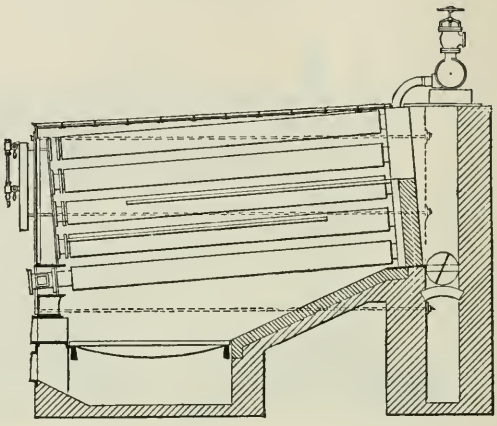
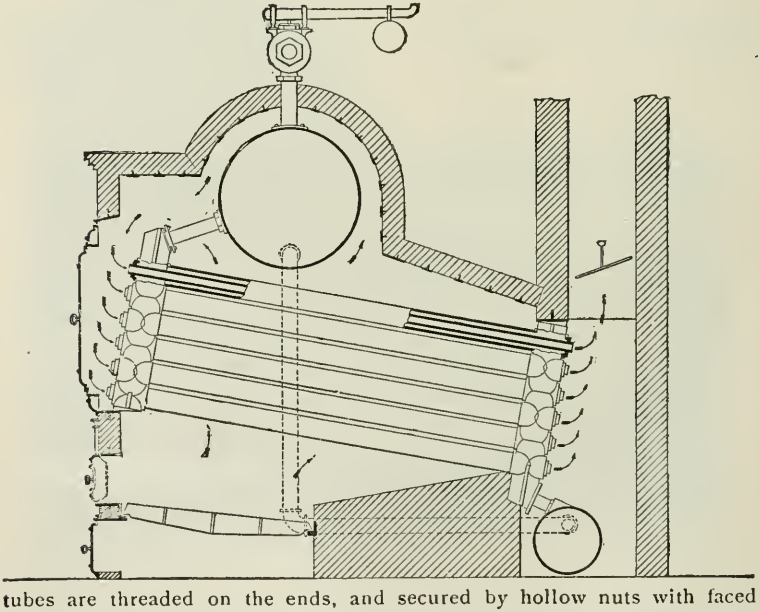


FIG. 621*f*.



tubes are threaded on the ends, and secured by hollow nuts with faced collars, which also draw the outer tubes firmly to their seats when screwed up. A vertical drum of considerable size is sometimes added, thus increasing the water room.

Name.		Total steam pressure in pounds per square inch.	Temperature, Fahrenheit degrees.					Quality of steam.		Pounds of combustible per hour.	Pounds of water evaporated per hour.	Pounds of water evaporated per pound of combustible from and at 212°.
			Air.	Fire room.	Steam.	Uptake.	Feed water.	Percentage of moisture in steam.	Number of degrees superheated.			
Wiegand	Capacity trial.	84·75	71·94	90·93	312·03	604·62	74·13	·810		613·91	4772·22	9·145
	Economy trial.	84·77	69·40	85·53	313·17	563·81	70·80		20·526	469·57	4311·74	10·834
Harrison	Capacity trial.	84 63	77·81	91·00	308·58	584·13	71·33	2·072		378·07	3170·39	9·889
	Economy trial.	84·90	75·35	95·94	310·76	517·50	71·16	·935		260·16	2410·29	10·930
Firmenich	Capacity trial.	84·77	74·71	91·17	357·91	418·00	68·92		9·067	213·18	1994·62	11·064
	Economy trial.	84·56	69·47	83·59	356·17	415·50	68·95		32·633	166·13	1685·27	11·988
Root	Capacity trial.	84·68	61·82	72·59	314·82		63·71		43·631	490·77	4315·95	10·441
	Economy trial.	84·41	72·23	88·35	312·50	393·33	64·59		41·350	341·71	3483·89	12·094
Kelly	Capacity trial.	84·73	55·31	83·91	308·91		61·26	18·566		410·32	2897·06	8·397
	Economy trial.	84·66	47·17	84·24	310·00		66·95	5·586		270·81	2358·89	10·312
Exeter	Capacity trial.	84·82	57·31	83·00	306·00	438·12	60·50	4·808		373·25	3126·91	9·974
	Economy trial.	84·80	44·81	84·06	308·00	429·94	68·93	4·165		248·59	2104·33	10·041
Babcock and Wilcox	Capacity trial.	84·62	63·25	90·13	305·44	472·81	57·60	·940		622·87	5390·81	10·330
	Economy trial.	84·75	41·00	86·81	289·06	295·82	63·98	2·670		395·57	3939·64	11·822
Anderson	Capacity trial.	84·58	56·65	56·65	320·47	534·00	54·75	·307		478·46	3826·31	9·568
	Economy trial.	84·64	55·13	55·13	322·75	417·00	54·00		15·668	318·41	2823·75	10·618

Each trial lasted eight hours, and a general summary of the most important results is contained in the table on page 151.

Efforts have been made to introduce sectional boilers into ocean steamers, but hitherto without success. An account of the experiments that have been made in this direction may be found in Chief-Engineer King's "Report on European Ships of War," Washington, 1877.]

A number of sectional boilers were tested at the Centennial Exhibition, and a summary of results, from the "General Report of the Judges o Group XX.," is appended.

The following are the principal dimensions of the boilers:

Name.	Grate surface, square feet.	Heating surface, sq. feet.			Draft area. sq. feet.	Ratio.		Water space, cubic feet.	Steam space, cubic feet.	Height of chimney above grate, feet.
		Water.	Steam.	Total.		Heating to grate surface.	Draft area to grate surface.			
Wiegand....	42·	1289·70	49·67	1339·37	5·	31·89	·119	181·36	44·18	84·75
Harrison. ...	23·	627·	274·	901·	6·54	39·17	·284	54·09	23·72	84·75
Firmenich....	15·41	1001·1	30·	1031·1	3·	66·91	·195	145·12	92·2	84·75
Root	42·	1451·77	146·66	1598·43	5·04	38·06	·120	116·68	45·69	84·75
Kelly...	27·5	575·06	60·48	635·54	4·08	23·11	·148	58·17	27·97	84·75
Exeter.....	30·	1005·06	557·94	1563·	4·3	52·1	·143	83·77	44·60	84·75
Babcock and Wilcox. ...	44·5	1676·32		1676·32	9·	37·67	·202	229·	137·85	84·75
Anderson....	36	630·	485·	1115·	11·	30·97	·300	66·9	55·75	79·

§ **404.—Heating Surface.**—The capacity of a boiler to generate steam depends chiefly upon the amount of the heating surface—*i.e.*, on that part of the surface of the steam boiler which is exposed to the action of the fire or of the heated gases before they enter the chimney. The rules for the extent of heating surface, corresponding to a certain amount of steam, are very different; according to the experiments of Cavé (see Bataille et Jullien, "Traité des machines à vapeur"), 19 kilogrammes of steam per hour can be allowed for every square meter of heating surface. In English measures, this gives 3·89 or about 4 lbs. of steam, or 110 cubic inches of water, for every square foot of heating surface.

Very often the steam-producing power of a boiler is expressed by horse-powers, or by the capacity for work of the steam produced. According to Grouvelle, we can allow for every horse-power, for high-pressure condensing steam engines, one square meter = about 10

square ft. heating surface, and for the same non-condensing, 1·3 square meter = 14 square ft.; for low-pressure steam engines, 1·4 square meter = 15 square ft. The latter statements are liable to considerable uncertainty, because an engine requires less steam the more perfect it is.

Engines, therefore, which utilize that part of the work which can be performed by the expansion of the steam, require less heating surface than engines without expansion. Moreover, the above statement relates only to stationary steam engines, for in marine boilers the quantity of steam per square meter is 30 to 35 kilogrammes, and in locomotive boilers 100 to 130 kilogrammes; giving thus, in the former case, about 6 to 7 lbs., and in the latter 21 to 26 lbs. steam on every square foot of heating surface. Also, the Cornish boiler, which has an exceedingly large heating surface, produces, according to Wicksteed's experiments, only about 1 lb. of steam per square foot. In the boiler which Dr. Brix employed in his experiments upon the heating power of the most important fuels, the heating surface was also unusually large, yet here where gases of combustion entered the chimney with a temperature of not more than 100 to 150 degrees of heat, one square foot of heating surface gave only 1·2 to 2·6 lbs. of steam.

The heating surface is of course only a portion of the whole surface of the boiler. In the wagon and cylindrical boiler, it is only about one half; in those with boiler tubes, it may rise to two thirds of the total area of all the surfaces.

Finally, it is readily seen that the generating power of a steam boiler depends also upon the thickness and upon the condition of the walls of the boiler, as also upon the position of the same with reference to the draught, and that it depends directly upon the difference of temperature between the boiler and furnace. Since the conducting power of copper (see § 367) is two and a half times as great as that of iron, this metal is especially suitable for the construction of steam boilers, the more so as its homogeneous texture ensures greater security. Its high price is therefore the only reason why sheet-iron is generally used.

Rapid combustion produces a more intense heat, and gives therefore a greater evaporative power. Thus in locomotive boilers, an artificial draught is employed.

We may distinguish also *direct* and *indirect* heating surfaces. The former is that part of the boiler surface which is situated directly above the furnace, and therefore is in contact with the flames. The remaining much more extensive part of the whole heating area is the indirect heating surface. The direct heating surface receives the heat chiefly by radiation, but the indirect heating surface only by conduction (see § 367). For equal surfaces and under similar conditions, the quantity of heat which is absorbed by the direct heating surface is about four to five times greater than that absorbed by the indirect heating surface. According to Fairbairn (see his "Useful Information for Engineers"), in well-constructed boilers the direct heating surface is $\frac{1}{11}$ of the total heating area. In the Cornish boiler, this proportion is only $\frac{1}{25}$, and in marine boilers, on the other hand, is from $\frac{1}{8}$ to $\frac{1}{6}$.

§ **405.—Water and Steam Space.**—The size of a steam boiler depends chiefly upon the extent of the heating surface, which is, in turn, determined by the quantity of steam which must be generated. Next to this, the proportion between the steam and water space exerts a great influence upon the size of the boiler. The water space should at least cover that part of the internal boiler surface which is externally exposed to the flames and the heated air in the flues, as otherwise that portion of the boiler would be overheated, and consequently weakened and liable to burst. For security, the level of the water within the boiler is generally kept about 4 inches above the heating pipes. The water space should also not be very small, as otherwise irregularities in the supply of feed water would produce considerable alterations of the temperature of the water in the boiler.

But, on the other hand, it is also necessary that the

steam space should not occupy too small a part of the boiler, lest part of the water should be carried away by the steam, and fluctuations of the steam pressure be caused. Generally, the steam space is made twelve times as large as the volume of steam taken from it per stroke. According to the Artisan Club (see their "Treatise on the Steam Engine"), for every nominal horse-power there should be allowed on the average a water space of 5 cubic ft., and a steam space of 3·2 cubic ft. The proportion of the latter to the whole capacity of the boiler is therefore $\frac{3 \cdot 2}{8 \cdot 2}$, or about 0·4. According to Tredgold, the steam space should be made so large that the variation in the pressure arising from the irregular use of the steam shall not exceed $\frac{1}{30}$. Adhering to this proportion, we can devise the following relation: Let V be the steam space, C the cylinder space, to be filled with saturated steam, at each stroke, and μ the ratio of the time of efflux to the time of an entire stroke; therefore, $1 - \mu$ the ratio of the time during which the steam is not drawn off to the time of stroke. Then the quantity of steam which collects during each stroke is

$$V_1 = (1 - \mu)\, C,$$

and taking $V_1 = \frac{1}{30}\, V$, we obtain finally,

$$V = 30\,(1 - \mu)\, C.$$

According to this, therefore, the steam space must be greater, the greater the amount of steam used per stroke, and the longer the interruption to the efflux of steam continues. This formula, moreover, applies only to simple expansion engines, where $\mu = \frac{1}{2}$ or less, but not to double-acting engines with crank action, if they work without expansion. For these latter engines, we have, according to the theory of the crank,

$$V = 30\, V_1 = 30 \times 0 \cdot 2105\, C = 6 \cdot 3\, C.$$

§ **406.—Size of Boiler.**—From the preceding, the dimensions of steam boilers may be easily calculated, especially if we are satisfied with approximate values, and if the relative dimensions are assumed.

1. For a wagon or trunk boiler, the calculation is as follows: Let the length of such a boiler be l, the mean diameter b, and the mean height h. Treating it as a parallelopipedon, we have for its capacity $= b\,h\,l$, and taking the steam space $= 0{\cdot}4$ of the total capacity, we obtain the water space $= 0{\cdot}6\ b\,h\,l$, and its mean height $= 0{\cdot}6\ h$. Assuming that the heating surface F occupies the whole lower surface up to the height $0{\cdot}6\ h$, we can put

$$F = \text{base } b\,l + \text{four lateral planes, } 2\,b \times 0{\cdot}6\,h + 2\,l \times 0{\cdot}6\,h$$

$$= b\,l + 1{\cdot}2\,(b + l)\,h.$$

Generally, $b = \frac{3}{4}\,h$ and $l = \frac{5}{2}\,h$ to $3\,h$. Taking the former proportion, we have

$$F = \tfrac{15}{8}\,h^2 + 1{\cdot}2 \times \tfrac{13}{4}\,h^2 = 5{\cdot}775\,h^2.$$

Hence the mean height of the boiler is

$$h = \sqrt{\frac{F}{5{\cdot}775}} = 0{\cdot}416\,\sqrt{F},$$

the mean diameter of the boiler is

$$b = 0{\cdot}312\,\sqrt{F},$$

and the length of the boiler is

$$l = 1{\cdot}040\,\sqrt{F}.$$

As the level of the water within the boiler must stand a few inches above the heating flues, and by the embedding of the boiler a portion of the heating surface is lost, we have to add somewhat to these dimensions, or else to take the steam space less than 0·4 of the total capacity.

2. For cylindrical boilers without boiler tubes and with external furnace, the calculation is as follows: Putting

again the steam space $= 0{\cdot}4$ of the whole capacity, then we can from a table of circular segments (see "Ingenieur," p. 218) find the angle at the centre which corresponds to the steam space $= 161^\circ\ 51'$, and, therefore, the angle at the centre which corresponds to the water space or to the fire surface $= 360^\circ - 161^\circ\ 51' = 198^\circ\ 9'$.

But now the arc which corresponds to this is for the radius 1, $= 3{\cdot}458$. Hence the cylindrical part of the heating surface, if r designates the radius and l the length of the boiler, is

$$F_1 = 3{\cdot}458\ r\ l.$$

For that part of the heating surface occupied by the segments of spheres, we can put

$$F_2 = 2 \times 0{\cdot}6\ \pi\ r^2 \left[1 + \left(\frac{h}{r}\right)^2\right],$$

if h denotes the height of each of these segments, and, therefore, the heating surface is

$$F = F_1 + F_2 = 3{\cdot}458\ r\ l + 1{\cdot}2\ \pi\ r^2 \left[1 + \left(\frac{h}{r}\right)^2\right].$$

But generally we have $l = 8\ r$ to $12\ r$; assuming $l = 10$, we obtain

$$F = 34{\cdot}58\ r^2 + 3{\cdot}80\ r^2 \left[1 + \left(\frac{h}{r}\right)^2\right] = 38{\cdot}35\ r^2 \left[1 + 0{\cdot}1 \left(\frac{h}{r}\right)^2\right].$$

Therefore the radius of the boiler is

$$r = \sqrt{\frac{F}{38{\cdot}35 \left[1 + 0{\cdot}1 \left(\frac{h}{r}\right)^2\right]}},$$

or more simply,

$$r = 0{\cdot}1615 \left[1 - 0{\cdot}05 \left(\frac{h}{r}\right)^2\right] \sqrt{F}.$$

For boilers with plain ends, $\frac{h}{r} = 0$, and for boilers with hemispherical ends, $\frac{h}{r} = 1$. For the above-mentioned reasons, we have to add something to these values for r and l, or else to make use of a smaller steam space.

3. For a cylindrical boiler with boiler tubes, we have, since in this case the latter are entirely and the former half enclosed by the heated gases,

$$F = \pi r l + 2 n \pi r_1 l_1,$$

where r and l denote the radius and the length of the main boiler, r_1 and l_1 the radius and length of the boiler tubes, and n the number of the latter. Generally

$$n = 2,\ r_1 = 0{\cdot}4\ r, \qquad \text{and} \qquad l = l_1 = 10\ r;$$

therefore,

$$F = 26\ \pi\ r^2,$$

and

$$r = \sqrt{\frac{F}{26\ \pi}} = 0{\cdot}1106\ \sqrt{F} \qquad \text{and} \qquad r_1 = 0{\cdot}04424\ \sqrt{F}.$$

In this case, however, the steam space is $= 0{\cdot}38$ of the whole capacity.

On account of the imperfect communication of heat from above, we may consider only $\frac{2}{3}$ to $\frac{5}{6}$ of the surface of the boiler tubes as effective.

4. In boilers with internal furnace, the whole inner surface is to be considered as heating area.

EXAMPLE.—Required the dimensions of a steam boiler which hourly converts 520 lbs. of water into steam. Reckoning for every square foot heating surface 4 lbs. of steam per hour, we obtain the required heating surface,

$$F = \frac{520}{4} = 130 \text{ sq. ft.}$$

For a trunk boiler, we have now the mean height $= 0{\cdot}416\ \sqrt{130} = 4{\cdot}74$ ft., the mean diameter $\frac{3}{4} \times 4{\cdot}74 = 3{\cdot}56$ ft., and the length $= \frac{5}{2} \times 4{\cdot}74 = 11{\cdot}85$ ft.

But for a cylindrical boiler, if we give to the segments the height $h = \frac{1}{2} r$, the radius

$$r = 0{\cdot}152\ (1 - 0{\cdot}0125)\ \sqrt{130} = 0{\cdot}150 \times 11{\cdot}4 = 1{\cdot}71 \text{ ft.}$$

Thus the diameter of the boiler = 3·42 ft., and the length of the middle portion = 17·1 ft., while the whole length of the boiler is 1·71 + 17·1 = 18·81 ft.

For a cylindrical boiler with two boiler tubes, the radius

$$r = 0{\cdot}1106\ \sqrt{130} = 1{\cdot}26 \text{ ft.},$$

therefore the diameter = 2·52, and, on the other hand, the diameter of a boiler tube = 0·4 × 2·52 = 1·008 ft.; consequently the length of the main boiler and that of the boiler tubes = 12·6 ft. By reason of the mounting, etc., these dimensions must be somewhat increased.

§ **407.—Thickness of the Boiler Sides.**—The thickness of the boiler is very important. We have already in Vol. I., § 363, deduced the relation between the thickness e, the diameter $2r$, and the pressure p, in the case of pipes, and can now apply the formula,

$$e = \frac{r\,p}{T},$$

found there to steam boilers or pipes. In such case, we may use instead of r the diameter $d = 2r$, instead of p the excess of pressure from within outwards in atmospheres, and for T the modulus of proof strength of the boiler plate, and add still a member e_1, which represents the thickness which the boiler sides must have in order to resist the effect of its own weight and contained water. According to the latest experiments of Fairbairn (see "Civilingenieur," Bd. 4), the modulus of rupture for wrought-iron is only at red heat noticeably diminished (compare also § 359), and for steam boilers, therefore, we may insert for T in the formula the modulus of proof strength as determined for ordinary temperatures.

In France, the thickness of boilers is by law

$$e = 1{\cdot}8\ p\ d + 3 \text{ millimeters},$$

where d must be given in meters.

In Prussia, the thickness prescribed by law is

$$e = (2 \cdot 71828^{0 \cdot 003 p} - 1) r + 0 \cdot 1 \text{ inch},$$

or approximately, and for ordinary cases sufficiently accurate,

$$e = 0 \cdot 0015 p d + 0 \cdot 1 \text{ inch},$$

where d is to be expressed in inches, and p in atmospheres.

To those parts of the boiler which come directly in contact with the fire, a greater thickness is often given.

Cast-iron boiler tubes, according to French practice, must be five times as thick as wrought-iron or copper tubes; according to Prussian practice, the thickness of such tubes is to be determined by the formula,

$$e = (2 \cdot 71828^{0 \cdot 01 p} - 1) r + \tfrac{1}{3} \text{ inch};$$

or, from the approximate expression,

$$e = 0 \cdot 005 p d + \tfrac{1}{3} \text{ inch}.$$

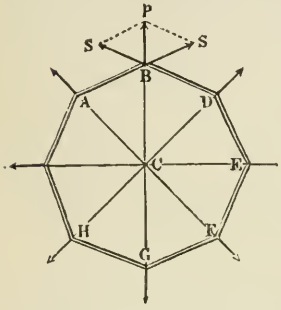
To facilitate the communication of heat through the boiler sides, and to avoid too great irregularity in the tension of the boiler plate, the thickness is not made over half an inch, and therefore rather narrower and longer, or two or more boilers instead of one may be employed. According to the French regulations, the thickness of the boiler must not exceed $1\frac{1}{2}$ centimeters (six tenths of an inch).

Steam boilers, before use, should undergo a hydrostatic test. According to Prussian regulations, a steam boiler is tested up to one and a half, and, according to French regulations, up to three times the pressure to which it is to be exposed when used.

§ **408.**—The formula $e = \frac{p r}{T}$, given in Vol. I., § 363, for the thickness of a steam boiler, can, if we consider the cross-section as a regular polygon, $A B D E \ldots$, Fig. 622, be developed as follows: Let us assume that at every

edge of the prismatic boiler a force P acts radially outward. Let us then decompose this force in the directions of the sides BA and BD, and indicating the central angles $BCA = BCD$ opposite to these sides by α, we obtain the tension of the sides

FIG. 622.



$$S = \frac{P}{2 \sin. \frac{\alpha}{2}},$$

or, when the number of sides is very great, and therefore α very small,

$$S = \frac{P}{\alpha}.$$

If p indicates the excess of the internal air, steam, or water pressure per square inch above the external pressure, l the length of boiler, and s a polygon side $\overline{AB} = \overline{BD}$, we have

$$P = l s p.$$

But now

$$s = 2 r \sin. \frac{\alpha}{2},$$

where r indicates the radius of the boiler $\overline{CA} = \overline{CB}$, therefore we have also

$$P = 2 r l \sin. \frac{\alpha}{2} p$$

and

$$S = r l p.$$

This tension must be sustained by the cross-section $l e$ of the boiler wall, consequently the tension which is to be placed equal to the modulus of proof strength T, is per square inch,

$$T = \frac{S}{l e} = \frac{r l p}{l e} = \frac{r p}{e},$$

and therefore, inversely, the required thickness of the wall is

$$e = \frac{r\,p}{T}.$$

In this formula, r indicates really the mean radius of the boiler, but if we understand by r the internal radius, we must put instead of r,

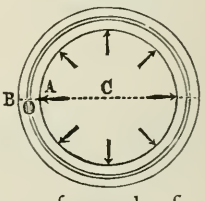
$$r + \frac{e}{2} = r\left(1 + \frac{e}{2\,r}\right),$$

and consequently,

$$e = \frac{p\,r}{T}\left(1 + \frac{e}{2\,r}\right) = \frac{p\,r}{T}\left(1 + \frac{p}{2\,T}\right).$$

§ **409.—Thick Boiler Sides.**—The formula $e = \frac{p\,r}{T}$ is only applicable to plate boilers, where $\frac{e}{r}$ is but a small number. For great thickness of the plate, this formula is not sufficient. Assuming that the thickness of the plate $\overline{A\,B} = e$, Fig. 623, is not changed by the expansion of the sides, in consequence of the internal pressure p per unit of surface, then we must also assume that hereby all the concentric rings, into which we can divide the boiler wall, are equally enlarged or expanded. If λ is this expansion, and x the radius $\overline{C\,O}$ of such a ring, and $d\,x$ the thickness of the same, we have, according to the known formulæ for elasticity (see Vol. I., § 204), the tension of this ring,

FIG. 623.



$$dS = \frac{\lambda}{2\,\pi\,x}\,l\,d\,x \cdot E = \frac{\lambda\,l\,E}{2\,\pi} \cdot \frac{d\,x}{x},$$

and consequently the tension of the entire boiler wall is

$$S = \frac{\lambda\,l\,E}{2\,\pi}\int\frac{d\,x}{x} = \frac{\lambda\,l\,E}{2\,\pi}\,log.\,nat.\left(\frac{x}{r}\right).$$

As the innermost ring of the radius $C\,A = r$ expands proportionally the most, and consequently is exposed to

the greatest tension, we put its tension per unit of the surface equal to the modulus of proof strength, thus,

$$\frac{\lambda}{2\pi r} E = T,$$

so that

$$S = l r T \log. nat. \left(\frac{x}{r}\right),$$

or, as $S = r l p$,

$$p = T \log nat. \left(\frac{x}{r}\right),$$

or inversely,

$$\frac{x}{r} = (2{\cdot}718 \ldots)^{\frac{p}{T}}.$$

If, finally, we take for the external radius of the boiler,

$$x = \overline{CB} = r + c.$$

we have

$$\frac{r+c}{r} = (2{\cdot}718 \ldots)^{\frac{p}{T}},$$

and therefore the thickness of the boiler is

$$c = r\left((2{\cdot}718 \ldots)^{\frac{p}{T}} - 1\right),$$

or approximately,

$$c = r\left[\frac{p}{T} + \tfrac{1}{2}\left(\frac{p}{T}\right)^2\right] = \frac{r p}{T}\left(1 + \frac{p}{2T}\right).$$

Lamé (see his "Traité de l'Elasticité") and Rankine ("Manual of Applied Mechanics") find, in an entirely different way,

$$c = r\left(\sqrt{\frac{T+p}{T-p}} - 1\right),$$

according to which, if T is much larger than p,

$$\frac{T+p}{T-p} = 1 + \frac{2p}{T} + 2\left(\frac{p}{T}\right)^2,$$

hence

$$\sqrt{\frac{T+p}{T-p}} = 1 + \frac{p}{T} + \tfrac{1}{2}\left(\frac{p}{T}\right)^2,$$

and, therefore,

$$e = \frac{r p}{T}\left(1 + \frac{p}{2T}\right).$$

According to E. Winkler's treatise upon the strength of tubes in the "Civilingenieur," Vol. 6, we have approximately for open tubes,

$$e = \frac{r p}{T}\left(1 + \tfrac{5}{6}\frac{p}{T}\right),$$

and, on the other hand, for tubes which are closed at their ends,

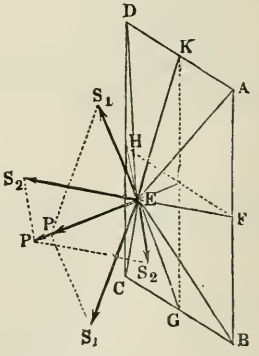
$$e = \tfrac{7}{8}\frac{r p}{T}\left(1 + \frac{91}{112}\frac{p}{T}\right).$$

According to D. F. Grashof ("Festigkeitslehre," Berlin, 1866), we have approximately,

$$e = \frac{r p}{T}\left(1 + \tfrac{3}{2}\frac{p}{T}\right).$$

§ **410.—End Surfaces.**—Cylindrical steam boilers are bounded on their ends by hemispheres, or by spherical or spheroidal segments, and the question arises as to what thickness is to be given to these parts of the boiler. Suppose the boiler in form a polyhedron, and assume that it is limited by three-sided plane surfaces, which unite to make a corner with four edges, as shown at E, Fig. 624; suppose, further, this corner situated above a rectangular base $A\,B\,C\,D$, whose side $A\,B = C\,D = s_1$, and whose side $A\,D = B\,C = s_2$, and denote again the pressure per unit of surface by p. Then we have for the entire corner the pressure,

Fig. 624.



$$P = s_1 s_2 p.$$

This force can be decomposed into two parts P_1 and P_2, of which the one balances the tensions S_1, S_1, of the surfaces $A\,D\,E$ and $B\,C\,E$, and the other the tensions S_2, S_2, of the surfaces $A\,B\,E$ and $C\,D\,E$. Hence

$$P_1 = \alpha_1 S_1 \quad \text{and} \quad P_2 = \alpha_2 S_2,$$

if α_1 and α_2 indicate the arcs corresponding to the angles α_1° and α_2°, which are the complements of the inclinations of the surfaces $A\,D\,E$ and $B\,C\,E$, as also $A\,B\,E$ and $C\,D\,E$—*i. e.*, of the angles $G\,E\,K$ and $F\,E\,H$ between the lines of altitude of these surfaces. Therefore,

$$P = P_1 + P_2, \quad \text{or} \quad s_1 s_2 p = \alpha_1 S_1 + \alpha_2 S_2.$$

Indicating further the radii of the circles through G, E and K, and through F, E, and H, by r_1 and r_2, we have

$$\alpha_1 = \frac{s_1}{r_1} \quad \text{and} \quad \alpha_2 = \frac{s_2}{r_2};$$

hence

$$s_1 s_2 p = \frac{s_1 S_1}{r_1} + \frac{s_2 S_2}{r_2}.$$

If, now, S is the tension per unit of surface, we can put the tension S_1, which for the thickness e_1 corresponds to the width $\overline{B\,C} = \overline{A\,D} = s_2$, $= e_1 s_2 S$, and the pressure S_2, which corresponds to the width $\overline{A\,B} = \overline{C\,D}$, $= e_1 s_1 S$, and thus obtain

$$s_1 s_2 p = \frac{e_1 s_1 s_2 S}{r_1} + \frac{e_1 s_1 s_2 S}{r_2},$$

whence

$$p = e_1 S \left(\frac{1}{r_1} + \frac{1}{r_2}\right),$$

or, if we insert instead of S the modulus of proof strength T,

$$p = e_1 T \left(\frac{1}{r_1} + \frac{1}{r_2}\right),$$

and therefore the desired thickness of the wall is

$$e_1 = \frac{p}{T\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}.$$

This formula can be applied to every form of boilers, if we only insert for r_1 the greatest and for r_2 the smallest radius of curvature of that portion of the boiler wall whose thickness (e_1) is given by this formula. Applying this formula to the ends of a cylindrical boiler, and assuming that the ends are formed by spheroids of the height h, we have for the thickness of such an end, since here each of the radii of curvature (see 'Ingenieur," p. 238) is $= \frac{r^2}{h}$,

$$e_1 = \frac{p}{T\left(\frac{h}{r^2} + \frac{h}{r^2}\right)} = \frac{p\,r^2}{2\,T\,h} = \frac{r}{2\,h} \cdot \frac{p\,r}{T}.$$

For hemispherical ends $h = r$, therefore $e_1 = \frac{1}{2}\,e$ (compare Vol. I., § 363); if, on the contrary, the thickness of the plates of the end segments were the same as for the intermediate cylindrical portion, then $e_1 = e$ and $h = \frac{r}{2}$, *i.e.*, we can take half the radius of the boiler for the height h. Compare the treatise of Lamé in the "Comptes rendus de l'Académie des Sciences," Tome 30, or the "Polytechnische Centralblatt," 1850, No. 19.

Example 1.—A cylindrical steam boiler with hemispherical ends, 4 ft. diameter and 22 ft. length, is to be constructed for a steam pressure of four atmospheres; required the thickness. The formula,

$$e = 0{\cdot}0015\,p\,d + 0{\cdot}1 \text{ inch},$$

gives, if we insert

$$p = 4 - 1 = 3 \quad \text{and} \quad d = 4 \times 12 = 48 \text{ inches},$$

the desired thickness of the boiler,

$$e = 0{\cdot}0015 \times 3 \times 48 + 0{\cdot}1 = 0{\cdot}316 \text{ inches}.$$

According to the above, the hemispherical ends could be only half as thick as the cylindrical part of the boiler, but for the sake of greater ease of con-

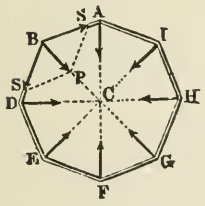
struction, and on account of the weakening by the bending, the thickness generally is not altered.

2. What thickness is to be given to a trunk boiler, 6 ft. in height, 4½ ft. in diameter, and 18 ft. long? Here we have to introduce instead of d the greatest diameter, which may be 7 ft. or 84 inches. Assuming the excess of the internal pressure over the external to be ¼ atmosphere, we obtain the desired strength of the wall,

$$e = 0{\cdot}0015 \times \tfrac{1}{4} \times 84 + 0{\cdot}1 = 0{\cdot}1315 \text{ inches.}$$

§ 411.—Fire Tubes.—We have still to consider the thickness of the fire tubes which allow the gases of combustion to pass through the boiler and are compressed externally by the steam. Let us first consider a boiler with polygonal cross-section $A\,E\,G$, Fig. 625, and assume that at each corner A, B, D . . . a force P acts from without inwardly. If we decompose this in the direction of the neighboring sides, we then obtain as above, in § 408, the force of compression for each side,

FIG. 625.



$$S = \frac{P}{2\ sin.\ \frac{\alpha}{2}},$$

or if we assume the central angle $\alpha^0 = A\,C\,B = B\,C\,D$ . . ., as very small, we have

$$S = \frac{P}{\alpha}.$$

But the pressure P at each corner, or rather upon every side, is $= \alpha\,r\,l\,p$, hence $S = r\,l\,p$. Any two of the forces S, S . . . compress the intervening portion, hence

$$S = e\,l\,T, \quad \text{or} \quad r\,l\,p = e\,l\,T,$$

and therefore the desired thickness of boiler is

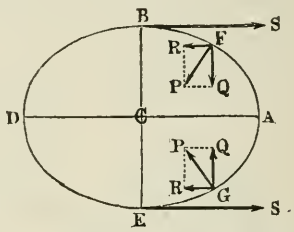
$$e = \frac{r\,p}{T}.$$

As the modulus of rupture for wrought-iron is almost twice as great for tearing as for crushing (see Vol. I.,

Table § 212), it follows that, with equal pressure and equal size, a boiler pressed from without must be twice as thick as one pressed from within. This remains correct so long as the boilers are perfectly round; but according to numerous observations (see "Traité des machines à vapeur, par Bataille, etc."), tubes under external pressure are easily flattened if they are not round, and it is therefore requisite to give to such tubes an accurate cylindrical form.

In order to investigate the phenomena of flattening of tubes, let us suppose a boiler with elliptical cross-section $A\,B\,D\,E$, Fig. 626, and determine the moments of the forces upon a quadrant of the same. The total pressure which acts upon this ellipse can be decomposed in two directions. If the major semi-axis $\overline{C\,A} = a$, the conjugate semi-axis $\overline{C\,B} = b$, the length of the boiler $= l$, and the pressure per square inch $= p$, then from known hydrostatic principles (see Vol. I., § 360), we have the force on $A\,B$ in the direction $B\,C = a\,l\,p$, and that in the direction $A\,C = b\,l\,p$. The forces upon the remaining three quadrants are the same. If we take, therefore, A as the point of support, we have to consider the following forces on $A\,B$: First, the force $S = b\,l\,p$, with the lever arm $\overline{C\,B} = b$ arising from the pressure on $B\,D$; second, the sum $a\,l\,p$ of the forces Q which act in the direction $B\,C$; and, third, the sum $b\,l\,p$ of the forces $R\ldots$, which act in the directions $A\,C$. The first sum consists of the components,

FIG. 626.



$$\frac{a\,l}{n}p,\quad \frac{a\,l}{n}p\ldots\frac{a\,l}{n}p,$$

with the lever arms

$$\frac{a}{n},\quad \frac{2\,a}{n}\ldots\frac{n\,a}{n},$$

$$\int p\,l\,x\,dx$$

and therefore has the moment,

$$\frac{a\,l}{n}\,p\left(\frac{a}{n}+\frac{2\,a}{n}+\cdot\cdot\cdot+\frac{n\,a}{n}\right)=\frac{a^2\,l\,p}{n^2}(1+2+3+\cdot\cdot\cdot+n),$$

or as the number of the components is to be taken as endless, the moment is

$$\frac{a^2\,l\,p}{n^2}\times\frac{n^2}{2}=\tfrac{1}{2}\,a^2\,l\,p.$$

For similar reasons, the second sum of forces has the moment $\frac{1}{2}\,b^2\,l\,p$. But now the moment $b^2\,l\,p$ of S acts contrary to the latter moment; the resultant moment for $A\,B$ is therefore

$$M=\frac{a^2\,l\,p}{2}+\frac{b^2\,l\,p}{2}-b^2\,l\,p=\frac{(a^2-b^2)\,l\,p}{2}.$$

If, now, e denotes the thickness of the boiler, then, in order that the latter shall sufficiently resist rupture at A, we must have

$$l\,e^2\,T=\tfrac{1}{2}\,(a^2-b^2)\,l\,p,$$

and hence

$$e=\sqrt{\left(\frac{a^2-b^2}{2\,T}\right)p}.$$

If the cross-section of the boiler is exactly spherical, then $b=a$, hence $e=0$, consequently a breakage does not then take place.

If the same boiler is pressed from within outwardly, then the moment of bending or rupture, and also the necessary thickness of the boiler, will indeed be just as great, but there is an essential difference, in the fact that a pressure from without will deform the boiler still more, and a pressure from within will bring the same more closely to the cylindrical form. If, then, by the connection of the tubes, a certain tension of the plates has been caused, this will be further augmented by the outer pressure and diminished by the pressure from within; hence, in the first

case, the tension may be brought nearer to bursting, and, in the second case, removed further from it.

According to experiments recently made by Fairbairn, the necessary thickness of tubes exposed to external pressure is also dependent upon the lengths l of these tubes, and may be taken approximately $e = \mu \sqrt{p\,d\,l}$, where μ indicates an empirical number dependent on the modulus of proof strength T (see "Civilingenieur," Vol. IV., p. 53).

According to Rankine, we have $p = 659720 \frac{e^2}{l\,d}$ atmospheres, whence $e = 0{\cdot}0012312 \sqrt{p\,l\,d}$ inches. Professor Grashof derives from these experiments the empirical formula, $p = 71917 \frac{e^{2{\cdot}081}}{l^{0{\cdot}564}\,d^{0{\cdot}889}}$ atmospheres, in which d indicates the diameter of the tube, l the length and e the thickness of the tube expressed in inches.

In France, tubes exposed to exterior pressure are made twice as thick as those subjected to interior pressure under otherwise similar conditions. But according to the regulations in Prussia, for heating pipes of sheet-iron, the thickness

$$e = 0{\cdot}0067\, d \sqrt[3]{p} + 0{\cdot}05 \text{ inches},$$

and for pipes of sheet-brass, but which must not be over 4 inches wide,

$$e = 0{\cdot}01\, d \sqrt[3]{p} + 0{\cdot}07 \text{ inches}.$$

EXAMPLE.—What thickness must we give to a 2-inch locomotive fire tube in order that it shall sustain an exterior pressure of 5 atmospheres? Putting $d = 2$ and $p = 5 - 1 = 4$, we obtain, according to Prussian regulations, the desired thickness for sheet-iron or copper,

$$e = 0{\cdot}0067 \times 2 \sqrt[3]{4} + 0{\cdot}05 = 0{\cdot}021 + 0{\cdot}05 = 0{\cdot}071 \text{ inches},$$

and for sheet-brass,

$$e = 0{\cdot}01 \times 2 \sqrt[3]{4} + 0{\cdot}07 = 0{\cdot}102 \text{ inches}.$$

If we insert in the formula, $e = \sqrt{\frac{(a^2 - b^2)\,p}{2\,T}}$, $a = 1$ and $b = 0{\cdot}9$ inches, furthermore, $p = 4 \times 14{\cdot}7 = 58{\cdot}8$ and $T = 6000$, we obtain

$$e = \sqrt{\frac{0{\cdot}19 \times 58{\cdot}8}{12000}} = \sqrt{0{\cdot}000895} = 0{\cdot}0299 \text{ inches}.$$

§ 412.—**Plane Boiler Walls.**—Simple plane boilers can with equal thickness stand much less pressure than when curved, and, therefore, they are only employed for low pressures, and for greater tensions, are stayed or strengthened with triangular gussets (Fr. goussets; Gr. Blechzwickel), as in *A B*, Fig. 627.

The theory of flexure for loaded plane plates leads to very complicated expressions, the development of which cannot be attempted here (see Navier's "Mechanik der Baukunst," § 641, etc.). but in the following manner we may attain, at least some data, for the estimation of the requisite thickness of such plates: Let *A B C D*, Fig. 628, be a rectangular sheet plate of the length $A\,B = l$ and width $A\,D = b$, which is enclosed by a frame or by a row of rivets, and which has to sustain a pressure p per unit of surface. Let us imagine this sheet to be cut into strips, whose ends are held fast in *A D* and *B C*, and let us assume that of the pressure p upon this plane surface, the portion p_1 causes the tension of these strips; then if we also indicate the breadth of such a strip by s and the thickness of the sheet by e, we can put (see Vol. I., § 246),

Fig. 627.

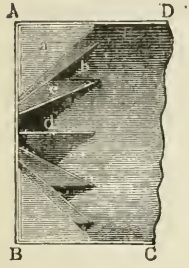
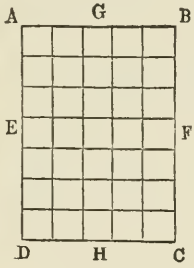


Fig. 628.



$$l\,s\,p_1 = \frac{12\,W\,T}{\frac{1}{2}\,l\,e} = 12\,\frac{s\,e^2}{l}\,\frac{T}{6} = 2\,\frac{s\,e^2}{l}\,T,$$

or

$$= l^2\,p_1 = 2\,e^2\,T,$$

and hence

$$e = l\,\sqrt{\frac{p_1}{2\,T}}.$$

If, on the other hand, we imagine the sheet cut up into lateral strips such as *G H*, and the ends held fast at *A B*

and CD, and assume that the tension of these strips requires the portion p_2 of the pressure $p = p_1 + p_2$, we find in the same manner,

$$e = b\sqrt{\frac{p_2}{2T}}.$$

As the deflection in the first case increases as $\frac{l^4 s p_1}{W} = \frac{l^4 s p_1}{s e^3} = \frac{l^4 p_1}{e^3}$, and in the other case as $\frac{b^4 p_2}{e^3}$ (see Vol. I., § 223), and as it is as great in the one case as in the other, we can put

$$l^4 p_1 = b^4 p_2,$$

and hence

$$p_2 = \frac{l^4}{b^4} p_1 \qquad \text{and} \qquad p_1\left(1 + \frac{l^4}{b^4}\right) = p,$$

or,

$$p_1 (l^4 + b^4) = b^4 p;$$

consequently

$$p_1 = \frac{b^4 p}{l^4 + b^4},$$

and the thickness of the plate is

$$e = b\sqrt{\frac{l^2 b^2}{l^4 + b^4} \cdot \frac{p}{2T}}.$$

Under the second supposition,

$$e = l\sqrt{\frac{l^2 b^2}{l^4 + b^4} \cdot \frac{p}{2T}}.$$

If, now, $b > l$, then we must find the thickness of the sheet according to the formula,

$$e = b\sqrt{\frac{l^2 b^2}{l^4 + b^4} \cdot \frac{p}{2T}}.$$

For square sheets, we have $l = b$, and, therefore,

$$e = b \sqrt{\frac{p}{4\,T}} = \frac{b}{2} \sqrt{\frac{p}{T}}.$$

In cylindrical boilers, the thickness of the wall is

$$e = \frac{d\,p}{2\,T} = 0{\cdot}0015\,p\,d + 0{\cdot}1 \text{ inch},$$

hence

$$\frac{1}{2\,T} = 0{\cdot}0015\,;$$

if, then, we insert this value for $\frac{1}{2\,T}$, we have for the thickness of plane boiler walls,

$$e = b \sqrt{\frac{l^2\,b^2}{l^4 + b^4} \cdot \frac{p}{2\,T}} = 0{\cdot}0387\,b \sqrt{\frac{l^2\,b^4}{l^4 + b^4} \cdot p},$$

and for $l = b$,

$$e = 0{\cdot}03\,b \sqrt{p} \text{ inches}.$$

EXAMPLE.—Plane plates which have to sustain a resultant pressure ($p = \frac{1}{4}$) of $\frac{1}{4}$ atmosphere, must, according to the above formula, have the thickness

$$e = 0{\cdot}0387\,b \sqrt{\frac{l^2\,b^2}{4\,(l^4 + b^4)}} = 0{\cdot}01935\,b \sqrt{\frac{l^2\,b^2}{l^3 + b^4}} \text{ inches},$$

and if the breadth of such a plate were $b = 72$ and the length $l = 60$ inches, therefore $\frac{b}{l} = \frac{72}{60} = \frac{6}{5}$, then the thickness would have to be

$$e = 0{\cdot}01935 \times 72 \sqrt{\frac{6^2 \times 5^2}{6^4 + 5^4}} = \frac{1{\cdot}393 \times 6 \times 5}{\sqrt{3921}} = \frac{41{\cdot}79}{62{\cdot}62} = \tfrac{2}{3} \text{ inches}.$$

§ 413.—**Stay Bolts.**—Plane boiler surfaces occur in locomotive and marine boilers. As these steam boilers produce steam of high pressure, stays, etc., are indispensable. Such plane surfaces are presented by the fire boxes (see § 403) of locomotives. To apply the heat engendered in such a space as much as possible to the production of steam, the fire box is composed of two plates, and the space between commu-

nicates with the water space of the boiler. The water filling this interspace now presses with the same force p as the steam above, upon the sides of the fire box, and, therefore, these must be united with each other by stays (Fr. entretoises; Ger. Stehbolzen). The inner or fire box proper consists generally of sheet copper of the thickness of $\frac{3}{8}$ of an inch, while the outer or water box is formed of sheet iron. The interspace has a width of from 3 to 4 inches, and the iron or copper stay bolts are 4 to 5 inches apart, and have a mean thickness of $\frac{3}{4}$ of an inch. According to the experiments of Fairbairn (see "Useful Information for Engineers"), the strength of iron plates with iron stays is about twice as great as that of copper plates with copper stays, and also that of bolts with heads, as $A\ A$, Fig. 629, is $\frac{1}{4}$ greater than that of simple screws.

Fig. 629.

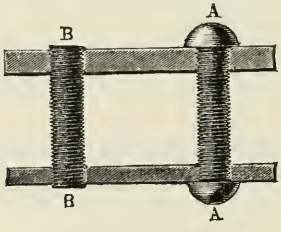
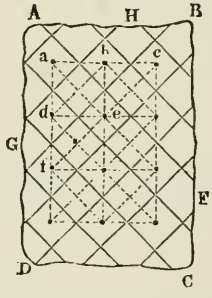


Fig. 630.



If we suppose the sheet $A\ B\ C\ D$, Fig. 630, supported by stays a, b, c . . . cut up into strips as $A\ F$ and $G\ H$, parallel to the diagonals $a\ c$ and $b\ d$ of the squares formed by any four stays, then we can directly apply the formula developed in the preceding paragraph,

$$e = \frac{b}{2}\sqrt{\frac{p}{T}},$$

to the determination of the necessary thickness of the sheet, if we only insert, instead of b, the diagonals of the square $b = a\sqrt{2}$, whose lateral length a is the distance between any two consecutive stays.

Accordingly, we have

$$e = \frac{a}{2}\sqrt{\frac{2p}{T}} = a\sqrt{\frac{p}{2T}},$$

and, therefore, for $\frac{1}{2T} = 0{\cdot}0015$,

$$e = 0{\cdot}0387\, a\sqrt{p} \text{ inches.}$$

This expression accords perfectly with the formula found by Brix (see "Die Verhandlungen des Vereins zur Beförderung des Gewerbfleisses in Preussen," 1849). The thickness of the inner plate facing the fire may be increased one fourth.

The thickness of a stay which has to sustain the pressure $a^2 p$ upon the square a^2 of its lateral length is determined by the equation,

$$\frac{\pi d^2}{4} T = a^2 p,$$

which gives

$$d = a\sqrt{\frac{4p}{\pi T}}.$$

Inserting here also $\frac{1}{2T} = 0{\cdot}0015$, we obtain

$$d = \sqrt{\frac{0{\cdot}012}{\pi}}\, a\sqrt{p} = 0{\cdot}0619\, a\sqrt{p}.$$

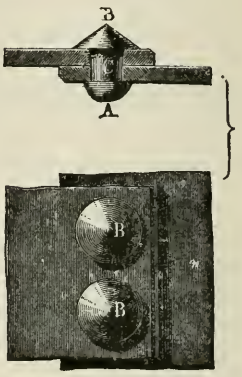
According to Brix, we have

$$d = 0{\cdot}069\, a\sqrt{p} + 0{\cdot}125 \text{ inches.}$$

The cover of the fire box consists of a plane plate and receives the necessary support from iron supporting rods, the thickness of which is to be calculated according to the known formulæ for relative strength (see Vol. I., § 240, etc.).

§ **414.—Rivet Connections.**—Plane and curved boiler-plate rivet connections are shown in cross-section and plan in Fig. 631. If e is again the thickness of the sheet, then the rivet bolt C receives the thickness

Fig. 631.



$$d = 2\,e,$$

the hemispherical head A the diameter

$$d_1 = 3\,e,$$

the conical head B the diameter

$$d_2 = 4\,e,$$

and the height

$$h_2 = \tfrac{5}{2}\,e,$$

so that the bolt necessary for the formation of the rivet must receive the length

$$l_2 = 2\,e.$$

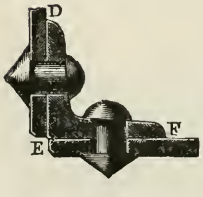
Furthermore, the distance of the axes of two bolts from each other is

$$a = 5\,e,$$

and the distance of these axes from the edges of the plate

$$a_1 = 3\,e.$$

Fig. 632.



The angular union of two sheets is accomplished by the angle iron DEF, Fig. 632, with two rows of rivets. The mean thickness of this angular sheet is equal to the thickness e of the plates to be united, but in the centre it is taken $\frac{1}{7}$ larger, and at the ends $\frac{1}{7}$ smaller than e. The width $\overline{ED} = \overline{EF}$ of each leg is taken = 1 inch + 4·5 e.

§ 415.—**The Furnace.**—Every steam boiler has also a furnace, which consists:

1. Of the fireplace or furnace proper (Fr. foyer; Ger. Herd).

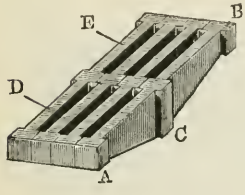
2. Of the flues (Fr. carneaux; Ger. Zügen, Feuerkanälen).

3. Of the chimney (Fr. cheminée; Ger. Esse or Schornstein).

In the first, the combustion of the fuel takes place; by the second, the products of combustion—fire, smoke, etc.—are conducted over the heating surface of the boiler, so as to impart their heat to the same; and, by the third, these products are expelled into the external air.

Considering first the fireplace, this is divided by the grate (Fr. la grille; Ger. Rost) into two compartments, the upper of which forms the real hearth, while the lower one serves for the reception of the ashes and other solid residues of combustion, and is therefore called the ashpit (Fr. le cendrier; Ger. Aschenraum). The grate is formed of iron bars with narrow slits between, which widen downwards and allow the air to enter and the ashes to fall through. These slits have a breadth of about $\frac{1}{2}$ inch for hard coal, but for wood or peat only $\frac{1}{5}$ of an inch, and in the first case occupy $\frac{1}{4}$, but in the second $\frac{1}{6}$ of the whole grate surface. In Fig. 633, we have represented a few grate bars. *A B C* is the foremost grate bar, and *D* and *E* are the slits between each pair.

Fig. 633.



In small boiler constructions, grates with movable bars are sometimes employed with advantage, where the grate bars terminate cylindrically, and are so placed that, by a simple mechanism, they can be turned to and fro, and may thus be easily kept clean.

The extent of the grate surface is of great importance. According to Cavé, it should be $\frac{1}{17}$ of the heating surface of the boiler. Moreover, we may calculate on an hourly consumption of about 14 lbs. of hard coal or 73 lbs. of wood

for each square foot of grate surface. In locomotive boilers where an artificial draught of air takes place, and where coke is burned, the conditions are altogether different. Here the grate surface is only from $\frac{1}{50}$ to $\frac{1}{60}$ of the heating surface. For the combustion of hard coal, the grate surface should be from 13 to 18 inches beneath the heating surface, and for wood fires from 18 to 24 inches. The ashpit beneath the grate should be at least $2\frac{1}{2}$ ft. in depth, so that the bars of the grate shall not be very greatly heated by the accumulated residue.

The air necessary for combustion passes by a door into the ashpit, and from there through the grate bars into the furnace. To regulate the draught of air, a special register may be applied, and, to heighten the same, the air may be brought on by a subterranean passage.

The fireplace above the hearth is provided with a door, which is only opened when it is required to stir the fire, to cleanse the grate, or to supply fresh fuel. To prevent cooling at the furnace door as much as possible, and to protect the door itself from the fire, it is well to provide it with double walls or to cover it inside with bricks.

§ 416.—**Combustion of Smoke.**—The smoke proceeding from combustion consists of unburned particles of carbon, and therefore occurs only during incomplete combustion and consequent loss of heat. For this reason, we should strive to attain a combustion as free from smoke as is possible. A great deal can here be at once effected by proper attention and maintenance of the fire, particularly in taking care that the fuel is not added in too great quantity and that it is spread as evenly as possible on the grate, and so served that the smoke formed by the addition of fresh fuel must pass over the fuel already in full combustion. Of course, it is here especially necessary that a sufficient quantity of atmospheric air shall be supplied to the furnace, and that sufficient chance shall be given to it to spread over all the fuel and to come in contact with the gases of combustion.

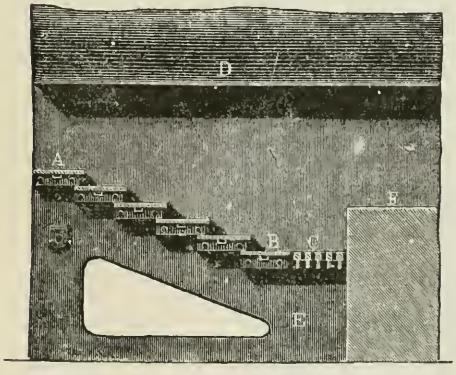
Double hearths are the best means for the production of a smokeless combustion.

Such a hearth is longitudinally divided by a partition into two equal parts, which nevertheless communicate with one and the same flue or fire canal. Now, if the fuel is alternately supplied to the one or the other compartment, then the gases of combustion loaded with smoke which arise from the freshly added fuel, and the gases which proceed from the fuels in process of complete combustion, and which are still mixed with atmospheric air, flow in common into and through the flues, and thereby produce thorough combustion.

As means for the consumption of smoke, special air channels, which terminate directly behind the fire bridge, have been employed. The atmospheric air supplied by these channels commingles with the gases of combustion directly upon their entrance to the flues, so that the smoke is completely burned. According to Fairbairn, by the use of these canals, if the cross-section of the same is $\frac{1}{11\frac{1}{3}}$ of the grate surface, the saving of fuel is at least $12\frac{1}{2}$ per cent. These air channels, however, have not been always found to be beneficial.

Another means for the combustion of smoke consists in the application of a grate with steps (Fr. grilles à gradins; Ger. Treppenrost). This is distinguished from the common grate in that the grate bars are replaced by iron plates of about eight inches width, which are placed above one another in steps at distances of from $1\frac{1}{2}$ to $2\frac{1}{2}$ inches from each other, and interlock about 2 inches over one another. The arrangement of such a furnace with

Fig. 634.



step grate is shown in Fig. 634. *A B* is the grate with steps consisting of six plates, *C* a short grate with bars, *D* the steam boiler, *E* the ashpit, and *F* the fire bridge.

Grates with steps are preferably used for fires of peat, brown coal, and the poorer sorts of hard or pit coal, where it is required to facilitate the ingress of atmospheric air. Instead of these, common inclined grates are also often employed.

§ **417.—Flues.**—In order that the fire may come in contact with the boiler, but especially that a more complete combustion may be effected through a more intimate contact with the air, it is necessary to construct a fire bridge (Fr. autel; Ger. Feuerbrücke) at the point of passage from the furnace into the flue, which only leaves an intermediate space of from 4 to 6 inches between it and the bottom of the boiler. The narrowing of the flue by the fire bridge answers the purpose of bringing the gases of combustion into closer contact with the inflowing air, and thereby effects a more complete combustion.

As to the fire tubes or flues, these consist either of a single channel passing once or several times about the boiler, or they may consist of one or more single channels or pipes, each of which separately conveys the smoke into the chimney. The latter kind of fire tubes are used only with interior furnaces, and especially with locomotive boilers. What these tubes lack in length is made up by the extent of the cross-section. Supposing, for instance, a single flue with spherical cross-section of the length l and the diameter d replaced by n flues in juxtaposition, each of the length l_1 and diameter d_1, we have the following equations:

$$\pi d l = n \pi d_1 l_1 \qquad \text{and} \qquad \frac{\pi d^2}{4} = \frac{n \pi d_1^2}{4},$$

and accordingly we obtain

$$d_1 = \frac{d}{\sqrt{n}} \qquad \text{and} \qquad l_1 = \frac{l}{\sqrt{n}}.$$

For instance, for $n = 64$,

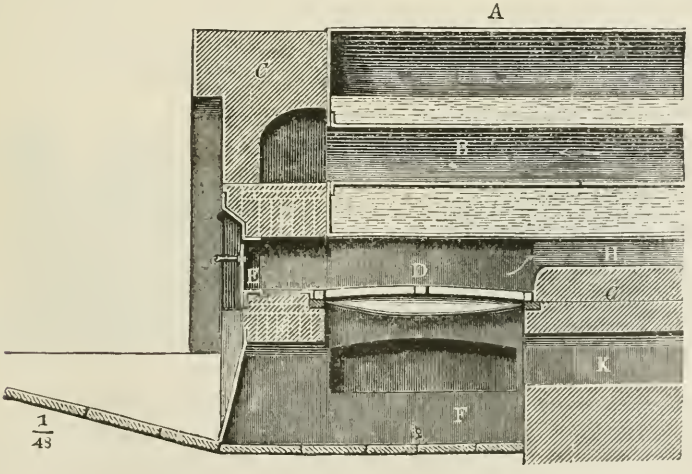
$$d_1 = \frac{d}{8} \qquad \text{and} \qquad l_1 = \frac{l}{8};$$

64 tubes, therefore, can be made eight times shorter and eight times narrower than a single flue.

Flues of the first kind consist of sheet pipes (compare § 403–5), but those of the second kind are constructed of fire-proof stones, and have a more or less rectangular cross-section, one side of which is bounded by the boiler. It is an empirical rule to make this cross-section $\frac{1}{4}$ to $\frac{1}{6}$ the grate area. The length of the flues must, moreover, not be too great, in any case not more than 90 ft. Generally it is considered satisfactory if the gases which enter the chimney do not retain more than from 250° to 300° of heat. At the extremity of the whole flue in the so-called smoke box between the boiler and the chimney, another door or damper (Fr. registre; Ger. Schieber) is used for the purpose of regulating the fire, and so as to be able to completely close the furnace. Besides this, the whole construction should be surrounded by a strong wall.

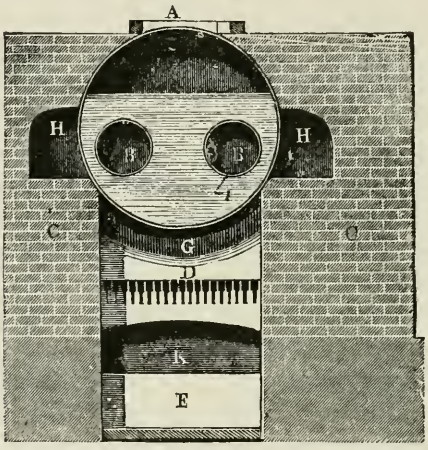
§ 418.—**Construction of Boiler.**—The principal arrange-

Fig. 635.



ment of a boiler construction with external furnace is shown in Fig. 635 in longitudinal, and in Fig. 636 in cross-section. Here *A* is the steam boiler with two flues *B B;* further, *C* is the brickwork, *D* the grate, *E* the furnace door, *F* the ash-pit, *G* that part of the flue through which the heated gases

Fig. 636.



pass beneath the boiler, and *H H* are the flues through which these gases pass along the side of the boiler after having returned by the flues *B B.* The atmospheric air passes through the air channel *K* from behind, but can also, as Fig. 635 shows, flow in from the sides.

A suitable construction for a boiler with double furnaces, according to Fairbairn, is shown in Figs. 637 to 640, in longitudinal, horizontal, and cross-section, and in front view. The steam boiler *A D* contains two internal flues *B C* and B_1 C_1, each with a separate furnace. These flues terminate at *E*, in the rear, in a common flue *E F G H*, which conducts the gases of combustion once exteriorly around the boiler, and then at *H* into the chimney. To attain a more perfect combustion, every fire bridge *K* is provided with an opening *a b*, which introduces heated air from the ashpit into the space directly behind the fire bridge; for this purpose, also,

one or the other furnace is alternately fed, to allow the smoke which is formed by the fresh supply of coal to be consumed at its entrance into the flue *E*.

Fig. 637.

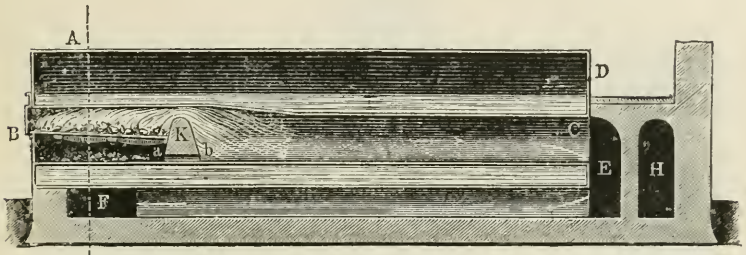


Fig. 638.

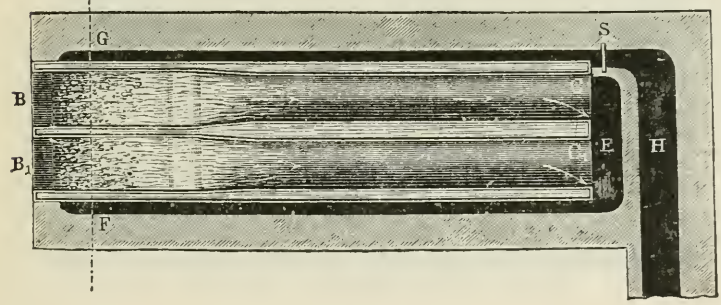


Fig. 639.

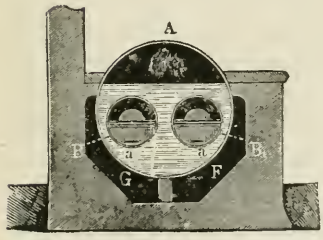
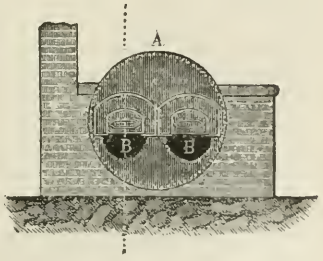


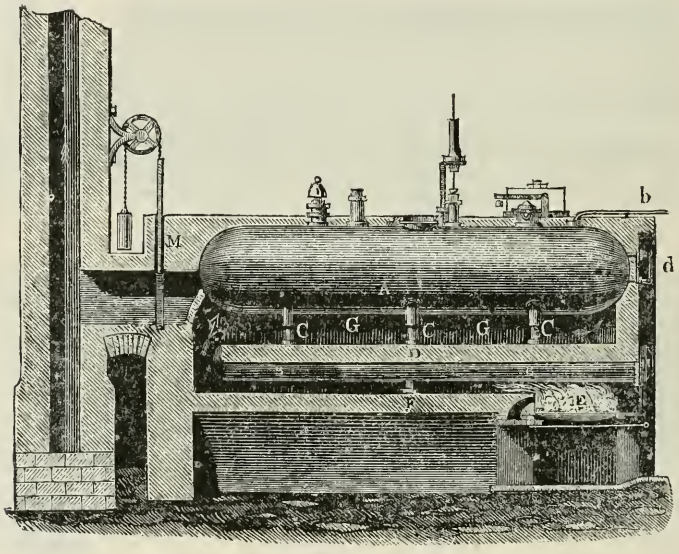
Fig. 640.



A boiler construction with boiler tubes is also shown in Fig. 641. Here *A* is the steam boiler, separated from the boiler tubes *B B* by a vault *D*, and the latter are wholly ex-

posed to the influence of the rearward flowing gases which come directly from the furnace, while the former is heated by the gases as they return through the flues *G G*, which may, if required, pass around the whole boiler.

Fig. 641.

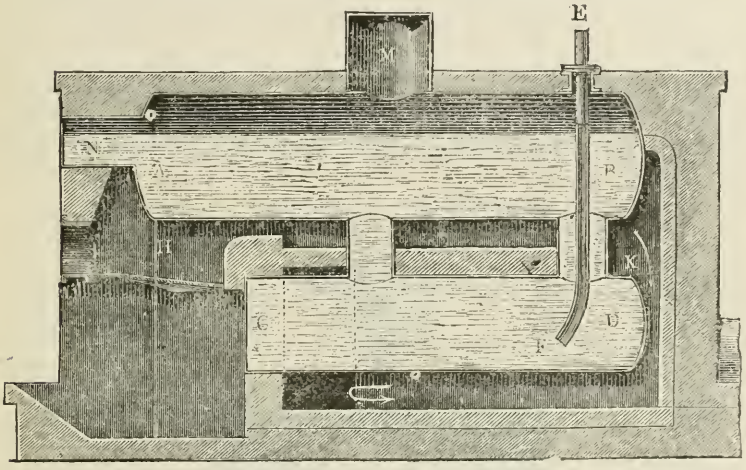


Between a steam boiler with boiler tubes and one with heating tubes, there is this difference: that in the first the furnace is situated beneath the tubes, and in the second beneath the boiler; consequently, in the first case the gases flow from the tubes to the boiler, and in the second they first heat the boiler and then the tubes.

To impart as much of the heat of the gases within the flues as possible to the steam generator, it is necessary that these gases should enter the chimney at that point where there is the least heat, where therefore the introduction of the feeding water and the motion of the water in the boiler begins; for this reason, steam boilers with heating tubes are to be preferred to those with boiler tubes. This principle is already shown in the Fairbairn boiler, Figs. 637 and 638.

A special form of boiler with boiler tube is shown in Fig. 642. Here *A B* is the steam boiler, *C D* the boiler

Fig. 642.



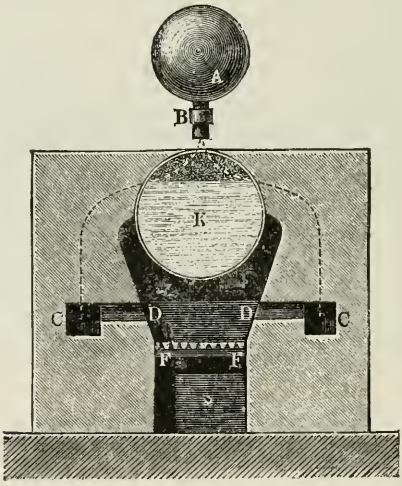
tube, and *E F* the feed pipe. The heated gases move first from the furnace *H* by way of *H K* beneath the boiler, then sink beneath the level of the boiler tube *C D*, and pass around the same before entering the chimney.

§ 419.—**Heating by Gas.**—Sometimes combustible gases are employed as heating materials. These gases can be burned either in a closed space and allowed to act directly on the piston of a special engine, or they can be burned in an ordinary furnace. The gases which serve as fuels are carbonic oxide gas, illuminating gas, blast-furnace gas, and the gas from puddling furnaces. Carbonic oxide gas is generated, like illuminating gas, in closed vessels, but blast-furnace gas is derived from the top of blast furnaces. The gas derived from the puddling furnace contains only a small quantity of carbonic oxide gas, and acts therefore chiefly by its own heat, while blast-furnace gas contains 2 per cent of hydrogen gas, as also 13 per cent of

carbonic oxide. While 1 lb. of good pit or hard coal as well as pure carbon furnish, by complete combustion, nearly 8000 heat-units, 1 lb. of carbonic oxide gas gives only 2400 heat-units, and 1 lb. of blast-furnace gas only 900 heat-units, while illuminating gas gives nearly 10,000 heat-units.

The arrangement of a furnace for the generation of steam by means of blast-furnace gases is shown in Fig. 643. The gas from the furnace top is first collected in the

Fig. 643.



reservoir *A*, then conducted through the branch tubes *B C*, *B C*, into the canals, *C*, *C*, and from these through a row of side canals as *C D*, *C D* into the furnace. The steam boiler *K* has its lower half surrounded by the gas, the combustion of which is maintained by a thin layer of coal spread over the grate *F F*. The boilers for the use of the puddling-furnace flame consist generally of a vertical tube, on the outer surface of which the gas flame plays; sometimes also horizontal tubular boilers similar to those of locomotives are employed.

§ **420.—Chimneys.**—The current of air necessary for combustion is caused chiefly by the chimney, which therefore is an important component part of the furnace. It is chiefly necessary that a sufficient height and diameter be given to the chimney, and that an appropriate material be chosen for the same. If the chimney cannot be made high enough, then the necessary draught must be produced by special means. In locomotives, the exhaust steam is therefore allowed to escape into the stack, and, in other cases, ventilators or blowers are employed, which either blow the air beneath the grate or suck it out of the flues.

Chimneys are constructed of stone or of metal. In the first case, bricks are preferably used, and in the second sheet-iron is employed. The external form of brick chimneys is generally quadrangular or octangular, while metal chimneys have always the shape of a truncated cone.

Usually an exterior batter of 0·015 to 0·025 per foot height is given to chimneys, while the walls have the usual width of the bricks (6 inches) above, and at the bottom they have a thickness of double or three times this width.

As to the height and diameter of chimneys, the one dimension depends upon the other. The higher a chimney is built, the more draught it gives, and the smaller therefore its diameter needs to be, for the removal of a given quantity of smoke. Besides this, the dimensions also depend upon the temperature of the smoke which enters the chimney, and for an equal quantity of smoke the dimensions must be so much larger the less the temperature of the smoke which is to be removed. According to this, an economical use of heat requires high and wide chimneys. The usual height of chimneys is from 60 to 120 ft.—we rarely find them 40 ft. or less. Chimneys of 300 or 400 ft. height are rarely constructed. It is a practical rule to give the chimney the same cross-section as the flues. In the following article, a special rule for finding the diameter of chimneys will be given. It is very necessary to place chimneys upon a solid base, as the least sinking may cause damage or even destruction. An exterior view and cross-

section of an octagonal chimney of bricks is given in Figs. 644 and 645, and an exterior view of a metal chimney is shown in Fig. 646. In the first, *A* is the foundation, *B* the

Fig. 644.

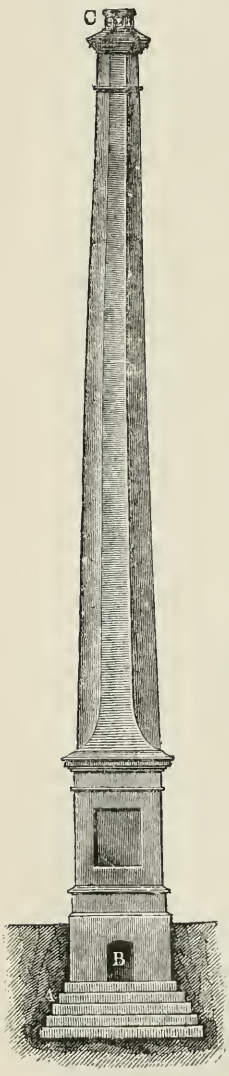


Fig. 645.

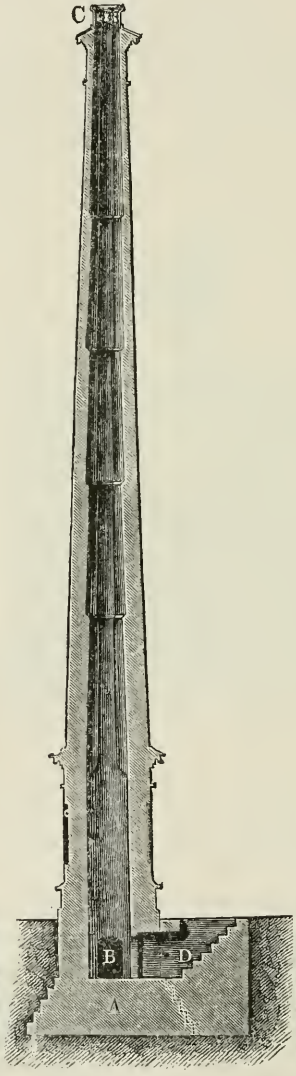
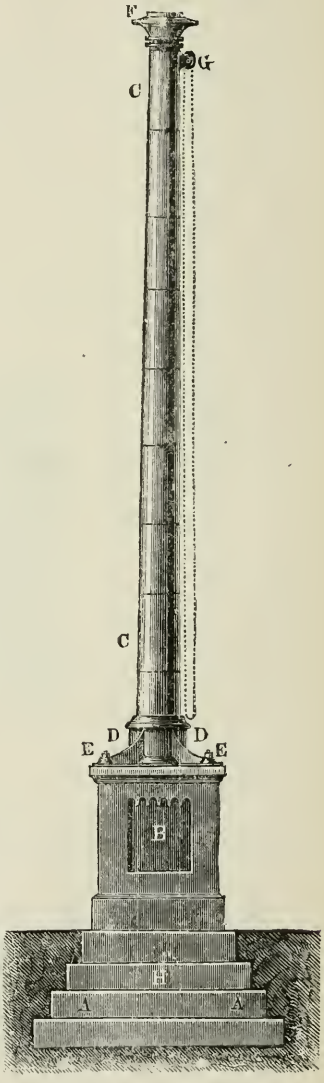


Fig. 646.



termination of the flues or smoke box, *C* the cast-iron cap of the chimney, and *D* a staircase which leads to an opening for cleaning the flues and chimney. To prevent the current of smoke from meeting resistance at its entrance into the chimney, the union between the flue and smoke box must be rounded off.

In Fig. 646, *A* represents the foundation, constructed of bricks and resting upon a solid base; *D*, *D* are anchor screws which connect the base of the chimney, by means of a plate *E E*, firmly with the foundation, and *G* is a pulley fixed below the head of the chimney *F*, over which passes a chain, by means of which a man can be pulled up for the purpose of cleaning and painting the chimney. We also see at *B* the termination of the smoke box, and at *H* the opening for cleaning. To prevent the overturning of such a chimney in a storm, not infrequently wires, stays, or cables are drawn in an inclined direction from the chimney to the ground and anchored.

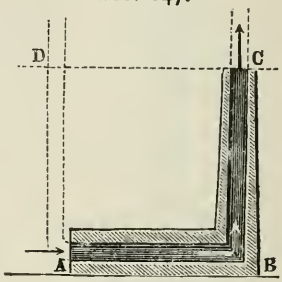
REMARK.—The famous chimney at St. Rollox, near Glasgow, of the height of 455½ ft., has the following dimensions (see "Verhandl. des Preuss. Gewerbevereins," 1845):

Division of the Chimney.	Height above Ground.	Outer Diameter in Feet.	Thickness of the Wall in Feet and Inches.
V.	435½	13½	1 2
IV.	350½	16¾	1 6
III.	210½	24	1 10½
II.	114½	30½	2 3
I.	54½	35	2 7½
	0	40	

The foundation of this chimney has a depth of 20 ft. and a diameter of 50 ft.

§ **421.—Theory of Chimney Draught.**—The theory of the motion of smoke in chimneys may be easily deduced from the laws of hydraulics, given in Vol. I., the more so since, on account of the very slight difference between the tension of the air within the chimney and without, the laws regulating the discharge of water may be directly applied. If γ is the density of the outer air, and h the height $A\,D$ of a chimney $A\,B\,C$, Fig. 647, together with its flue, the excess of pressure at A above that at C is $q = h\gamma$. This excess of pressure opposes the pressure q_1 of the heated air or smoke column. If, then, we denote the density of this column by γ_1, we have for the pressure causing the velocity v of the smoke,

FIG. 647.



$$q - q_1 = h\gamma - h\gamma_1 = h(\gamma - \gamma_1),$$

and, therefore, without reference to resistances,

$$\frac{v^2}{2g}\gamma_1 = h(\gamma - \gamma_1) \text{ or } v = \sqrt{\frac{2gh(\gamma - \gamma_1)}{\gamma_1}} \text{ (see Vol. I., § 399).}$$

If, now, t is the mean outer and t_1 the mean inner temperature, or that of the smoke, we have from Vol. I., § 393,

$$\gamma = \frac{0{\cdot}005494\,p}{1 + 0{\cdot}00367\,t} \quad \text{and} \quad \gamma_1 = \frac{0{\cdot}005494\,p_1}{1 + 0{\cdot}00367\,t_1},$$

hence

$$\frac{\gamma}{\gamma_1} = \frac{1 + 0{\cdot}00367\,t_1}{1 + 0{\cdot}00367\,t} \cdot \frac{p}{p_1};$$

or, since the tensions p and p_1 of the outer and inner air cannot be very different, on account of the moderate velocity of the smoke, we have approximately,

$$\frac{\gamma}{\gamma_1} = \frac{1 + 0{\cdot}00367\,t_1}{1 + 0{\cdot}00367\,t},$$

and hence the smoke velocity at its exit is

$$v' = \sqrt{2gh\left(\frac{1 + 0{\cdot}00367\,t_1}{1 + 0{\cdot}00367\,t} - 1\right)} = \sqrt{\frac{0{\cdot}00367\,(t_1 - t)}{1 + 0{\cdot}00367\,t} \cdot 2gh},$$

or approximately,

$$v = \sqrt{0{\cdot}00367\,(t_1 - t) \cdot 2gh} = 0{\cdot}485\,\sqrt{(t_1 - t)\,h} \text{ ft.}$$

This velocity will, indeed, be considerably diminished by the resistances due to the contraction of the fire space, friction in the chimney, etc. These resistances are to be estimated in entire accordance with the rules of hydraulics. From the height h and breadth d of the chimney, we have, according to Vol. I., § 466, for the loss of head by reason of friction,

$$h_1 = \zeta \cdot \frac{h}{d} \cdot \frac{v^2}{2g}.$$

Although, according to the above article, $\zeta = 0{\cdot}024$, yet, for the sake of safety, we may take, in accordance with the observations of Péclet, for a chimney coated with soot,

$$\zeta = 0{\cdot}025 \times 1{\cdot}962 = 0{\cdot}049 \quad \text{or} \quad 0{\cdot}05.$$

For the remaining losses of head, arising from friction in the flues, grate, etc., we may, according to Péclet, take $\zeta_1 = 30$, and hence

$$\zeta_1 = \frac{v^2}{2g} = 0{\cdot}00367\,(t_1 - t)\,h - 0{\cdot}05\,\frac{h}{d}\frac{v^2}{2g} - 30\,\frac{v^2}{2g},$$

or

$$\frac{v^2}{2g}\left(30 + 0{\cdot}05\,\frac{h}{d}\right) = 0{\cdot}00367\,(t_1 - t)\,h.$$

If, finally, we suppose that the density of the air in the chimney is about 1·044 times that of the outside air, we have

$$v = \sqrt{\frac{0{\cdot}00367\,(t_1 - t)\,2\,g\,h}{1{\cdot}044\left(30 + 0{\cdot}05\,\frac{h}{d}\right)}} = 0{\cdot}0595\,\sqrt{\frac{(t_1 - t)\,2\,g\,h}{30 + 0{\cdot}05\,\frac{h}{d}}}$$

$$= 0{\cdot}477\,\sqrt{\frac{(t_1 - t)\,h\,d}{30\,d + 0{\cdot}05\,h}} \text{ ft.}$$

§ **422.—Dimensions of Chimneys.**—It is now easy, by the aid of the foregoing formulæ, to determine the cross-section S and the dimensions of a chimney which shall afford passage to a quantity Q of air or smoke per second. Thus,

$$Q = S\,v = 0{\cdot}47\,S\,\sqrt{\frac{(t_1 - t)\,h\,d}{30\,d + 0{\cdot}05\,h}} \text{ cubic ft.,}$$

and, therefore, the cross-section is

$$S = \frac{Q}{v} = 2{\cdot}13\,Q\,\sqrt{\frac{30\,d + 0{\cdot}05\,h}{(t_1 - t)\,h\,d}} \text{ square ft.}$$

For a chimney with circular cross-section, $S = \frac{\pi\,d^2}{4}$, and hence

$$d^{\frac{5}{2}} = 2{\cdot}13 \cdot \frac{4}{\pi}\,Q\,\sqrt{\frac{30\,d + 0{\cdot}05\,h}{(t_1 - t)\,h}},$$

or

$$d = 1{\cdot}49\,\sqrt{\frac{30\,d + 0{\cdot}05\,h}{(t_1 - t)\,h}}\,Q^2 \text{ ft.}$$

For a chimney with square cross-section, $S = b^2$, and hence

$$b = 1{\cdot}353\,\sqrt[5]{\frac{30\,d + 0{\cdot}05\,h}{(t_1 - t)\,h} \cdot Q^2}.$$

If we take $h = 100\,d$, we have

$$v = 0{\cdot}08\sqrt{(t_1 - t)h} \text{ ft.} = 0{\cdot}044\sqrt{(t_1 - t)h} \text{ meters,}$$

and

$$S = \frac{Q}{v} = \frac{12{\cdot}5\,Q}{\sqrt{(t_1 - t)h}} \text{ square ft.,}$$

hence

$$d = \frac{4\sqrt{Q}}{\sqrt[4]{(t_1 - t)h}} \quad \text{or} \quad b = \frac{3{\cdot}54\sqrt{Q}}{\sqrt[4]{(t_1 - t)h}} \text{ ft.}$$

The quantity of smoke $Q = S\,v = 0{\cdot}08\,S\sqrt{(t_1 - t)h}$, reduced to the outer temperature t, becomes

$$Q_1 = \left(\frac{1 + \delta t}{1 + \delta t_1}\right) S\,v,$$

or approximately, since t_1 is much greater than t,

$$Q_1 = \frac{S\,v}{1 + \delta t_1} = 0{\cdot}08\,S\sqrt{\frac{(t_1 - t)h}{(1 + \delta t_1)^2}}$$

$$= 0{\cdot}08\,S\sqrt{\frac{t_1 h}{(1 + 0{\cdot}00367\,t_1)^2}},$$

and is a maximum when $\frac{t_1}{(1 + 0{\cdot}00367\,t_1)^2}$ is a maximum. This is a maximum for $1 + 0{\cdot}00367\,t_1 = 2 \times 0{\cdot}00367\,t_1$, or $0{\cdot}00367\,t_1 = 1$; *i. e.*, when $t_1 = \frac{1}{0{\cdot}00367} = 273°$. If we take approximately $t_1 - t = 270°$, we have

$$v = 1{\cdot}32\sqrt{h} \text{ ft.} \quad \text{and} \quad S = \frac{0{\cdot}76\,Q}{\sqrt{h}} \text{ square ft.}$$

But the quantity of air Q can be easily found from the heating surface F as well as from the weight K of the fuel. (See § 400.)

If K is the carbon consumed per hour, and if we assume that every pound of carbon requires 600 cubic ft. of air, then

$$Q = \frac{600\,K}{60 \times 60} = \frac{K}{6},$$

and we obtain

$$S = 0{\cdot}128\,\frac{K}{\sqrt{h}} \text{ square ft.},$$

and

$$h = 0{\cdot}0164\left(\frac{K}{S}\right)^2 \text{ ft.}$$

For $\frac{K}{S} = 75$, we should have, therefore, for the height of chimney,

$$h = 0{\cdot}0164 \times (75)^2 = 92{\cdot}2 \text{ ft.}$$

The usual heights of chimneys are, in fact, from 60 to 120 ft. From the conditions of stability, we can determine the height as follows: If the velocity of the wind is c, and γ is its density, further, if h is the height and b the mean outer breadth of chimney, then the pressure of the wind is

$$P = 3\,\frac{c^2}{2\,g}\,b\,h\,\gamma \text{ (§ 344)},$$

and the moment of this force with reference to the foot of chimney is

$$\frac{P\,h}{2} = 3\,\frac{c^2}{2\,g} \cdot \frac{b\,h^2}{2}\,\gamma.$$

If, now, e is the mean thickness of the walls, and γ_1 the density of the masonry, we have for the weight of the chimney,

$$G = 4\,(b - e)\,e\,h\,\gamma_1,$$

and for its moment,

$$\frac{G\,b}{2} = \frac{4\,(b - e)\,e\,h\,b\,\gamma_1}{2} = 2\left(1 - \frac{e}{b}\right)e\,h\,b^2\,\gamma_1.$$

Placing these two moments equal, we have

$$3\frac{c^2}{2g}\frac{b\,h^2}{2}\gamma = 2\left(1-\frac{e}{b}\right)e\,h\,b^2\,\gamma,$$

and hence for the ratio of the height of chimney to its mean breadth,

$$\frac{h}{b} = \tfrac{4}{3}\left(1-\frac{e}{b}\right)\frac{2ge}{c^2}\cdot\frac{\gamma_1}{\gamma}.$$

This formula holds good only for chimneys with square cross-section. For a circular cross-section, we can increase $\frac{h}{b}$ by one half, or take

$$\frac{h}{b} = 2\left(1-\frac{e}{b}\right)\frac{2ge}{c^2}\cdot\frac{\gamma_1}{\gamma}.$$

For an octagonal chimney, we may take the mean of the preceding, or

$$\frac{h}{b} = \tfrac{5}{3}\left(1-\frac{e}{b}\right)\frac{2ge}{c^2}\cdot\frac{\gamma_1}{\gamma}.$$

EXAMPLE 1.—What breadth must a chimney have which is 100 ft. high, and discharges the smoke arising from the combustion of 120 lbs. of hard coal per hour? From what has preceded, we may assume that for the combustion of 120 lbs. of hard coal, with a mean temperature of 300° in the chimney, $120 \times 584 = 70080$ cubic ft. of air are required. We have then, per second,

$$Q = \frac{70080}{60\times 60} = 19\tfrac{1}{2} \text{ cubic ft. about.}$$

Assuming $t_1 - t = 300 - 10 = 290$, and $h = 100$, we have for the interior diameter,

$$d = 1{\cdot}49\sqrt[5]{\frac{30\,d + 0{\cdot}05\times 100}{290\times 100}\cdot(19{\cdot}5)^2} = 0{\cdot}627\sqrt[5]{30\,d+5}.$$

Substituting $d = 1{\cdot}25$ under the radical, we have as a first approximation,

$$d = 0{\cdot}627\sqrt[5]{42{\cdot}5} = 1{\cdot}33 \text{ ft.},$$

and substituting this value, we have as a second approximation,

$$d = 0{\cdot}627\sqrt[5]{44{\cdot}5} = 1{\cdot}34 \text{ ft.}$$

If we wish the chimney to be only 40 ft. high, then we should have

$$d = 1{\cdot}49\sqrt[5]{\frac{30\,d + 0{\cdot}05 \times 40}{290 \times 40} \cdot (19{\cdot}5)^2} = 0{\cdot}753\sqrt[5]{30\,d + 2} = 1{\cdot}67 \text{ ft.}$$

Example 2.—If we assume the greatest wind velocity $c = 100$ ft., $\gamma = 0{\cdot}0766$ and $\gamma_1 = 62{\cdot}5 \times 1{\cdot}6 = 100$ lbs., we have for a square chimney,

$$\frac{h}{b} = \tfrac{4}{3}\left(1 - \frac{e}{b}\right)\frac{e}{0{\cdot}017 \times 10000} \cdot \frac{100}{0{\cdot}0766} = \frac{400\left(1 - \frac{e}{b}\right)e}{39{\cdot}066} = 10{\cdot}23\left(1 - \frac{e}{b}\right)e.$$

If we take $e = 1$ ft. and $\frac{e}{b} = \frac{1}{4}$, we have

$$\frac{h}{b} = \tfrac{3}{4} \times 10{\cdot}23 = 7{\cdot}67.$$

In order to withstand the force of the wind, then, the mean outer breadth must be one eighth of the height of the chimney. If the chimney were round, we should have $\frac{h}{b} = 12$, or the mean diameter one twelfth of the height. We can conclude, therefore, that there are many unsupported chimneys standing, unable to withstand the force of a tempest of 100 ft. velocity.

Remark.—From the formula

$$d = 1{\cdot}49\sqrt[5]{\frac{30\,d + 0{\cdot}05\,h}{(t_1 - t)\,h}\,Q^2},$$

we see that the breadth decreases as the height increases, and that, inversely, if the height is diminished the width must be increased.

Strictly speaking, according to the principles of hydraulics (Vol. I., § 425) in the formula

$$Q = 0{\cdot}47\,S\sqrt{\frac{(t_1 - t)\,h\,d}{30\,d + 0{\cdot}05\,h}},$$

we should insert for S, not the mean cross-section, but the cross-section of the mouth, and hence we see at once that, other things being the same, a chimney which gradually widens towards the top can discharge more smoke than one which gradually diminishes.

§ **423.—Efficiency of the Steam Boiler.**—According to the observations of Péclet, the mean temperature t_1 in the chimney can be taken $= 300°$. The temperature t_2, on the other hand, of the air in the furnace can be easily determined from the heat W generated by a pound of fuel, and the quantity of air in cubic feet V necessary for combus-

tion, if we take the specific heat of air $\omega = \frac{1}{4}$ of that of water, and the weight of a cubic foot $\gamma = 0{\cdot}080$ lbs.

Thus

$$W = \omega V \gamma (t_2 - t_0) = \tfrac{1}{4} \times 0{\cdot}080\, V (t_2 - t_0),$$

and hence,

$$t_2 = \frac{4 W}{0{\cdot}080\, V} + t_0 = 50 \frac{W}{V} + t_0,$$

where t_0 is the temperature of the entering air.

Finally, the loss of heat in the chimney is

$$W_1 = \frac{t_1 - t_0}{t_2 - t_0} W.$$

If we take for W the mean value of 6000 heat-units, for V 225 cubic feet, and $t_0 = 0$ degrees, we have for the heat in the furnace,

$$t_2 = \frac{50 \times 6000}{225} = 1333^\circ,$$

and the loss of heat in the chimney,

$$W_1 = \frac{300}{1333} W = \frac{300 \times 6000}{1333} = 1350 \text{ heat-units},$$

or about one quarter of the total heat generated by the fuel.

Under the assumption that the amount of heat which goes to the production of steam is proportional to the difference of temperatures (§ 368), we can determine also the temperature t_1 of the air upon its entrance to the chimney.

If x is the temperature at any point of the flue, Y the extent of heating surface up to this point, and κ the amount of heat per square foot of heating surface, which for one degree difference of temperature passes into the water in the boiler, then for the elementary area of heating surface

dY and the temperature difference $z - t$, the quantity of heat imparted to the water is

$$\kappa (z - t)\, dY = -\omega V \gamma\, dz,$$

and hence

$$Y = -\frac{\omega V \gamma}{\kappa} \int \frac{dz}{z - t} = -\frac{\omega V \gamma}{\kappa} \textit{ log. nat. } (z - t) + \text{constant.}$$

For $Y = 0$, we have $z = t_2$, and for $Y = F$ (the entire heating surface), $z = t_1$, hence

$$F = \frac{\omega V \gamma}{\kappa} \textit{ log. nat. } \left(\frac{t_2 - t}{t_1 - t}\right),$$

and the temperature of the air upon entrance to the chimney is

$$t_1 = t + (t_2 - t)\, e^{-\frac{\kappa F}{\omega V \gamma}},$$

where $e = 2{\cdot}71828$ denotes the base of the Naperian system of logarithms. The heat carried away by the chimney is thus

$$W_1 = \frac{t_1 - t_0}{t_2 - t_0} W = \frac{t - t_0 + (t_2 - t) e^{-\frac{\kappa F}{\omega V \gamma}}}{t_2 - t_0} W,$$

and hence the *efficiency of the boiler*, or the ratio of the heat received by it to the total heat, is

$$\eta = 1 - \frac{W_1}{W} = \left(\frac{t_2 - t}{t_2 - t}\right)\left(1 - e^{-\frac{\kappa F}{\omega V \gamma}}\right),$$

or, since $t_2 - t_0 = \dfrac{W}{\omega V \gamma}$,

$$\eta = \left(1 - \frac{t - t_0}{t_2 - t_0}\right)\left(1 - e^{-\frac{\kappa F}{\omega V \gamma}}\right)$$

$$= \left(1 - (t - t_0)\frac{\omega V \gamma}{W}\right)\left(1 - e^{-\frac{\kappa F}{\omega V \gamma}}\right).$$

If we put $t_2 - t_0 = 1200$, we have

$$\eta = \left(1 - \frac{t - t_0}{1200}\right)\left(1 - e^{-\frac{\kappa F}{\omega V \gamma}}\right).$$

In this again,

$$\omega\gamma = \tfrac{1}{4} \times 0{\cdot}086 = 0{\cdot}0215, \kappa = 0{\cdot}0007,$$

and

$$\frac{F}{V} = \frac{60 \times 60}{22} f = 163 f,$$

where f is the heating surface which gives one pound of steam per hour; hence

$$\eta = \left(1 - \frac{t - t_0}{1200}\right)(1 - e^{-5{\cdot}3f}).$$

Thus, for example, for $t - t_0 = 120°$,

$$\eta = 0{\cdot}9(1 - e^{-5{\cdot}3f}).$$

We have already (§ 404) reckoned four pounds of steam per hour for each foot of heating surface, hence $f = \frac{1}{4}$, and

$$\eta = 0{\cdot}9\,(1 - e^{-1{\cdot}33}) = 0{\cdot}9(1 - 0{\cdot}2645) = 0{\cdot}66.$$

If we take the heating surface twice as great, hence $f = \frac{1}{2}$,

$$\eta = 0{\cdot}9(1 - e^{-2{\cdot}66}) = 0{\cdot}9(1 - 0{\cdot}093) = 0{\cdot}81.$$

If we take the heating surface only half as large as at first, or $f = \frac{1}{8}$, we have, on the other hand,

$$\eta = 0{\cdot}9\,(1 - e^{-0{\cdot}665}) = 0{\cdot}9(1 - 0{\cdot}514) = 0{\cdot}9 \times 0{\cdot}486 = 0{\cdot}44.$$

We see from this that for advantageous generation of steam, a large heating surface is required.

If we take the temperature in the boiler $t = 140°$, we have in the first case the temperature in the chimney,

$$t_1 = t + (t_2 - t)\, e^{-1.33} = 140 + 1060 \times 0.2645$$
$$= 140 + 280 = 420°;$$

in the second case,

$$t_1 = 140 + 1060 \times 0.093 = 140 + 99 = 239°,$$

and, in the third case,

$$t_1 = 140 + 1060 \times 0.514 = 140 + 545 = 685°.$$

Naturally, these temperatures have a great influence upon the necessary dimensions of the chimney, and it is easy to see that it may be advantageous, for a very low temperature of the departing air, to increase the draught by a blower. (See an article upon this point by Prof. Zeuner in the "Civilingenieur," Bd. 4.)

§ **424.—Feed Apparatus.**—To a steam boiler belong also special apparatus for the supply of the feed water, for drawing off the steam, for regulating the generation of steam, for guarding against explosion, etc.

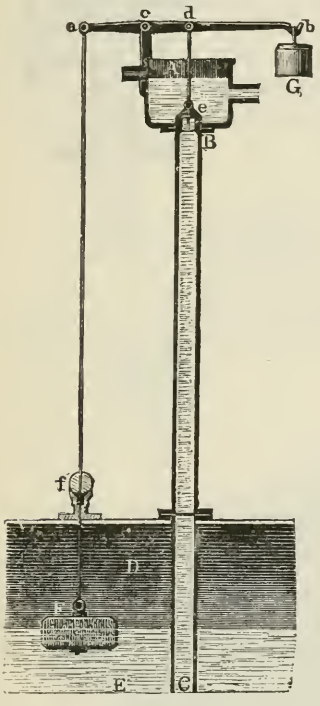
The feed water should be supplied as uniformly as possible, not in too great quantities at once, and should be as pure and warm as possible. To effect the last, the water is passed through pipes in the smoke box or chimney, or a part of the water of condensation is used as feed. If steam of low pressure, having a tension not much greater than the atmosphere, is generated in the boiler, a simple pipe is sufficient for the introduction of the feed water. If the steam is of high pressure, the water must be forced in by pumps, as a simple feed pipe would have to be too long.

The feed pipe (Fr. tube d'alimentation; Ger. Speiserohr) enters from above and ends within about one half a foot of the bottom of the boiler, as far as possible from the fireplace. In order to regulate the feed, so that just as

much water may enter as is evaporated, a float is generally used, which rises and falls with the water level, and thus admits or shuts off the feed.

The arrangement of such a feed apparatus for low-pressure boilers is shown in Fig. 648. *A* is the reservoir which is kept full of water, *B C* the feed pipe, about 8 feet long, *D* the steam and *E* the water in the boiler, and *F* the float of lime or sandstone, immersed somewhat more than half in the water. There is also a lever *a b* turning about *c*, to one end of which the float is hung and to the other end a weight *G*, while at *d* is attached the stem of a conical valve *e*. If, now, the water level sinks, the end *a* of the lever is drawn down by the rod *a F*, which passes through the steam-tight stuffing-box *f* in the top of the boiler. The valve *e* is thus lifted, so that water can enter. If the level of the water rises, the weight *G* gradually overbalances *F* and closes the valve *e*.

Fig. 648.

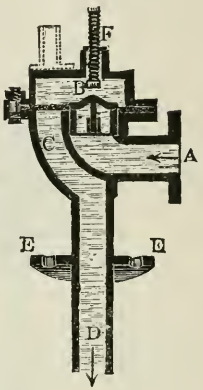


In high-pressure boilers, the introduction of the feed is more difficult, as a considerable pressure has to be overcome. A special pump called the feed pump (Fr. pompe d'alimentation ; Ger. Speisepumpe) is therefore used. As we shall treat specially of pumps in another place, it will be sufficient to remark here that the pump consists of a simple force pump with plunger piston. The feed pipe is shown in Fig. 649. At *A* the water is forced in by the pump. *B* is a valve which the water must raise in order to enter the feed pipe proper, *C D*. In order to regulate the

lift of the valve, we have a set screw *F*, against the end of which the valve strikes when lifted.

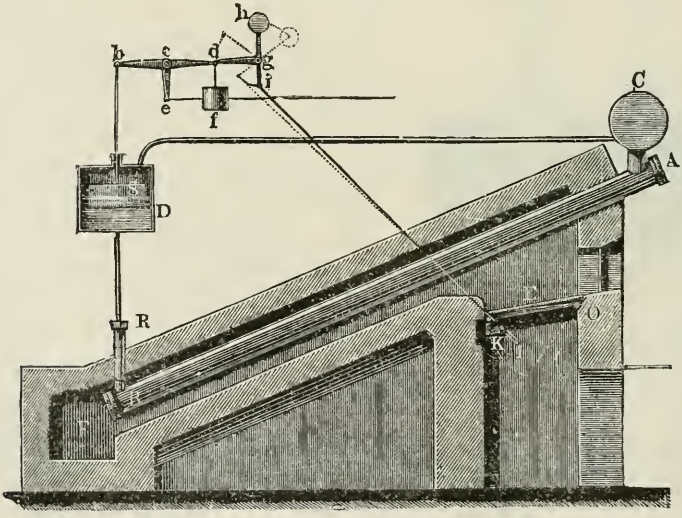
The feed is not, as a general thing, regulated by the machine, but by the attendant, who opens or closes a cock according to the level of the water, and thus increases or diminishes the feed. Floats have indeed been used in high-pressure boilers for the regulation of the feed, but as they require constant attention and often fail to work, the regulation is generally performed by hand.

FIG. 649.



REMARK.—In Henschel's boiler, the feed is regulated by a float. The arrangement is shown in Fig. 650. *A B* is a boiler tube 6 to 12 inches wide and 10 to 20 feet long, and side by side with it are any desired number of similar tubes. At *B* the feed water enters, and *C* is the horizontal tube in which the steam collects. The hot air in the fireplace passes through the

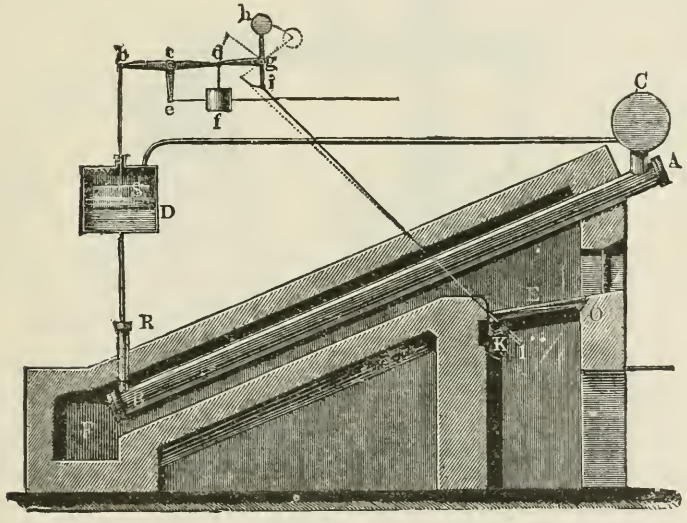
FIG. 650.



flue *E F* inclined at an angle of 24°, surrounds the boiler tubes completely, and enters the chimney at *F*. The grate *E* turns about an axis at *O*, the other end being supported by a small angle lever *K*. *R* is one of the pipes which

convey the feed water to the separate boiler tubes. For the regulation of the feed we have a float *S*, which is buoyed up by the feed water in the cast-iron reservoir *D*. This float is attached, as shown in the figure, to the end *b* of the double lever *b e d*, whose end *d* bears the counterpoise *f*, while the other end *e* is connected with the suction valve of the feed pump. As the float *S* sinks, by means of *c e*, the suction valve is made to work ; as the float *S* rises, the arm *c e* raises the suction valve and prevents the action of the pump. If, finally, the generation of steam is very violent, and exceeds a certain limit, the end *d* acts upon the arm *d g* of an angle lever *d g i*, which is furnished with a counterpoise at *h*, and by means of the rod *l i* turns the small lever at *K*. This removes the support of the grate, which falls inward and throws the fuel into the ashpit, thus preventing any further overheating. According to Henschel, such a boiler presents many advantages, of which some are as

FIG. 651

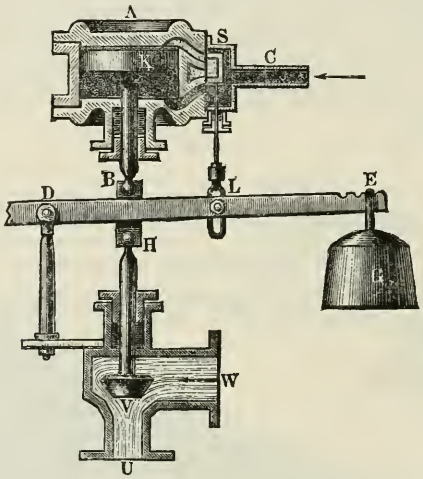


follows: The apparatus needs only a small heating surface of 4 sq. ft. per horse-power, the generation of steam is rapid, the care and cleaning of the boiler easy, and its safety is very great, especially since no great amount of superheated steam can be formed, and the surface where superheating can occur is small. Upon the other hand, it is objected that, on account of the small water area, the steam carries with it much of the water.

§ **425.—New Feed Apparatus.**—In recent times, instead of the ordinary feed apparatus with feed pump, various self-acting arrangements have been devised. Among

others, we have the feed apparatus of Auld, of Jolly, and of Brière, and especially of Giffard. The self-acting feed apparatus of Jolly (see Armengaud's "Génie industriel," July, 1865; also Dingler's Journal, Bd. 178) consists essentially of a small steam cylinder *A B C*, Fig. 652, whose slide

Fig. 652.

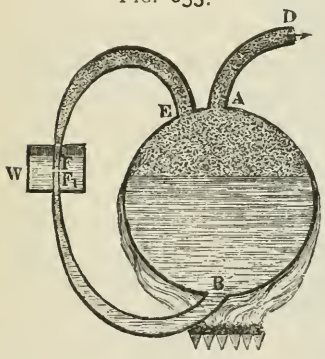


S is fastened by the adjustable rod *S L* to a lever *D E* which turns about *D*, and whose further end is attached to a float. The piston *K* raises the valve *V* by means of the rods *K B* and *H V*. The weight *G* counterpoises the float (not shown in the figure). As this float sinks, owing to lack of water in the boiler, the arm *D E* rises and moves the slide *S* upwards. The steam then enters below the piston *K*, forces it up, and thus raises the valve *V* and allows the water driven by the feed pump to enter. As the float rises, the arm *D E* sinks, moves the slide *S* downwards, and allows the steam to enter above the piston, thus closing the valve *V*

Another self-acting feed apparatus of Brière is described in Armengaud's "Génie industriel," 1866, as also in Dingler's Journal, Bd. 180.

§ **426. Giffard's Injector.**—If from the boiler $A\,B$, Fig. 653, we draw off steam not only by the steam pipe $A\,D$, but also by a second pipe $E\,F$, we can by means of this last force the feed water into the boiler. For this it is only necessary that the pipe $E\,F$ should end in a conical mouthpiece, that the feed pipe $F_1 B$ should end in a very little wider conical mouthpiece, and that, finally, both these mouthpieces should be under water, and so opposed that only a small space shall be left between them. The steam then flows with so great a velocity through F into the pipe $F_1 B$, that it carries not only the water formed by its partial condensation, but also the water in the vessel W with it into the boiler. If $Q\gamma$ is the weight of the water passing through the space between F and F_1, which passes as steam through $A\,D$, and $Q_1\gamma$ the weight of the steam passing through $E\,F$, and h the head of water measuring the pressure in the boiler, we have for the expenditure of work necessary to force the water into the boiler,

Fig. 653.



$$L = (Q + Q_1)\,h\,\gamma.$$

The work inherent in the steam is approximately

$$L_1 = Q_1\gamma\frac{v^2}{2g} = Q_1\gamma\,\mu\,h = Q_1\,\mu\,h\,\gamma,$$

where μ is the specific volume, and v the velocity of the steam flowing out under the head h. If, now, we put $L_1 = L$, we have

$$\mu\,Q_1 = Q + Q_1, \qquad \text{and hence} \qquad Q_1 = \frac{Q}{\mu - 1},$$

for which we may write $Q_1 = \dfrac{Q}{u}$.

By reason of the cooling of the steam at exit, and by contact with the feed water, Q_1 is much less than $\frac{Q}{\mu}$. If the feed water Q which enters at W has the temperature t, and the total quantity of water $Q + Q_1$ entering at F_1 has the temperature t_1, we have, since the loss of heat of Q_1 is equal to the gain of heat of $Q + Q_1$, and the total heat of the steam is approximately $= 640°$,

$$Q_1 (640 - t_1) = Q (t_1 - t).$$

Hence

$$Q_1 = \left(\frac{t_1 - t}{640 - t_1}\right) Q.$$

If, for example, the temperature of the feed water $t = 15°$, and that of the feed water which enters the pipe F_1, and has been warmed by the steam, is $t_1 = 60°$, we have for the circulating steam,

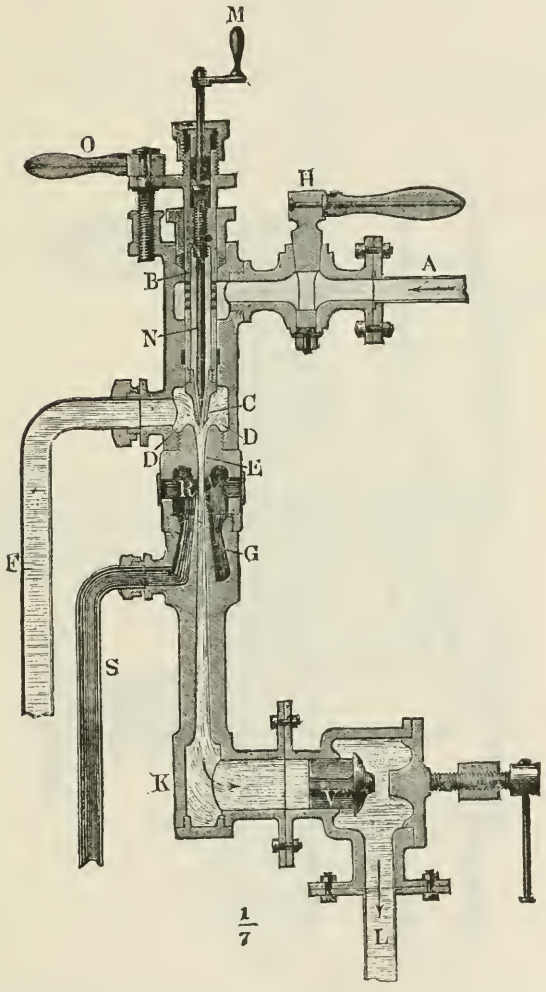
$$Q_1 = \frac{45}{580} Q = \frac{Q}{13}.$$

The further development of the theory of the injector is the province of the mechanical heat theory.

The special arrangement of such a feed apparatus is shown in Fig. 654. The pipe A communicates with the steam space in the boiler, and when the cock H is opened, conducts the steam through a number of holes into the pipe $B\,C$ which has the conical mouth-piece C. This last opens into a chamber $D\,D$, which acts as a condenser, and communicates through the suction pipe F with the feed-water reservoir. It has also a conical opening E, through which passes not only the water sucked up through the pipe F, but also the water formed by the condensation of the steam. Another conical mouth-piece G receives the water passing through E, and conducts it into the pipe K, which communicates by L with the water space of the boiler. The regulation of the quantity of steam is effected by a crank M, by turning which the tapered point of the rod

N can be pushed down into the mouth-piece *C* as far as may be desired, while the feed delivery is regulated by

Fig. 654.



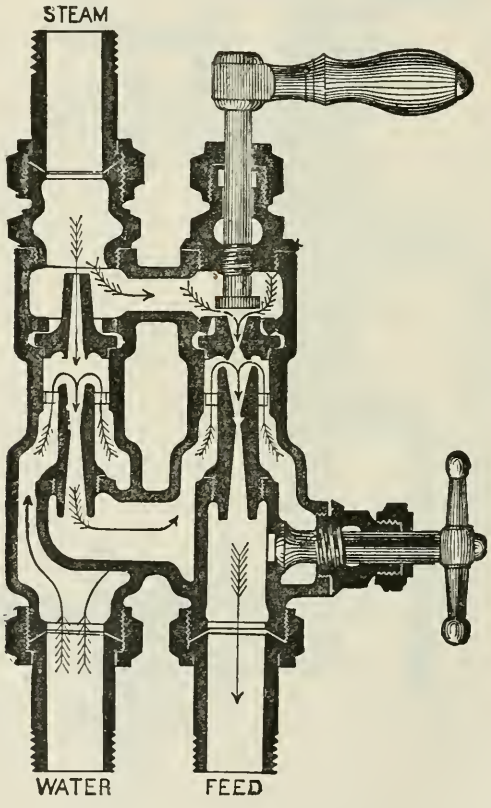
another crank *O*, by means of which the pipe *B C* can be raised or lowered, and thus the distance of its orifice from the bottom of the chamber *D D* increased or diminished.

The superfluous feed water which does not enter *G* collects in the chamber *R* and runs off through *S*.

REMARK.—The improved injector of Türk is described by Gagg in the "Civilingenieur," Bd. XI. The patented injector of Schäffer and Budenberg is described in Dingler's Journal, Bd. 182.

Fig. 654*a* is a sectional view of the Hancock "Inspirator," which has a separate attachment for raising the water and delivering it to the injector proper, or forcing attachment.

FIG. 654*a*.

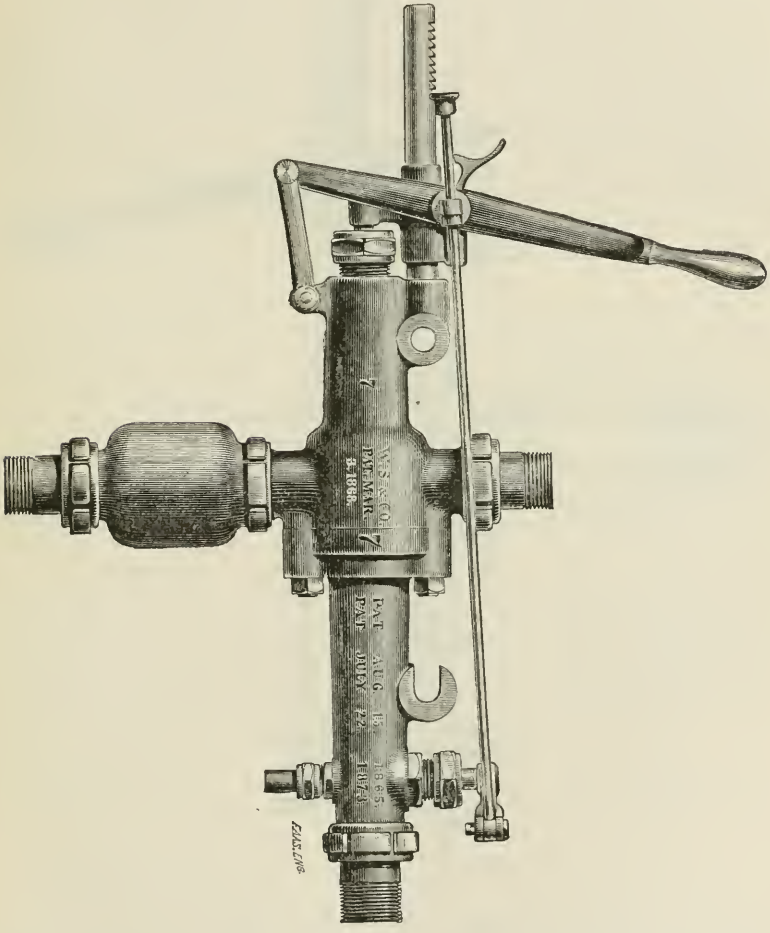


The operation of the machine will be readily understood by an inspection of the sketch, the arrows indicating the course of the water and steam respectively. In a lifting attachment, it is desirable that the steam should expand on entering the combining tube, and this is generally accomplished as

in the present machine, by enlarging the steam-supply tube from the point of admission to the point of discharge. With such an instrument the water can be lifted as high as with an ordinary suction pump.

The Sellers' self-adjusting injector, Figs. 654*b*, 654*c*, is provided with a

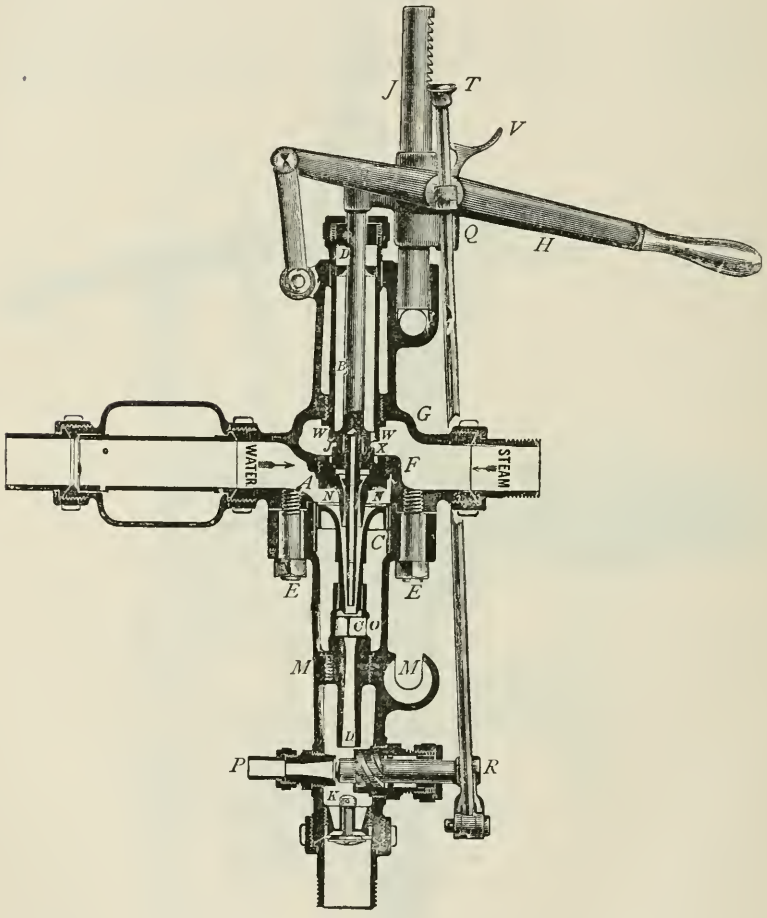
FIG. 654*b*.



lifting attachment, and is so arranged that the water supply is self-adjusting, no waste overflow taking place except at the time of starting the instrument. By reference to Fig. 654*c*, it will be seen that the case consists of two parts, united by the bolts E E. The upper portion contains inlets for steam and

water, separated by a plate F F, into which is screwed the nozzle A, for the steam jet. The supply of steam to this nozzle is regulated by a tapered plug. The tube C, within the case, has a piston at the upper end, working freely, and the lower end of this tube slides in the delivery tube D. This latter tube is

FIG. 654*c*.



stationary. P R is an overflow valve and pipe, through which the water can run to waste when the valve is open. X is a valve through which steam is admitted to the tube A, and W is a valve which, when opened, permits steam to pass through the centre of the spindle. A single handle H controls the movement of all these valves. If this handle is raised to its full extent, the waste valve will be closed, and will remain closed for any movement of the

handle between the stops T and Q. A click V drops into the top notch on the rod J, when the waste valve is closed, thus holding the handle in the point suitable for the maximum capacity of the injector. After passing the lower notch, which fixes the adjustment for the minimum capacity, the latch is thrown out of action, any further downward movement of the handle opening the waste valve and shutting off the steam supply.

The action of this injector is thus described by the manufacturers:

"The cut shows the instrument with valves W X closed, so that no steam can enter the receiving tube A. By drawing back the lever H a short distance, or until a collar on the taper spindle beyond valve X comes in contact with valve X, steam will enter and pass down through the hole in centre of the tapered spindle, and flow out of waste valve at P. As soon as water shows itself at this opening P, the lever may be drawn entirely back, taking with it the valve X, and admitting a free flow of steam through A, accelerating the escaping flow at P, then closing the waste valve by rod L, lowering latch V into teeth of ratchet, and starting the instrument at full capacity; after which the lever H, being pushed in to any required point between the two stops T Q, the required delivery can be obtained.

"If now the water supply is or becomes too great, a portion of the water escapes by the opening O in the upper part of the delivery tube, and, accumulating in the surrounding chamber, forces back the piston N N, which of course carries with it the combining tube, thus diminishing the annular space through which the water enters the combining tube, and so limits its supply. If, on the other hand, there be not sufficient water admitted, the velocity of the steam jet will be increased and a partial vacuum will be produced in the chamber, and the piston N N will consequently be brought forward, thus opening the space for water supply and correcting the defect."

The following table shows the maximum amount of water delivered by various sizes of these injectors, the minimum capacity being 40 per cent of the maximum. The numbers distinguishing the sizes express the diameters of the smallest part of the delivery tubes, in millimeters. Thus a No. 4 injector has an opening four millimeters in diameter. Experiments in regard to the maximum temperature of feed water allowable gave, according to the manufacturer's statements, the following results:

Pressure of steam in pounds per square inch,	20	40	60	80	100	120	140	150
Admissible temperature (Fahrenheit degrees) of feed water before entering injector.	138	135	130	130	132	133	127	128

The injector, considered as a pumping engine, is not an economical machine, consuming about five times as much steam as direct-acting steam pumps. As a boiler feeder, however, it is more economical than a steam pump, when cold feed water is used, since, although but little of the heat of the steam is converted into useful work, nearly all the remainder is returned to

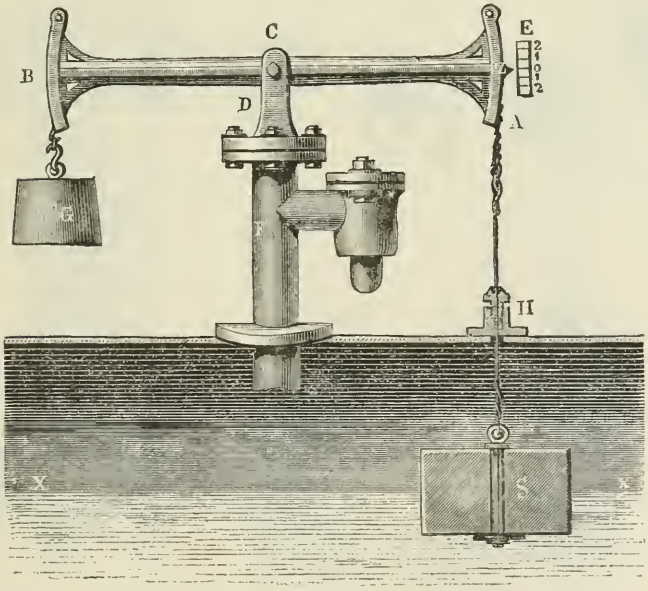
MAXIMUM CAPACITIES (CUBIC FEET OF WATER DISCHARGED PER HOUR) OF SELLERS' SELF-ADJUSTING INJECTORS.

Size.	Diameter of pipe for connections, inches.	Pressure of Steam, in Pounds, per square Inch.														
		10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
No. 2	0·5	8	9	10	10	11	12	13	13	14	15	15	16	17	17	18
" 3	0·75	19	21	23	25	26	28	30	32	33	35	37	39	41	42	44
" 4	1·	37	40	43	46	49	52	55	58	62	65	68	71	74	77	80
" 5	1·25	58	63	67	72	77	82	87	92	97	102	107	112	117	122	126
" 6	1·25	83	91	98	105	112	120	126	133	140	148	155	162	169	176	183
" 7	1·5	114	124	133	143	153	163	172	182	192	202	211	221	231	240	250
" 8	2·	149	162	175	188	200	213	226	239	252	264	277	290	303	316	328
" 9	2·	189	205	222	238	254	270	286	302	318	335	351	367	383	399	415
" 10	2·	234	254	274	294	314	334	354	374	394	413	433	453	473	493	513
" 12	2·5	337	366	395	424	452	481	510	539	568	596	625	654	683	712	740
" 14	2·5	451	491	531	571	611	651	691	731	771	811	851	891	931	971	1011
" 16	3·	603	652	703	784	805	857	908	959	1011	1062	1113	1164	1216	1267	1318
" 18	3·	760	825	890	955	1020	1085	1150	1215	1280	1344	1409	1474	1539	1604	1669
" 20	3·5	939	1019	1099	1179	1259	1340	1420	1450	1583	1660	1740	1821	1901	1981	2061

the boiler, having raised the temperature of the feed water. An account of some comparative experiments with injectors and locomotive feed pumps was published in the "Report of the Master Mechanics' Convention," 1876, several runs being made, under approximately the same conditions, feeding with the injector and the pump respectively, with about the same results in each case, so far as the expenditure of fuel was concerned.]

§ **427.— Water-Level Indicator.**—In every boiler, we should also have some arrangement by which the position of the water level may be indicated. Such are floats, test cocks, and glass gauges. The float gauge (Fr. niveau au flotteur; Ger. Schwimmniveau) consists of a two-armed lever *A B C*, Fig. 655, which carries at one end an iron or

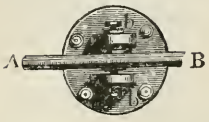
FIG. 655.



stone float, and at the other a weight *G*. The axis of rotation *C*, Fig. 656, is either a knife edge, as in the ordinary balance, or two steel points which support *A B* by means of a depression, into which they enter. The standard *D* is generally set on the feed apparatus *F*. In order to tell the

exact level of the water, a pointer Z is fixed to the lever and moves over a graduated scale. The water level is shown at XX, and at H is the stuffing-box for the rod which suspends the float.

Fig. 656.



Sometimes a steam whistle (Fr. siflet à vapeur; Ger. Sicherheitspfeife) is combined with the float, which is thus made to sound when the water sinks too low.

The test or gauge cocks (Fr. robinets de niveau; Ger. Probirhähne) only give the level with any certainty when the water in the boiler is not subject to very great disturbances, which occur in large boilers and under a low steam

Fig. 657.

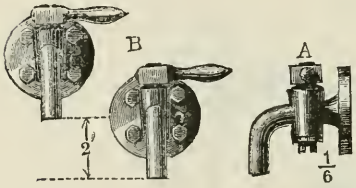


Fig. 658.

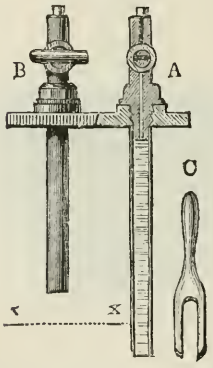
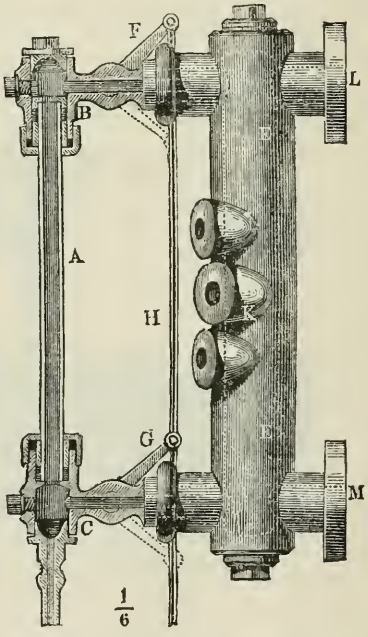


Fig. 659.



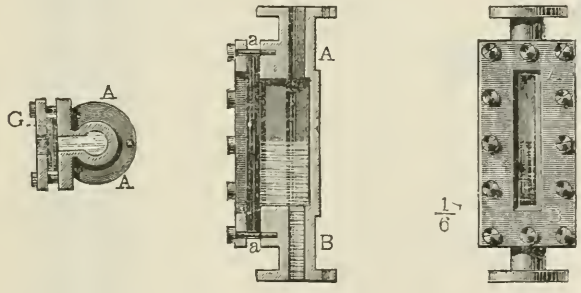
pressure. There are always two, and often three of these cocks, one about two inches below and the other as much above the water level. As long, therefore, as the water level is between them, steam will flow through one and water through the other. We may have both horizontal

and vertical gauge cocks, the first in the face of the boiler and the latter in the top. Fig. 657 shows at *A* a side and at *B* a front view of gauge cocks of the first kind. Fig. 658 represents two vertical gauge cocks with the accompanying wooden key at *C*. It will be seen that *B* opens above and *A* below the water level at *X*.

The water level is best indicated by the glass gauge (Fr. niveau à tube de verre: Ger. Wasserstandsröhre). The arrangement of such a gauge is shown in Fig. 659. *A* is the glass tube, *B* and *C* are metal communicating pipes, one entering the boiler below and the other above the water level. *F* and *G* are two levers united by the rod *H*, by means of which the cocks are moved and the communications of the glass tube with the boiler opened and closed. Finally, in the pipe *E E*, which unites the pipes *L* and *M*, are the places for three test valves.

On account of the fragility of the glass tube and the ease with which it becomes stopped up or discolored, this gauge is not so often used as it would otherwise deserve to be. Scholl recommends, in his "Führer des Maschinisten," a gauge of which Fig. 660 gives a horizontal and Fig. 661

Fig. 660. Fig. 661. Fig. 662.



a vertical section, and Fig. 662 a front view. The whole forms a brass chest *A B* communicating below with the water and above with the steam space, and having only in front two thick glass plates. Recently, prisms have been used in place of these plates.

§ **428.—Manometer.**—In every boiler there is also requisite an apparatus which shall indicate the tension of the steam, in order that the firing may be advantageously regulated. Such an apparatus is the manometer (Fr. manomètre; Ger. Manometer).

Manometers are either open to the air (Fr. à air libre; Ger. Offene), or with compressed air (Fr. à air comprimé; Ger. Verschlossene). Both classes have already been noticed in Vol. I., § 386 and § 394, so that here we shall only add a little with reference to their special application to steam. Glass tubes are not advisable for this instrument, because of their fragility and because they soon become discolored. Iron tubes are best, and the level of the mercury is indicated by a float. Fig. 663 gives a section of a cistern manometer. *A B* is the iron mercury reservoir or cistern, *C* the pipe communicating with the boiler, *D E* the iron manometer tube, *S* the float, and *Z* the index which is united with the float by a cord passing over the pulley *R*, and rises or falls with the mercury in the tube *D E*, thus indicating its height on the scale.

Fig. 663.

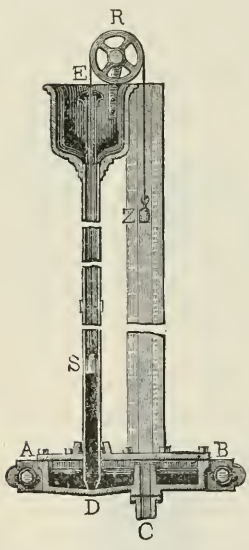
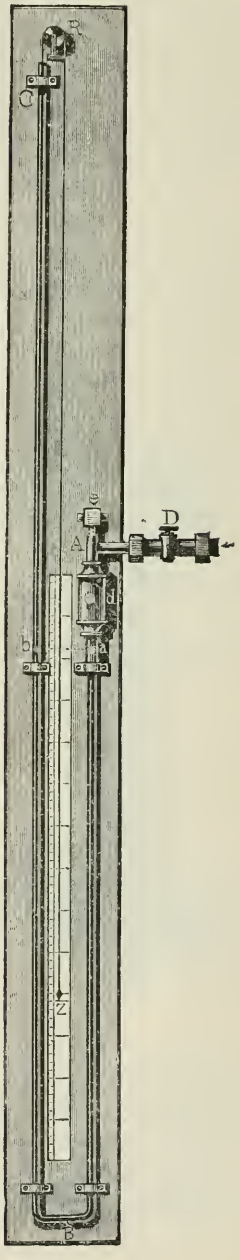


Fig. 664.



A syphon manometer is shown in Fig. 664. $A\,B\,C$ is the syphon-shaped tube, which at one end communicates with the water-filled vessel $A\,a$, and at the other end is open to the air, the portion from a to b being filled with mercury. The steam enters through $D\,A$ above the water in $A\,a$, and pressing upon its surface causes the mercury in the leg $a\,B$ to sink and in $B\,C$ to rise. The height in this leg is indicated upon a scale by means of the index Z, which is joined to the small metal float in the tube by means of a cord passing over the pulley R.

The question arises, through what distance x does the mercury rise in $B\,C$ or the index Z sink, when the steam presses with a certain force p upon the water surface in the leg $a\,B$? If both legs have equal bores, the mercury will sink as much in one leg as it rises in the other, and hence the distance between the levels of the mercury will be $= 2\,x$. If, now, the height of the barometer is b, the pressure is that of a column of mercury whose height is $2\,x + b$. The equal opposite pressure is that of the water column h in the vessel, of the column of water x in the first leg, and of the steam pressure p measured by the height of a column of mercury; or, if ε is the specific weight of mercury, it is equal to

$$p + \frac{h + x}{\varepsilon}.$$

We have, therefore,

$$2\,x + b = p + \frac{h + x}{\varepsilon},$$

and hence,

$$x = \frac{\varepsilon\,(p - b) + h}{2\,\varepsilon - 1}.$$

If we express p in atmospheres, and h and x in inches, we have, since $\varepsilon = 13{\cdot}6$,

$$x = \frac{13{\cdot}6 \times 30\,(p - 1) + h}{26{\cdot}2} = 15{\cdot}57\,(p - 1) + 0{\cdot}0382\,h \text{ inches.}$$

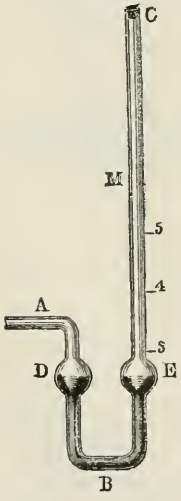
Therefore, if we take our zero point 0·0382 h above the point b of the pipe $B\,C$, we have

for $p =$	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2	3	4	atmos.
$x =$	0	3·89	7·78	11·67	15·57	30·14	46·71	inches.

The instrument is filled with mercury, and the water poured in by means of an opening at c, closed by means of a stopper. In order that the proper amounts of mercury and water may be poured in, the hole a is opened during the pouring in of the first and the hole d during the pouring in of the second.

§ 429. **—Air Manometer.**—The manometer with float just described is applied more especially to low-pressure boilers, because in such case the manometer tube can be tolerably short; it may, however, be fitted to boilers of moderate pressure, containing steam of 3 to 4 atmospheres, since a tube of $2 \times 30 = 60$ to $3 \times 30 = 90$ inches is still amply sufficient. For high-pressure steam, this manometer becomes too large, and we have to make use of a different instrument.

FIG. 665.



The air manometer, the theory of which has been given already in Vol. I., § 394, may be used for all steam tensions, but, on account of the uncertainty of its indications by reason of the oxidation of the mercury, it is not used very often on stationary engines. In order that for the higher steam tensions, the changes of the mercury column may not be too small, a reservoir E is combined with the manometer tube $B\,C$, Fig. 665, from which all the air will be expelled only at high pressures. If, for example, the mercury stands just above E at 3 atmospheres tension, at 6 atmospheres it will stand at the middle point M of $C\,E$, and by graduation of $C\,E$ all tensions between 3 and 6 atmospheres can be read off. A simi-

lar purpose is answered by the hyperbolic manometer of Delaveye (Dingler's "Journal," Bd. 93), which gradually contracts towards one end and terminates in a bulb. It indicates equal changes of pressure by equal differences in the mercury column.

Fig. 666.

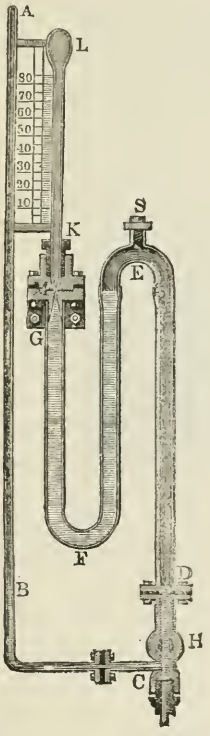
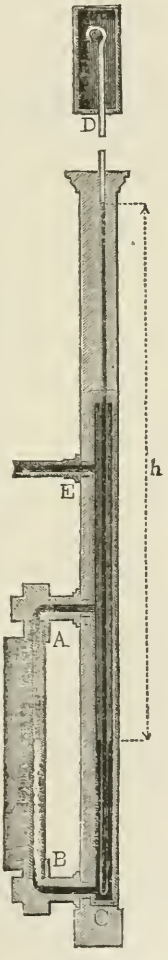


Fig. 667.



A more complicated arrangement is the air manometer of Hofmann in Breslau (see "Verhandlungen des Vereins

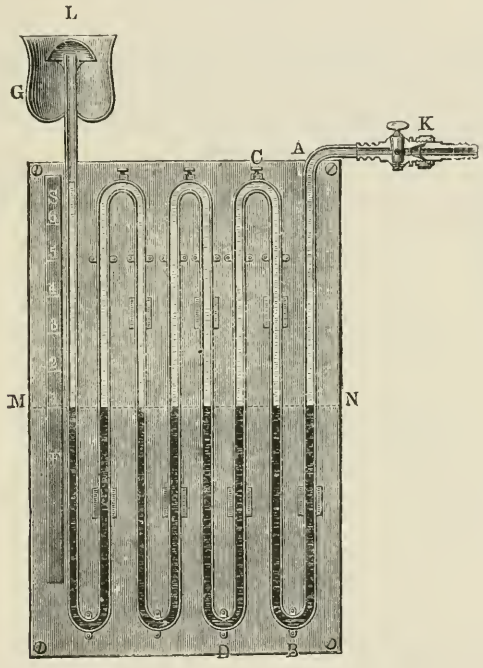
zur Beförderung des Gewerbefleisses in Preussen," 1849). The essential features of such an instrument are to be seen from Fig. 666. *A B C* is a copper pipe communicating with the boiler, *C H D* a cock, *D E F G* a copper pipe with two turns, and *K L* a glass tube ending in a pear-shaped bulb. The portion *E F G* is filled with alcohol, and *B C D* with water which receives the pressure of the steam and transmits it through the air column *D E* to the alcohol, which again compresses the air in the manometer tube *K L*. The alcohol is poured in through *S* until it just flows out of a small hole at *M*, which is afterwards stopped up. When it is desired to know the pressure, the steam cock is opened, and the level of the alcohol in *K L* is observed upon a graduated scale. The scale is of course to be graduated experimentally.

On account of their accuracy, *open* syphon manometers are now used even for high steam pressures; in order, however, to render the scale small, that portion *A B*, Fig. 667, in which the level of the mercury is observed, is made of larger diameter. If, for example, the diameter of this portion is three times that of the rest of the tube, the motion of the mercury column in it is only one ninth as great as in *C D*, but since the pressure is measured by the fall of the mercury in one leg plus the rise in the other, the motion of the mercury in this case is in the wider portion one tenth of that in the other. In the manometer of Decoudun, the wider portion *A B* is below, and the steam entering at *E* presses upon the mercury in it. In the manometer of Desbordes, on the other hand, *A B* is above and the air presses directly upon the mercury in it.

§ **430.— Differential Manometer.** — The differential manometer is well suited to the measurement of high steam pressures. Such an instrument consists of a system of parallel tubes *A B*, *B C*, *C D* . . ., Fig. 668, the lower portion of each being filled up to *M N* with mercury, and the upper portions with water. If, now, one end *K* communi-

cates with the steam and the other end L with the air, the mercury sinks in the first, third, fifth, etc., legs, and rises in the second, fourth, sixth, etc., until the steam pressure at one end and the air pressure at the other are in equili-

FIG. 668.



brium with the combined mercury and water pressure. If all the pipes have the same diameter, which is necessary for the success of the instrument, then any rise of the mercury in the first tube is equal to its fall in the second, and the difference of level is therefore $2x$, and the same difference exists between the third and fourth, fifth and sixth, etc. On the other hand, the water column in the first is shorter by $2x$ than in the second, in the third than in the fourth, and so on. If ε denotes the specific weight of the mercury, then the height of a mercury column which balances a water column $2x$ in height is $\frac{2x}{\varepsilon}$, and

hence the pressure which causes the difference of level $2x$ corresponds to a column of mercury $= 2x - \frac{2x}{\varepsilon}$ in height, or

$$= \left(1 - \frac{1}{\varepsilon}\right) 2x = \frac{2(\varepsilon - 1)}{\varepsilon} x.$$

This pressure is doubled by the difference of level between the third and fourth, and so on, so that if n is the number of legs, p the pressure of the steam upon the first leg, and b the barometer height, we have

$$p = b + \frac{n}{2} \cdot \frac{2(\varepsilon - 1)}{\varepsilon} x,$$

or

$$p = b + \frac{n(\varepsilon - 1)}{\varepsilon} x = b + 0{\cdot}9266\, n\, x,$$

also

$$x = \frac{\varepsilon(p - b)}{(\varepsilon - 1)\, n} = 1{\cdot}079 \frac{(p - b)}{n} \text{ inches.}$$

If we take p in atmospheres, and let therefore $b = 1$,

$$x = 32{\cdot}37 \frac{p - 1}{n} \text{ inches.}$$

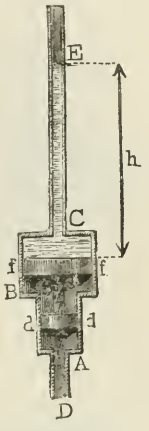
Thus, for an instrument with 8 pipes, we have for $p = 1$, $1\frac{1}{2}$, 2, 3, 4, 5, 6 atmospheres,

$$x = 0,\ 2{\cdot}02,\ 4{\cdot}046,\ 8{\cdot}09,\ 12{\cdot}13,\ 16{\cdot}18,\ 20{\cdot}23 \text{ inches.}$$

The leg $F L$ is of glass, and furnished with a scale $M S$, upon which the heights may be read off. In order that the mercury may not be thrown out by a sudden access of steam, the top of the glass tube has a cap L and a vessel G, in which the mercury can be caught. Further information and description of such an instrument by Richards may be found in Jahrgang 44 (1845) of the "Bulletin de la société d'encour.," as also in "Annales des mines," T. VII., 1845.

§ **431.—Piston Manometer.**—In recent times, still other manometers have been devised and used for the determinations of high steam pressures. The more important are the open manometer of Galy-Cazalat or of Journeux, and the manometer of Bourdon. (See "Annales des mines," IV. Sér., T. XVI., 1849; or "Der Ingenieur," Bd. II.)

Fig. 669.



The principle of the first mentioned is as follows: In the vessel *A B C*, Fig. 669, are two pistons *d d* and *f f*, which are united by a rod so as to move together, of different diameters, of which one sustains the pressure of the steam which enters at *D*, and the other that of a column of liquid *C E*. If r and r_1 are the radii of *d d* and *f f*, p the steam pressure, h the manometer height or the height of the liquid column whose density is γ, we have for the equilibrium,

Fig. 670.

$$\pi r^2 p = \pi r_1^2 h \gamma,$$

or

$$h = \left(\frac{r}{r_1}\right)^2 \frac{p}{\gamma}.$$

Thus for $\frac{r}{r_1} = \frac{1}{3}$,

$$h = \tfrac{1}{9} \frac{p}{\gamma};$$

so that a pressure of one atmosphere or 30 inches will sustain a column of liquid $\frac{28}{9}$ or $3\frac{1}{9}$ inches high in *C E*.

In the manometer of Journeux (Fig. 670), in order to get rid of the piston friction, the piston is replaced by metal plates *d d* and *f f*, and the pressure is conveyed from one of these plates to the other by a special form of piston *g*.

In order to shut off the steam from the mercury, both plates are furnished with disks of vulcanized rubber; and in order that the air may press upon both sides of *f f*, there is a hole *o* in the lower part of the vessel. The mercury is filled in by means of a funnel at *D*.

§ 432.— Metal Manometer, or Bourdon's Gauge.— Bourdon's manometer consists of a curved brass pipe *B E F*, Fig. 671, of elliptical cross-section, which, under the pressure of the fluid in it alters its shape. One end *B* of the

FIG. 671.

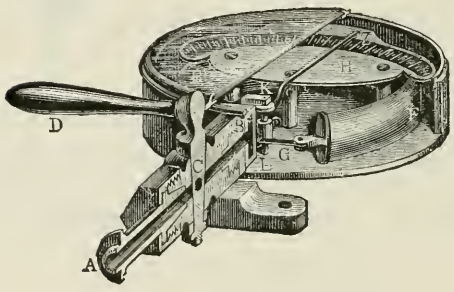
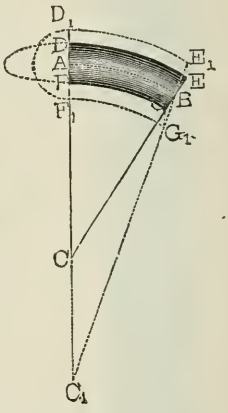


FIG. 672.



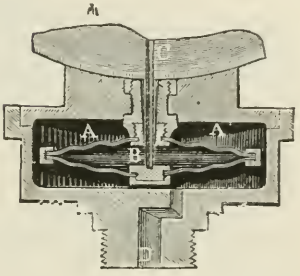
pipe is open and communicates with the steam-pipe *A B*, the other end *F* is closed and free to move, and a pointer *Z*, joined to this end by means of an axle *K L*, moves back and forth over the scale *H*, when the pipe end moves by reason of the change of steam pressure.

Since, in consequence of the steam pressure, the elliptical cross-section of the pipe approaches a circle (§ 411), the breadth *D F*, Fig. 672, becomes $D_1 F_1$, so that the sides *D E* and *F G* take the positions $D_1 E_1$ and $F_1 G_1$, thus giving the cross-section *E G* the position $E_1 G_1$, and changing the radius of curvature from $C A = C B$ to $C_1 A = C_1 B$.

In the metal manometers of Schäfer and Budenberg, the spiral tube is replaced by an axle-shaped steel plate, and

in those of Gäbler and Veitshaus by a pair of plates *A A*, Fig. 673. The steam entering at *D* presses these plates together and shoves the pin *B C* upwards, thus setting in action a pointer and indicating the amount of steam pressure.

Fig. 673.



Finally, the tension of the steam can be determined by the *thermometer*, since, by means of formulæ or tables, the tension corresponding to an observed temperature can be determined. The thermometer can be hung in the boiler through a stuffing-box, and protected from breaking by a metal shield. (See Dr. Scheffler's Monograph, "Die Ursachen der Dampfkesselexplosionen und das Dampfkesselthermometer.")

§ **433.—Safety-Valves.**—The safety-valve (Fr. soupape de sûreté; Ger. Sicherheitsventil) is the most important adjunct of the steam boiler. We may distinguish inner and outer safety-valves. Outer valves open outwards when the steam pressure within the boiler exceeds a certain amount, and thus allow the steam to escape until the pressure sinks again to the required amount, when they close.

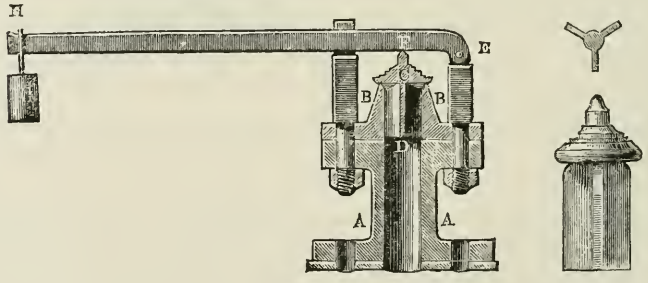
Inner valves, or air valves, or vacuum valves, or atmospheric safety-valves, on the other hand, open inwards, when the pressure within the boiler, owing, it may be, to condensation of the steam or irregularity in the firing, falls below a certain limit, and thus allows air to enter from without until the interior pressure is nearly equal to that of the atmosphere. While the outer safety-valve prevents explosion of the boiler by excessive steam pressure, the inner valve prevents its collapse under the atmospheric pressure. We see at once that the inner or vacuum valve is only necessary when the steam is liable to be condensed.

We can also distinguish valves with direct from those

with indirect loading, according to the manner in which the weight which holds the steam pressure in equilibrium is applied. Valves of the first kind are applied especially for moderate steam tensions, while the latter are used for higher tensions, in order to diminish the necessary counter-weight. In the first kind, the weight is applied directly over the valve; in the latter it is suspended to the longer arm of a lever. We also have valves pressed down by springs, but, on account of the variable force of the spring, they do not answer as well.

To facilitate the opening, the valve is not conical but flat, and rests upon a circular seat. According to Belgian regulations, the breadth of the circular area of contact should be only 2 millimeters (0·08 inch); in France it must be $\frac{1}{30}$ of the diameter of the inner valve surface, if this diameter is 30 or more millimeters (1·2 inches); if, however, it is smaller, the breadth should be one millimeter. Fig. 674

Fig. 674.



shows a safety-valve with indirect loading. *A A* is the valve box, which is screwed on to the boiler, *B B* the valve seat, *C D* the valve, *C* being a plate, and *D* a guide stem to insure vertical motion of the valve. *E F H* is the lever, having its fulcrum at *E* and loaded at *H* with the weight *G*, which is raised by the pressure on the valve at *F*.

A later form of valve, shown by *A*, Fig. 675, is a turned cylinder moving vertically between four guides *B*, *B*, and attached by the bolt *C* and rod *D* to the lever above.

A more complete apparatus is shown in Fig. 676. Both valves *A* and *B* have, as in Fig. 675, exterior guides. The

Fig. 675.

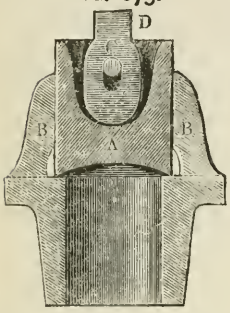
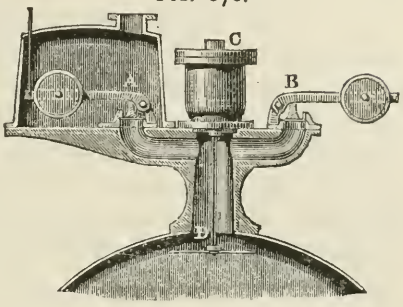
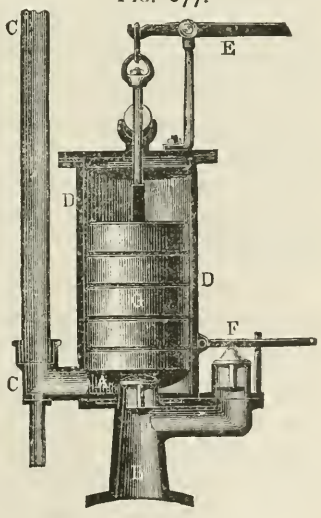


Fig. 676.



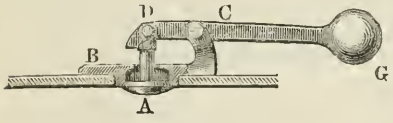
valve *A* is enclosed in a chest, and therefore cannot be meddled with by the driver while the other is under his control. In *C* is the stop valve, and at *D* is the baffle plate which prevents water from being carried with the steam into the steam pipe. Fig. 677 gives a representation of a safety-valve with direct loading. *A* is the valve, *G* the weights strung upon the square valve stem, *C C* is the steam pipe, *D D* the box protecting the valve from the driver, *E* a lever for testing the freedom of the valve, and *F* a second valve under the control of the driver.

Fig. 677.



In Fig. 678, we have an air or vacuum valve. Here the

Fig. 678.



valve *A* is fastened by a link *D* with the lever *D G*, and, by means of a moderate weight *G*, the valve is pressed upwards against its seat.

§ 434.—**Theory of the Safety-Valve.**—The outer safety-valve must not only be loaded with a certain weight so that it cannot open until the steam pressure arrives at a certain point, but it must also have a certain size, in order that when opened the escape of steam may be sufficiently rapid. The condition, at least, must be fulfilled that the steam shall escape faster than it is generated in the boiler. As to the determination of the loading of a safety-valve, we have already spoken in Vol. I., § 386. If p is the steam pressure, b the atmospheric pressure, and r the inner radius of the valve, then the upward pressure upon it is

$$P = \pi r^2 (p - b).$$

If the load is directly applied, the weight G of the entire valve must equal this pressure. If indirectly applied with the lever arm a, we have

$$G = \frac{P d - Q s}{a},$$

where d is the lever arm of the pressure P and $Q s$ the moment of the unloaded valve. There is some uncertainty in this determination, especially when the ring-shaped surface of contact is not very small, because the surfaces of contact may then be separated to some extent by air on the outside and by steam from within, so that the area of upward pressure may be somewhat greater than πr^2. (See an article by Cato, in the "Polytechnischen Centralblatt," Bd. VIII., 1846.)

In order to determine the necessary area of valve surface, we assume, in accordance with the mechanical theory of heat, that through an opening of F square meters, under the pressure of p atmospheres, besides a large quantity of hot water, the steam quantity $Q = 20\, p\, F$ kilogrammes per second escapes. (See Zeuner's "Grundzüge der mechanischen Wärmetheorie," p. 421.)

In English measures, taking F in square feet, this becomes $Q = 4{\cdot}1\, p\, F$ pounds per second. If we also assume, according to the preceding (§ 404), that F_1 square meters heating surface of boiler furnishes the steam quantity,

$$Q = \frac{19\,F_1}{60 \times 60} = 0{\cdot}00528\,F_1 \text{ kilogrammes per second;}$$

or, taking F_1 in square feet,

$$Q = 0{\cdot}00108\,F_1 \text{ pounds per second,}$$

we have, for the required ratio of the valve area F to the heating surface of boiler F_1,

$$\frac{F}{F_1} = \frac{0{\cdot}00528}{20\,p} = \frac{0{\cdot}000264}{p}.$$

From this formula, we have the following table:

Steam Tension $p =$	$\frac{4}{3}$	2	3	4	5	6 atmos.
Ratio $\frac{F}{F_1} =$	0·0001980	0·0001320	0·0000880	0·0000660	0·0000528	0·0000440

The greater, therefore, the steam pressure, the less the necessary area of the safety-valve.

According to the Prussian regulations, $\frac{F}{F_1}$ must be at least $\frac{1}{3000}$; therefore, for a low pressure of $\frac{4}{3}$ atmospheres, we have $(\frac{1}{3000} : 0{\cdot}000198) = \frac{1}{0{\cdot}594} = 1{\cdot}68$, or nearly $\frac{5}{3}$-fold security, or the valve is $\frac{5}{3}$ greater than it need be, and for higher tensions the security is still greater.

The French "Ordinances" prescribe the valve diameter according to the empirical formula of Thrémery,

$$d = 2{\cdot}6\sqrt{\frac{F_1}{p - 0{\cdot}412}} \text{ centimeters,}$$

so that, since F_1 here is in square meters,

$$\frac{F}{F_1} = \frac{(0{\cdot}026)^2\,\pi}{4\,(p - 0{\cdot}412)} = \frac{0{\cdot}000531}{p - 0{\cdot}412}.$$

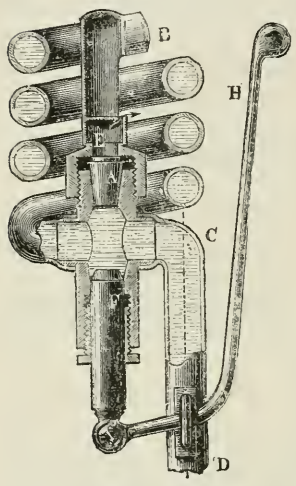
Hence, for $p = \frac{5}{4}$,

$$\frac{F}{F_1} = 0{\cdot}000634 = \frac{1}{1577}.$$

In order to still further increase the security, two valves are used, each of the prescribed size, and set on opposite ends of the boiler.

In order to protect the valve from the driver, the lever may, as recommended by Fairbairn, be hung inside of the boiler.

Fig. 679.



Safety plugs, or easily melting plugs, of lead, bismuth, and zinc, inserted in the boiler, are inconvenient and unsatisfactory safeguards.

In Fig. 679, we have represented Black's "warner," which consists of a plug which melts at 100° C, and gives warning when the water level is too low. *B C D* is a copper pipe which passes into the boiler below, and is closed above by a conical plug *A*. When the water level sinks so that the mouth *D* is above water, the water in the pipe *C D* runs out, and steam enters by which the plug is melted. The steam then issues through a whistle *E*, and gives warning of the state of the water. By reason of the spiral form of *B C*, the water possesses only a temperature of 40 to 50 degrees. By raising the piston *F* by means of the lever *H*, a new plug can be inserted.

Example.—What dimensions must be given to the two valves of a boiler which generates 500 lbs. of steam per hour? The required heating area is

$$F_1 = \tfrac{1}{4}\,500 = 125 \text{ sq. ft.},$$

and hence, according to Prussian regulations, each valve area should be

$$F = \frac{F_1}{3000} = \tfrac{125}{3000} = 0{\cdot}04167 \text{ sq. ft.}$$

The diameter is then

$$d = \sqrt{\frac{4 \times 0{\cdot}04167}{\pi}} = 0{\cdot}23 \text{ ft.} = 2\tfrac{3}{4} \text{ inches.}$$

According to the French regulations,

$$d = 2{\cdot}6\sqrt{\frac{125 \times 0{\cdot}0985}{4{\cdot}000 - 0{\cdot}412}} = 2{\cdot}6\sqrt{\frac{12{\cdot}3125}{3{\cdot}588}} = 4{\cdot}82 \text{ centimeters,}$$

or about 2 inches. Our formula gives for threefold security,

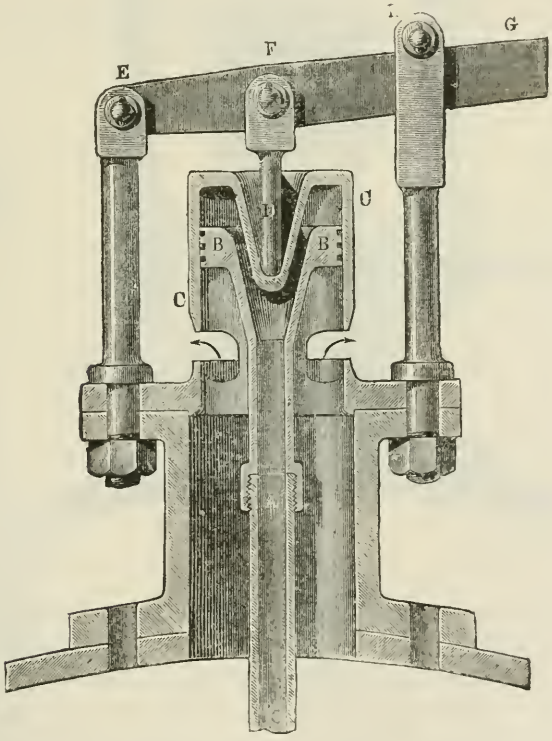
$$\frac{F}{F_1} = \frac{3 \times 0{\cdot}000264}{p} = 0{\cdot}000198\ ;$$

hence,

$F = 0{\cdot}000198 \times 125 = 0{\cdot}02475$, and, accordingly, $d = 0{\cdot}178$ ft. $= 2\frac{1}{8}$ inches.

§ 435. — **More Recent Safety-Valves.** — Many experiments with safety-valves have shown that during the flow of steam they are lifted but little, and, therefore, not so

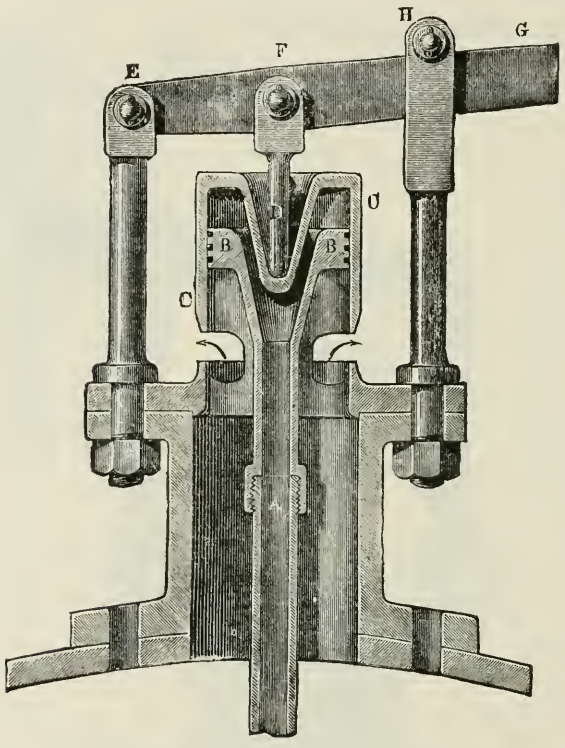
Fig. 680.



much steam escapes as was to have been expected from their cross-section. Thus it has been found by v. Burg that ordinary valves open only from $\frac{1}{8}$ to $\frac{1}{3}$ of a line (see his article "Ueber die Wirksamkeit der Sicherheitsven-

tile," Wien, 1863). In the new improved safety-valves, this defect of ordinary valves is removed, or, at least, diminished. Among others, we have the safety-valves of Hartley, Bodmer, etc. In the safety-valve of Hartley, the

Fig. 681.



usual circular orifice is replaced by two ring-shaped orifices, and also the valve plate by two valve rings, while the loading is below the valve, and placed, therefore, within the boiler. The safety-valve of Bodmer, *C C*, Fig. 680, is not raised directly by the steam pressure, but by the boiler water. For this purpose, we have a pipe *A B*, which widens above and conducts the boiler water to the interior of the valve. To facilitate the opening, the cylindrical valve *C C* is turned true, and the outer circumference of the pipe end

is grooved instead of being furnished with washers. The lever EFG turns about E and presses the valve down by means of the pin DF which butts against the conical valve cover.

§ **436.—Safety-Valves with Spring.**—In locomotives and other motors, the unavoidable vibrations prevent the use of loaded valves, and hence the less accurate spring valves must be used.

The arrangement of an ordinary safety-valve with spring is shown in Fig. 682. The end E of the lever DCE, to which the safety-valve A is attached, bears the rod FF_1,

Fig. 683.

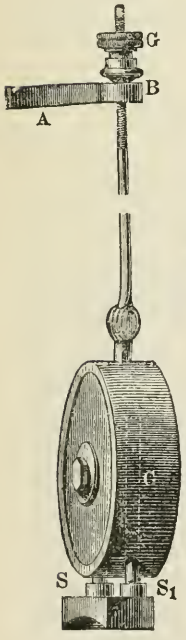


Fig. 682.

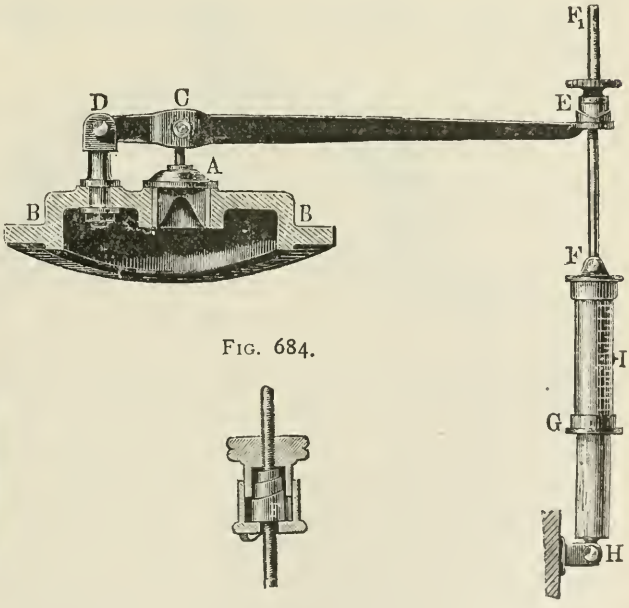


Fig. 684.

which is attached to a spiral spring contained in the cylinder FGH. While the lever end E is raised by the steam pressure, the spring is compressed, and, by screwing down

E, equilibrium is produced. A pointer *I* shows on a scale upon the cylinder *F G* the amount of steam pressure.

Safety-valves with springs possess the defect that the force opposed to the steam pressure is not constant, but increases with the opening of the valve. To avoid this defect, Meggenhofen does not allow the spring to act directly upon the lever, but only by means of an angle lever, the ratio of whose arms varies with the steam pressure.

In order, finally, to apply safety-valves with weights to locomotives, Kirchweger hangs the weight *G*, Fig. 683, by means of a spiral spring, to the lever *A B*. This spring *F* is enclosed in a box *H*, as shown in Fig. 684. To guide the weight, there are two pins *S*, S_1, which enter the weight.

§ **437.—Emptying and Opening the Boiler.**—In every steam boiler we have still

(1) The steam pipe, for drawing off the steam ;
(2) The man-hole, for entrance to the boiler ;
(3) The discharge pipe, for drawing off the water ;
(4) The blow-off pipe, for blowing off the water.

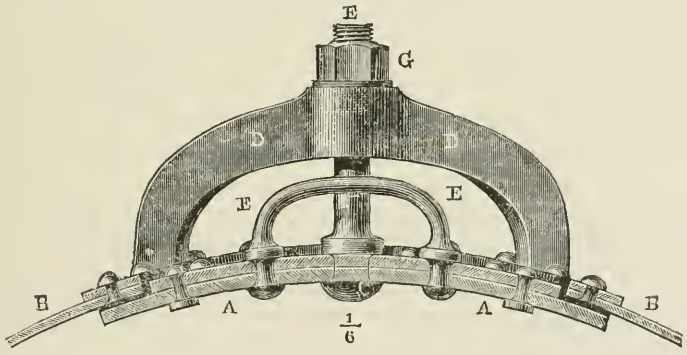
The steam pipe will be treated of in a following chapter. The man-hole is an opening of from 16 to 18 inches long and 13 inches wide, in the top of the boiler, and is closed, as shown in Fig. 685, by a cast-iron plate *A A*. In the crevice between this plate and the boiler is a ring of hemp and putty. The plate is furnished with the handle *E E* and is pressed down by the screw bolt *C F*. In recent times, stationary boilers have been furnished, like locomotive boilers, with a steam dome, in which not only the man-hole is placed, but also the feed pipe, the steam pipe, the pipes for the safety-valves, etc., so that the boiler is weakened less than when all these parts are screwed into it.

According to the Prussian regulations, this dome cannot consist of cast-iron, but must consist, like the boiler, of sheet-iron and be riveted to it.

The hole for drawing off the water is at the bottom of the boiler and is closed by a conical steel plug from within.

The feed water is never entirely pure, and, therefore, the boiler water soon becomes muddy, and it is necessary to clean out the boiler from time to time. In order to prevent this mud from collecting in the boiler, we have the blow-off cock, which opens a pipe reaching nearly to the bottom and expanded at its lower end. If this cock is opened after the firing has ceased and the steam has only a moderate tension, the water will be discharged by the pressure of the steam without danger. This blowing out is especially necessary in marine boilers which are fed with

FIG. 685.



sea water. The constituents of such water, such as lime, salt, etc., are especially injurious and form upon the interior of the boiler a hard crust called *scale*. This stony crust not only prevents the passage of heat, but acts to weaken the boiler, since the portions thus covered may be easily made red hot. In order that such deposits may not take place over the furnace, the feed water is introduced as far off as possible, and the boiler is here one to three inches deeper than at the furnace. Bottom and side plates or catch basins are also used in order to prevent the deposit of scale, or at least to prevent its deposit over the furnace. It is, of course, necessary to remove the scale

from time to time, or by chemical means (hydrochloric acid) to loosen it. By the use of soda, the deposit of scale from fatty substances is prevented.

§ **438.—Boiler Tests.**—Every boiler should be tested before it is used. Generally it is subjected to hydrostatic pressure, the safety-valve being doubly loaded. If the water at most only sweats through at the joints, the test is satisfactory. The pressure should not be carried too far, as the material may easily be thus overstrained and thus weakened by the test. According to Jobard, we can heat a boiler, entirely filled with water, until the manometer shows an excess of two to three atmospheres above the normal pressure which it is designed to sustain. This test carefully carried out is at least not so dangerous as a test by steam, and is, at the same time, more suitable than the hydraulic test, as the boiler, by the heat, is placed in a condition more nearly resembling its state in use.

In spite of all tests and all safety regulations, explosions occur, and not only the boiler and furnace, but also the building, neighboring machines, as well as the lives of the firemen, are destroyed. Unfortunately, we know thus far only the general causes of such occurrences, and are not able to assign the conditions and causes of many of those explosions which have taken place. Among the general causes of explosions, we may enumerate:

(1) Excessive steam pressure, especially combined with shocks or vibrations of the boiler.
(2) Lack of water, so that the boiler plates become red hot and steam is either generated too quickly or decomposed.
(3) Poor construction as well as bad condition of the boiler, and too severe demand upon it. For example, lack of strength in the man-hole and steam-dome flanges.
(4) Lack of proper care.
(5) Loosening of scale from the boiler plates.

(6) Too rapid introduction of feed water after a deficiency, so that the exposed plates have become red hot, thus causing a too rapid generation of steam.

(7) Sudden opening of the safety-valve, by which the equilibrium of the water and steam is destroyed and the water set into fierce commotion.

(8) Sudden evolution of steam by reason of sudden diminution of pressure.

The air introduced in the feed-water, and which is supposed to form an explosive mixture with the gas from the decomposed water, has been by some considered as the cause of explosions. According to others, explosions are caused by violent commotion of the water, and especially by the formation of water jets in the boiler, by which water instead of steam is forced through the valve and other openings.

This subject cannot be pursued further here, and we content ourselves with reference to the following literature.

Closing Remark.—Upon heating, and especially upon the generation of steam, we can recommend the following: Péclet treats the subject generally and fully in his "Traité de la chaleur," etc., II. Tom., 2d Edit., Paris, 1843. To be recommended from a practical point of view is Grouvelle et Jaunez's "Guide du chauffeur et du propriétaire des machines à vapeur," etc., 4th Edit., Paris, 1858; also Verdam's "Theory of the Steam Engine," published in German under the title, "Die Grundsätze, nach welchen alle Arten von Dampfmaschinen zu beurtheilen und zu erbauen sind." We may also refer to "Traité des machines à vapeur, par Bataille et Jullien;" or to the English original, "A Treatise on the Steam Engine," by the Artizan Club, edited by J. Bourne, London, 1846—new edition, 1861. A short discussion of this subject is given by Claudel in his "Formules, Tables, etc.," and especially by Scholl in his "Führer des Maschinisten," and Baumgartner in his "Anleitung zum Heizen der Dampfkessel." Upon economy of fuel, see E. Bède, in "Civilingenieur," Bd. 4. For experiments upon steam boilers, see E. Burnat, "Civilingenieur," Bd. 9. See also "Ordonnances du roi relat. aux appareils à vapeur, etc.," par C. E. Jullien, Paris, 1843; "Machines à vapeur, arrêtés et instructions," Bruxelles, 1844; laws and ordinances of the German states upon the setting up of boilers and engines, as, for example, the "Königl. Preuss. Regulativ," or the "Oesterr. Verordnung" (see "Polytech. Centralblatt," Bd. VI., 1845). Upon steam-boiler explosions, see "Annales des ponts et chaussées," T. IV., Paris, 1842, et seq.; "Verhand-

lungen des Preuss. Gewerbevereins," Jahrg. 20 and 21, Berlin, 1841 and 1842; "Annales des mines," T. VII., Paris, 1845, et seq.; Dingler's "Polytech. Journal," Bd. 94; also the treatises of Arago cited in the following paragraphs. Grimburg gives in the "Civilingenieur," Bd. 11, an extract from Dufour's "Sur l'ebullition de l'eau et sur une cause probable d'explosion des chaudières à vapeur." Upon safety-valves, see a treatise by Thrémery in the "Annales des mines," T. XX., 1841. Upon chimneys, see "Verhandlungen des Preuss. Gewerbevereins," Jahrg. 19, Berlin, 1840; also "Useful Information for Engineers, etc.," by W. Fairbairn, London, 1856.

Upon gas-firing, "Die Wärmemesskunst," by Schinz. In Claudel's "Collection of Formules, Tables, etc.," troisième édition, 1854, will be found data upon the heating of boilers by blast-furnace gases. From a scientific tandpoint may be recommended Th. Weiss's "AllgemeineTheorie der Feuerungsanlagen," Leipzig, 1868; "Theorie der Zugerzeugung," by Prof. F. Grashof, Berlin, 1866.

Steam-boiler explosions, especially the English association for their prevention, are treated by Prof. Hartig in a monograph, Leipzig, 1867. See also Blum, "Die Dampfkesselexplosionen," Chemnitz, 1867. Upon the causes of boiler explosions, see C. Kayser, in the "Zeitschrift des Vereins deutscher Ingenieure," Bd. IX., X., and XI. See also "Die Ursachen der Dampfkesselexplosionen," by Dr. H. Scheffler, Berlin, 1867.

In designing boilers, the results of previous careful experiments furnish valuable data. The number of really thorough experiments on record in which the time of trial was long enough to secure average conditions, and in which all necessary data were noted, is comparatively small. The results of a very extended series of boiler experiments are contained in the "Report of 'The Board of Engineers,' on the Experiments tried with the Horizontal Fire-Tube and the Vertical Water-Tube Boilers, designed for the purpose of Comparison," Philadelphia, 1868. Two boilers were designed for these tests, with the proportions generally regarded as the best in practice, and these proportions were varied greatly throughout the trials. Subsequent experiments, with boilers of various forms, tend to corroborate the statement that, other things being equal, the proportions and rate of combustion, rather than the designs, determine the efficiency of a boiler of particular design. In this view of the case, the experiments referred to, covering as they do a wide range of proportions and rates of combustion, furnish very valuable data for comparing the performance of other boilers, and for determining the proper proportions for boilers to fulfil most conditions that occur in practice. In the tables that follow, a summary of the most important features of these experiments is presented, and two other tables are added, for facilitating the comparison of the results with each other, or with those obtained from other boilers.

Good anthracite coal was used in making these experiments, and the quality was maintained uniform, as far as possible, throughout the whole series. All the steam formed was discharged at atmospheric pressure, and,

with two or three exceptions, each experiment lasted for forty-eight hours or longer. The principal dimensions of the boilers were as follows:

	Horizontal Fire-Tube Boiler.	Vertical Water-Tube Boiler.
Weight in pounds	39,480	42,088
Number of tubes	162	748
External diameter of tubes, in.	2½	2
Thickness " " "	0·109	0·109
Number of rows " "	9	34
Number of tubes in a row	18	22
Number of tube boxes	2	2
Number of furnaces	2	2
Length of each furnace, feet	6	6·5
Width " " " "	3	3
Heating surface, furnace, sq. ft.	96·57	96·76
" " back connection, "	116·66	73·97
" " tubes, "	701·80	929·80
" " smoke box, "	34·90	17·21
" " tube boxes, "	.....	147·07
" " total, "	949·93	1264·81
Length of boiler	10 ft. 4 in.	10 ft. 6 in
Width " "	7 " 5½ "	7 " 5½ "
Height " "	9 " 7 "	9 " 7 "

It will be observed that the rate of evaporation in the following tables is referred to the combustible (or pounds of coal consumed less ashes withdrawn) rather than to the coal consumed. This is done in order to make the experiments more strictly comparative. Of course, a smaller per cent of ashes in any particular case is favorable to that experiment, since some heat is carried off by the refuse, and the accumulation of ashes in a furnace retards combustion, so that the amount of refuse should be the same, other things being equal in experiments that are absolutely comparative. This is not possible in practice, however, and a comparison of the effects produced by the combustible is the most accurate that can be made.

In making experiments on boilers, it is generally important to test the quality of the steam, for the purpose of ascertaining whether or not it has water entrained with it. Although no such tests were made in the case of the experiments under consideration, it is reasonable to conclude that the steam was dry, since all the circumstances were favorable to this result, the evaporation taking place at atmospheric pressure, and in boilers with ample steam room.

SUMMARY OF EXPERIMENTS WITH HORIZONTAL FIRE-TUBE BOILER, 1865, 1866.

Number for reference.	Pounds of combustible per hour.			Pounds of water evaporated from and at 212°.			Ratio.		Per cent of refuse.	Draught.	Per cent of evaporation for draught.	Temperature of uptake, Fahrenheit degrees.	Remarks.	Number for reference.
	Total.	Per sq. foot of grate surface.	Per sq. foot of heating surface.	Per pound of combustible.	Per. sq. foot of grate surface per hour.	Per sq. foot of heating surface per hour.	Heating to grate surface.	Draught area to grate surface.						
1	699	19·43	·736	9·55	224·4	8·5	26·4	·128	17·4	Natural.	....		Boiler as built.	1
2	711	19·75	·748	9·43	232·9	8·82	26·4	·128	20·	"	....		" " "	2
3	704	19·54	·746	9·52	185·8	7·04	26·4	·128	21·3	"	. .		" " "	3
4	718	18·77	·716	9·55	179·3	6·79	26·4	·128	23·	"	....		" " "	4
5	473	13·13	·497	11·50	150·7	5·71	26·4	·128	22·3	"	...		" " "	5
6	608	16·9	·64	10·71	180·8	6·85	26·4	·128	23·2	"	....		" " "	6
7	282	7·84	·297	12·45	98·	3·71	26·4	·128	23·7	"	....	512		7
8	535	17·82	·562	10·79	192·2	6·06	31·7	·153	21 7	"	....	Lead melts.	G. S. reduced to 30·	8
9	260	8·65	·273	12·2	105·5	3·33	31·7	·153	18·9	"	....	510	" " " 30·	9
10	424	17·68	·448	11·52	203·6	5·15	39·5	·192	22·	"	....	Lead melts.	" " " 24·	10
11	209	8·71	·221	11·65	101·9	2·58	39·5	·192	16·4	"	....	431	" " " 24·	11
12	344	19·1	·362	1·52	219·7	4·16	52·8	·256	22·8	"	....	570	" " " 18·	12
13	152	8·43	·16	12·29	103·7	1·96	52·8	·256	20·3	"	....	399	" " " 18·	13
14	257	19·02	·27	11·26	214·7	3·05	70·4	·341	20·4	"	....	478	" " " 13·5.	14
15	109	8·08	·115	12·53	101·	1·43	70·4	·341	23·3	"	. .	360	" " " 13·5.	15
16	256	7·12	·27	12·71	90·4	3·43	26·4	·128	19·6	"	....	510	Boiler as built.	16
17	645	17·92	·679	9·99	178·8	6·77	26·4	·128	16·1	"	....	Lead melts.	" " "	17
18	135	3·75	·142	13·08	48·8	1·85	26·4	·128	13·5	"	....	371	" " "	18
19	601	16·69	·632	9·96	166·3	6·31	26·4	·128	18·6	"	....	Lead melts.	" " "	19
20	437	12·15	·49	11·05	134 3	5·09	26·4	·128	16·7	"	....	478	" " "	20
21	635	17·64	·668	10·45	184·8	7·01	26·4	·128	17·6	"	. ..	Lead melts.	" " "	21
22	565	15·68	·594	10·60	166·4	6·31	26·4	·128	19·4	"	...	" "	" " "	22
23	438	15·2	·461	11·52	174·8	5·3	33·	·16	20·9	"	....	" "	G. S. reduced to 28·8.	23
24	400	18·51	·421	11·62	214·6	4·88	44·	·213	22·5	"	....	" "	" " " 21·6.	24
25	423	14·6	·111	11·59	169·4	1·3	131·	·635	20·2	"	....	" "	" " " 7·25.	25
26	663	18·43	·698	10 44	191·4	7·25	26·4	·128	17·	"	....	Lead and zinc melt.	Boiler as built.	26
27	733	20·35	·771	7·51	153·2	5·81	26·4	·128	18·1	"	....	Zinc melts.	" " "	27
28	668	18·56	·703	9·47	170·1	6·67	26·4	·128	15·7	"	....	" "	" " "	28
29	772	21·43	·812	7·41	158·6	7·03	26·4	·128	18·2	"	....	" "	" " "	29
30	671	18·64	·706	9·75	181·4	6·87	26·4	·128	16·9	"	....	" "	" " "	30
31	482	21·92	·128	11·20	245·3	1·43	171·	·83	16·3	"	....	Lead melts.	G. S. reduced to 5·54.	31
32	669	18·58	·704	10·78	199·8	7·57	26·4	·116	16·2	"	....	Zinc "	Ferrules in 4 upper rows of tubes.	32
33	665	18·49	·701	10·78	199·8	7·57	26·4	·102	16·3	"	...	Lead and zinc melt.	" " all tubes.	33
34	671	18·64	·706	10·91	202·7	7·68	26·4	·095	16·4	"	....	" " "	" " " "	34
35	597	16·59	·694	11·42	189·2	7·92	23·9	·114	19·4	"	....	Zinc melts.	Upper row of tubes plugged.	35
36	600	16·67	·779	11·19	185·9	8·69	21·4	·1	19·5	"	....	" "	More tubes plugged.	36
37	628	17·44	·661	11·26	196·6	7·45	26.4	·128	22·2	"	....	" "	Boiler as built.	37

SUMMARY OF EXPERIMENTS WITH HORIZONTAL FIRE-TUBE BOILER, 1865, 1866—(*Continued*).

Number for reference.	Pounds of combustible per hour.			Pounds of water evaporated from and at 212°.			Ratio.		Per cent of refuse.	Draught.	Per cent of evaporation for dra'ght.	Temperature of uptake, Fahrenheit degrees.	Remarks.	Number for reference.
	Total.	Per sq. foot of grate surface.	Per sq. foot of heating surface.	Per pound of combustible.	Per sq. foot of grate surface per hour.	Per sq. foot of heating surface per hour.	Heating to grate surface.	Dra'ght area to grate surface.						
38	669	18·59	·704	10 68	198·	7·5	26·4	·128	17·2	Natural.	....	Lead melts.	Flat deflecting plate in back connection.	38
39	616	17·12	·649	10·54	179·6	6·8	26·4	·128	20·7	"	...	Lead and zinc melt.	Concave " " " " "	39
40	675	18·74	·71	10·56	198·2	7·51	26·4	·128	17·5	"	....	Zinc melts.	Two " plates " " "	40
41	805	22·37	·847	10·	223·7	8·47	26·4	·128	16·3	Jet.	4 01	" "	Boiler as built.	41
42	780	21·67	·821	9·88	214·4	8·12	26·4	·128	18·5	Fan.	3 82	" "	" " "	42
43	783	21·76	·824	9·25	201·4	7·63	26·4	·128	17·7	"	4·49	" "	" " "	43
44	778	21·62	·819	11·06	239·8	9·08	26·4	·128	20·8	"	9·38	" "	" " "	44
45	797	22·15	·839	11·36	251·9	9·54	26·4	·128	18·9	"	9·25	" "	" " "	45
46	546	15·16	·634	12·05	182·7	7·64	23·9	·114	20·8	Natural.	....	Lead "	Upper row of tubes plugged.	46
47	536	14·88	·564	12·75	189·2	7·17	26·4	·128	21·5	"	....	" "	Boiler as built.	47
48	549	15·24	·577	12·17	185·4	7·02	26·4	·128	20·3	"	....	" "	" " "	48
49	780	21·66	·821	11·18	241 9	9·16	26·4	·128	24·2	Fan.	11·2	Zinc "	" " "	49
50	513	19·	·54	12 92	245·1	6·96	35·2	·148	22·4	"	10·1	....	G. S. reduced to 27. Ferrules in 4 upper rows	50
51	659	18·3	·847	11·23	205·	7·76	26·4	·128	18·2	Jet.	5 05	....	Boiler as built. [of tubes.	51
52	671	18·63	3·15	5·86	109·	18·5	5·9	·067	19·3	Natural.	....	....	Tubes closed.	52
53	259	7·88	1·33	8·24	64·9	11·	5·9	·067	17·	"	....	....	" "	53
54	494	13·73	2·32	6·89	94·4	15·9	5·9	·067	17·9	"	....	....	" "	54
55	556	15·45	·585	11·47	177·1	6·71	26·4	·087	18·9	"	....	....	Ferrules in tubes.	55
56	558	15·49	·587	11·37	175·6	6·65	26·4	·111	18·6	"	...	....	" " "	56
57	505	14·03	·635	11·58	162·4	7·35	22·1	·099	20·2	"	....	Lead melts.	Two upper rows of tubes plugged.	57
58	482	13·4	·673	11·67	156·8	7·88	19·9	·085	18·9	"	....	" "	Three " " " " "	58
59	417	11·57	·654	12·77	147·2	8·32	17·7	·071	21·	"	....	" "	Four " " " " "	59
60	403	11·21	·723	11·81	132·2	8 53	15·5	·057	18·7	"	....	" "	Five " " " " "	60
61	310	8 62	·643	12·28	106·	7·91	13·4	·043	21·4	"	...	" "	Six " " " " "	61
62	210	5·82	·52	12·38	84·6	7·55	11·2	·028	19·6	"	....	409	Seven " " " " "	62
63	88	2·43	·267	13·6	33 1	3·63	9·1	·014	21·3	"	....	383	Eight " " " " "	63
64	507	14·09	·534	11·72	165·	6·26	26·4	·083	19·5	"	....	Lead and zinc melt.	Ferrules in tubes.	64
65	546	15·18	·575	11·55	174·8	6 62	26·4	·083	18 9	"	. .	Lead melts.	" " "	65
66	519	14·42	·546	11·75	168·5	6 38	26·4	·083	18·	"	...	" "	" " "	66
67	356	11·4	·282	11 60	132·2	3·27	40·4	·142	15·4	"	....	" "	G. S. reduced to 31·25.	67
68	571	18·28	·452	11·01	201·3	4·98	40·4	·142	15·	"	....	" "	" " " 31·25.	68
69	453	14·51	·359	11·47	166·8	4·13	40·4	·142	13·9	"	....	" "	" " " 31·25.	69
70	157	5·03	·125	12·63	63·4	1·57	40·4	·142	17·6	"	....	428	" " " 31·25.	70
71	123	3·42	·13	11·25	38·3	1·45	26·4	·1	15·	"	....	337	Ferrules in tubes.	71
72	244	6·79	·257	11·96	81·5	3·09	26·4	·1	13·6	"	....	427	" " "	72
73	370	10·26	·389	11·81	121·5	4·6	26 4	·1	13·6	"	....	477	" " "	73

SUMMARY OF EXPERIMENTS WITH VERTICAL WATER-TUBE BOILER, 1865, 1866.

Number for Reference.	Pounds of combustible per hour.			Pounds of water evaporated from and at 212°.			Ratio.		Per cent of refuse.	Draught.	Per cent of evaporation for dra'ght.	Temperature of uptake, Fahrenheit degrees.	Remarks.	Number for Reference.
	Total.	Per sq. foot of grate surface.	Per sq. foot of heating surface.	Per pound of combustible.	Per sq. foot of grate surface per hour.	Per sq. foot of heating surface per hour.	Heating to grate surface.	Dra'ght area to grate surface.						
1	500	12·81	·395	10·83	175·0	5·41	32·4	·142	21·2	Natural.	. . .	....	Boiler as built.	1
2	480	12·31	·380	12·13	148·8	4·59	32·4	·142	21·7	"	....	....	" " "	2
3	494	12·66	·391	11·88	150·6	4·65	32·4	·142	23·6	"	...	459	" " "	3
4	489	12·55	·387	11·79	147·9	4·57	32·4	·142	23·5	"	....	437	" " "	4
5	462	11·84	·365	11·99	141·6	4·37	32·4	·142	23·9	"	...	409		5
6	414	11·51	·355	12·29	123·1	3·80	35·1	·154	23·5	"	...	339	G. S. reduced to 36.	6
7	285	7·30	·225	13·27	97·1	2·99	32·4	·142	22·9	"	....	324	Boiler as built.	7
8	348	11·61	·275	13·01	150·8	3·57	42·2	·185	24·3	"	....	320	G. S. reduced to 30·0.	8
9	258	8·60	·204	13·33	114·4	2·71	42·2	·185	19·4	"	....	294	" " " 30·0.	9
10	292	12·17	·231	13·55	164·7	3·13	52·7	·231	23·9	"	....	287	" " " 24·0.	10
11	207	8·64	·164	12·70	109·7	2·08	52·7	·231	17·0	"	....	266	" " " 24·0.	11
12	228	12·66	·180	13·28	168·9	2·40	70·3	·308	25·3	"	....	264	" " " 18·0.	12
13	149	8·27	·118	13·44	110·8	1·58	70·3	·308	21·7	"	....	242	" " " 18·0.	13
14	162	12·02	·128	12·84	153·6	1·64	93·7	·411	23·1	"	....	236	" " " 13·5.	14
15	108	8·00	·085	13·79	110·4	1·18	93·7	·411	24·2	"	....	232	" " " 13·5.	15
16	254	6·51	·201	13·55	87·9	2·71	32·4	·142	20·4	"	....	299	Boiler as built.	16
17	633	16·23	·501	10·54	170·1	5·25	32·4	·142	17·4	Jet.	10·18	about 500, variable.	" " "	17
18	134	3·44	·106	13·70	47·1	1·45	32·4	·142	15·3	Natural.	....	269	" " "	18
19	370	10·27	·296	12·70	130·8	3·73	35·1	·154	21·3	"	....	389	G. S. reduced to 36·0.	19
20	428	11·88	·339	12·16	145·2	4·13	35·1	·154	18·4	"	....	375	" " " 36·0.	20
21	626	16·03	·495	11·02	176·6	5·45	32·4	·142	18·8	Jet.	5·37	489	Boiler as built.	21
22	545	15·15	·468	11·00	166·7	4·75	35·1	·154	22·0	"	7·17	447	G. S. reduced to 36·0.	22
23	299	9·59	·237	13·02	124·7	3·08	40·5	·178	24·5	Natural.	....	314	" " " 31·2.	23
24	285	12·19	·225	13·25	161·0	2·98	54·1	·237	26·0	"	....	323	" " " 23·4.	24
25	327	10·42	·091	13·12	136·2	3·38	40·3	·177	23·3	"	....	354	" " " 31·4.	25
26	661	18·36	·523	10·64	195·0	5·56	35·1	·154	16·7	Jet	8·68	Lead melts, and lower temperature.	" " " 36·0.	26
27	450	12·49	·385	12·76	160·0	4·94	35·1	·154	19·9	"	7·74	382	" " " 36·0.	27
28	530	14·71	·454	13·02	199·8	6·17	35·1	·154	19·8	"	4·99	548	" " " 36·0.	28
29	497	13·81	·426	12·88	178·0	5·07	35·1	·154	19·5	"	4·29	437	" " " 36·0.	29
30	531	14·75	·455	12·94	190·9	5·89	35·1	·154	19·5	"	3·49	464	" " " 36·0.	30
31	577	14·78	·456	11·82	174·6	5·39	32·4	·142	18·3	"	3·54	484	Boiler as built.	31
32	626	17·39	·496	11·62	201·8	5·75	35·1	·154	16·7	"	6·09	Lead melts.	G. S. reduced to 36·	32
33	620	15·89	·491	12·10	192·4	5·94	32·4	·142	16·8	"	5·85	"	Boiler as built.	33
34	627	16·07	·496	11·97	191·6	5·91	32·4	·142	16·8	"	4·69	"	" " "	34
35	591	16·42	·468	12·30	201·7	5·75	35·1	·154	17·3	"	4·93	585	G. S. reduced to 36·	35

SUMMARY OF EXPERIMENTS WITH VERTICAL WATER-TUBE BOILER, 1865, 1866—(*Continued*).

Number for Reference.	Pounds of combustible per hour.			Pounds of water evaporated from and at 212°.			Ratio.		Per cent of refuse.	Draught.	Per cent of evaporation for draught.	Temperature of uptake, Fahrenheit degrees.	Remarks.	Number for Reference.
	Total.	Per sq. foot of grate surface.	Per sq. foot of heating surface.	Per pound of combustible.	Per sq. foot of grate surface per hour.	Per sq. foot of heating surface per hour.	Heating to grate surface.	Draught area to grate surface.						
36	521	14·47	·412	12·00	174·0	4·96	35·1	·154	19·7	Jet.	7·93	495	G. S. reduced to 36.	36
37	545	15·14	·431	12·81	193·3	5·50	35·1	·154	20·8	"	5·95	506	" " " 36.	37
38	587	16·31	·465	12·57	205·4	5·85	35·1	·154	16·4	"	4·34	522	" " " 36.	38
39	496	12·71	·392	13·11	166·6	4·45	32·4	·142	15·9	"	3·77	379	Boiler as built.	39
40	379	15·92	·081	12·74	201·9	3·8	53·1	·233	15·9	Natural.	....	454	G. S. reduced to 23·8.	40
41	675	17·30	·534	11·32	195·5	6·03	32·4	·129	15·3	Jet.	5·66	596	Draught area reduced by plates.	41
42	662	16·97	·524	11·54	195·5	6·04	32·4	·114	16·5	"	7·39	Lead melts.	" " " " rods in tube spaces.	42
43	646	16·56	·511	11·85	195·9	6·04	32·4	·142	16·9	"	5·86	524	Smaller chimney.	43
44	667	17·10	·528	11·39	194·9	6·02	32·4	·114	16·6	"	5·25	Lead melts.	Draught area reduced by rods in tube spaces.	44
45	558	14·32	·408	12·01	171·6	5·3	32·4	·125	18·8	Natural.	....	448	" " " " " " " "	45
46	505	12·94	·369	12·39	160·0	4·94	32·4	·111	17·6	"	....	415	" " " " " " " "	46
47	486	12·45	·355	12·61	157·5	4·86	32·4	·100	18·4	"	....	414	" " " " " " " "	47
48	599	15·37	·781	11·22	172·5	8·76	19·7	·126	18·8	Jet.	5·43	487	Bars in upper tube spaces.	48
49	596	15·29	·899	11·01	168·3	9·90	17·0	·110	19·1	"	5·55	497	" " " " "	49
50	369	9·47	·202	13·30	126·0	3·89	32·4	·100	18·6	Natural.	....	352	" between tubes.	50
51	449	11·51	·355	13·15	151·8	4·69	32·4	·100	22·2	"	....	395	" " "	51
52	248	6·36	·196	13·40	85·2	2·63	32·4	·100	17·4	"	....	291	" " "	52
53	637	17·69	·504	11·23	198·2	5·65	35·1	·154	20·1	Jet.	5·99	484	" " " G. S. reduced to 36.	53
54	666	17·08	·527	11·15	189·8	5·86	32·4	·142	17·0	"	4·30	481	Boiler as built.	54
55	602	15·44	·477	11·70	180·2	5·56	32·4	·142	22·5	"	5·95	573	" " "	55
56	671	17·22	·532	11·03	189·2	5·84	32·4	·142	17·4	"	4·90	588	" " "	56
57	657	16·83	·519	11·76	198·2	6·12	32·4	·142	18·2	"	4·59	488	" " "	57
58	782	20·06	·619	9·80	197·0	6·08	32·4	·142	18·2	Fan.	3·98	Lead and zinc melt.	" " "	58
59	779	21·65	·617	9·23	200·3	5·70	35·1	·154	17·8	"	4·56	" " " "	G. S. reduced to 36.	59
60	1030	26·93	·831	7·17	192·9	5·95	32·4	·142	16·8	"	6·37	Zinc melts.	Boiler as built.	60
61	167	4·27	·132	14·37	61·5	1·90	32·4	·100	12·1	Natural.	....	199	Bars between tubes.	61
62	679	17·40	·537	12·51	217·5	6·71	32·4	·142	16·7	Fan.	6·65	501	Boiler as built.	62
63	760	19·49	·602	12·47	242·5	7·48	32·4	·142	17·4	"	7·33	Lead melts.	" " "	63
64	736	18·87	·582	13·32	230·6	7·12	32·4	·100	17·3	"	8·19	" "	Bars between tubes.	64
65	337	8·65	·258	14·06	117·7	3·63	32·4	·111	23·8	Natural.	....	361	" " "	65
66	550	15·29	·436	11·33	172·9	4·93	35·1	·154	19·6	Jet.	5·80	426	G. S. reduced to 36.	66
67	304	7·78	·240	14·86	115·9	3·58	32·4	·100	20·8	Natural.	....	322	Bars between tubes.	67
68	545	13·97	·431	13·39	187·6	5·79	32·4	·111	20·9	Jet.	8·81	440	" " "	68
69	324	8·31	·257	13·75	114·7	3·54	32·4	·100	16·1	Natural.	....	328	" " "	69
70	701	17·98	·555	12·14	217·8	6·72	32·4	·142	20·9	Fan.	12·08	524	Boiler as built.	70
71	668	17·12	·529	12·98	222·3	6·86	32·4	·100	19·1	"	8·06	507	Bars between tubes.	71

SUMMARY OF EXPERIMENTS WITH VERTICAL WATER-TUBE BOILER, 1865, 1866—(*Continued*).

Number for Reference.	Pounds of combustible per hour.			Pounds of water evaporated from and at 212°.			Ratio.		Per cent of refuse.	Draught.	Per cent of evaporation for dra'ght.	Temperature of uptake, Fahrenheit degrees.	Remarks.	Number for Reference.
	Total.	Per sq. foot of grate surface.	Per sq. foot of heating surface.	Per pound of combustible.	Per sq. foot of grate surface per hour.	Per sq. foot of heating surface per hour.	Heating to grate surface.	Dra'ght area to grate surface.						
72	488	12·51	·385	12·95	148·4	4·59	32·4	·142	18·7	Jet.	4·70	413	Boiler as built.	72
73	492	12·61	·389	13·10	165·1	5·10	32·4	·142	20·2	"	7·36	415	" " "	73
74	661	16·95	3·66	6·13	104·2	22·50	4·63	·068	19·5	Natural.	....	....	Tube spaces closed.	74
75	261	6·70	1·45	8·14	54·5	11·80	4·63	·068	16·3	"	....	....	" " "	75
76	495	12·70	2·74	7·03	89·3	19·00	4·63	·068	17·8	"	....	....	" " "	76
77	483	17·50	·540	12·75	204·8	6·32	32·4	·100	19·1	Fan.	8·05	442	Smaller chimney. Plates in tube spaces.	77
78	665	17·05	·526	12·43	194·9	6·02	32·4	·100	18·3	"	7·97	470	" " " " " "	78
79	246	13·64	·194	12·66	162·7	2·31	70·3	·111	18·0	Jet.	10·07	355	G. S. reduced to 18· Draught area reduced.	79
80	570	14·62	·451	12·19	178·2	5·53	32·4	·100	18·3	"	4·65	533	Draught area reduced.	80
81	180	9·96	·142	13·31	132·5	1·88	70·3	·111	19·5	Natural.	....	316	G. S. reduced to 18· Draught area reduced.	81
82	103	5·94	·084	13·44	79·6	1·13	70·3	·111	14·9	"	....	268	" " " 18· " " "	82
83	495	12·68	·391	12·09	152·5	4·71	32·4	·142	16·4	"	....	607	Deflecting plates in tube box.	83
84	400	10·26	·317	13·16	134·6	4·16	32·4	·142	14·0	"	...	407	" " " " " "	84
85	403	10·32	·319	13·11	134·9	4·16	32·4	·142	13·6	"	....	420	" " " " " "	85
86	572	14·66	·453	12·59	184·0	5·68	32·4	·142	16·1	Jet.	5·14	488	" " " " " "	86
87	390	9·99	·308	13·23	131·9	4·07	32·4	·142	16·2	Natural.	....	403	" " " " " "	87
88	779	19·96	·616	12·03	240·0	7·41	32·4	·142	16·1	Fan.	7·97	571	" " " " " "	88
89	393	10·08	·323	13·20	132·0	4·23	31·2	·142	15·2	Natural.	....	396	3 rows of tubes removed.	89
90	427	11·72	·376	12·55	147·4	4·73	31·2	·142	17·2	"	....	416	3 " " " " "	90
91	677	17·37	·557	12·16	211·1	6·76	31·2	·100	17·5	Fan.	7·77	509	3 " " " " " Draught area reduced.	91
92	392	10·05	·345	12·89	129·0	4·43	29·1	·142	15·5	Natural.	....	421	5 " " " " "	92
93	431	12·58	·432	12·63	158·8	5·46	29·1	·142	17·9	"	....	480	5 " " " " "	93
94	401	10·28	·381	12·68	129·5	4·80	27·0	·142	13·8	"	....	432	9 " " " " "	94
95	509	13·06	·484	12·05	157·2	5·82	27·0	·142	16·6	"	....	491	9 " " " " "	95
96	395	10·14	·407	12·50	126·3	5·07	24·9	·142	14·6	"	. ..	461	12 " " " " "	96
97	568	14·55	·585	11·77	171·1	6·87	24·9	·142	16·4	"	.. .	600	12 " " " " "	97
98	402	10·32	·452	12·08	124·6	5·47	22·8	·142	14·2	"	....	514	15 " " " " "	98
99	543	13·93	·611	11·69	162·6	7·13	22·8	·142	16·5	"	...	Lead and zinc melt.	15 " " " " "	99
100	401	10·37	·501	12·19	124·4	6·01	20·7	·142	13·7	"	....	504	18 " " " " "	100
101	620	15·89	·767	11·20	177·0	8·55	20·7	·142	18·3	"	....	Lead and zinc melt.	18 " " " " "	101
102	396	10·15	·540	11·64	118·3	6·29	18·8	·142	15·8	"	....	" " " "	21 " " " " "	102
103	636	16·31	·867	10·69	174·4	9·28	18·8	·142	16·8	"	...	" " " "	21 " " " " "	103
104	394	10·10	·612	11·41	115·1	6·98	16·5	·142	16·0	"	....	" " " "	24 " " " "	104

In the two tables that follow, the principal results of the experiments are arranged in the order of their relative standing. These tables will be found useful in making comparisons with other experiments, and also in determining the effect of different proportions and different rates of combustion:

EXPERIMENTS WITH HORIZONTAL FIRE-TUBE BOILER, ARRANGED WITH REFERENCE TO ECONOMY, CAPACITY, ETC.

Relative number.	Combustible per hour.		Combustible per sq. ft. G.S. per hour.		Combustible per sq. ft. H. S. per hour.		Equivalent evaporation pound of combustible.		Equivalent evaporation per sq. ft. G. S. per hour.		Equivalent evaporation per sq. ft. H. S. per hour.		Relative number.
	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	
1	63	88	63	2·43	25	·111	52	5·86	63	33·1	15	1·43	1
2	15	109	71	3·42	15	·115	54	6·89	71	38·3	71	1·45	2
3	71	123	18	3·75	70	·125	29	7·41	18	48·8	70	1·57	3
4	18	135	70	5·03	31	·128	27	7·51	70	63·4	18	1·85	4
5	13	152	62	5·82	71	·13	53	8·24	53	64·9	13	1·96	5
6	70	157	72	6·79	18	·142	43	9·25	72	81·5	11	2·58	6
7	11	209	16	7·12	13	·160	2	9·43	62	84·0	14	3·05	7
8	62	2 0	7	7·84	11	·221	28	9·47	16	90·4	72	3·09	8
9	72	244	53	7·88	72	·257	3	9·52	54	94·4	67	3·27	9
10	16	256	15	8·08	63	·207	1	9·55	7	98·	9	3·33	10
11	14	257	13	8·43	11	·27	4	9·55	15	101·	16	3·43	11
12	53	259	61	8·62	16	·27	30	9·75	11	101·9	63	3·03	12
13	9	260	9	8·63	9	·273	42	9·88	13	103·7	7	3·71	13
14	7	282	11	8·71	67	·282	19	9·96	9	105·5	69	4·13	14
15	61	310	73	1 ·26	7	·297	17	9 99	61	106·	12	4·16	15
16	12	344	60	11·21	69	·359	41	10·	52	109·	73	4·6	16
17	67	356	67	11·6	12	·362	26	10·44	73	121·5	24	4·88	17
18	73	370	59	11·87	73	·389	21	10·45	60	132·2	68	4·93	18
19	24	400	20	12·15	24	·421	39	10·54	67	132·2	20	5·09	19
20	60	403	5	13·13	10	·448	40	10·56	20	134·3	10	5·13	20
21	59	417	58	13·4	68	·452	22	10·6	59	147·2	25	5·16	21
22	25	423	54	13·73	20	·46	38	10·68	5	150·7	23	5·3	22
23	10	424	57	14·03	23	·461	6	10·71	27	153·2	31	5·63	23
24	20	437	64	14·09	5	·497	32	10·78	58	156·8	5	5·71	24
25	23	438	66	14·42	62	·52	33	10·78	29	158·6	27	5·81	25
26	69	453	69	14·51	64	·534	8	10·79	57	162·4	29	6·01	26
27	5	473	25	14·6	50	·54	34	10·91	64	165·	8	6·06	27
28	31	482	47	14·88	66	·546	68	11·01	19	166·3	64	6·26	28
29	58	482	40	15·16	8	·562	20	11·05	22	166·4	19	6·31	29
30	54	494	65	15·18	47	·564	44	11·06	69	166·8	21	6·31	30
31	57	505	23	15·2	65	·575	49	11·18	66	168·5	65	6·38	31
32	64	507	48	15·24	48	·577	36	11·19	25	169·4	65	6·62	32
33	50	513	55	15·45	55	·585	31	11·2	23	174·8	56	6 65	33
34	66	519	56	15·49	56	·587	51	11·23	65	174·8	28	6·67	34
35	8	535	22	15·68	22	·594	71	11·25	56	175·6	53	6·71	35
36	47	536	35	16·59	19	·632	37	11·29	28	176·1	17	6·77	36
37	46	546	36	16·67	46	·634	14	11·27	55	177·1	4	6·79	37
38	65	546	19	16·69	57	·635	45	11 36	17	178·8	33	6·8	38
39	48	549	6	16·9	6	·64	56	11·37	4	179·3	6	6·85	39
40	55	556	39	17·12	61	·643	35	11·42	39	179·6	30	6·87	40
41	56	558	37	17·44	39	·649	55	11·47	0	180·8	50	6·96	41
42	22	565	21	17·64	59	·654	69	11·47	30	181·4	21	7·01	42
43	63	571	10	17·68	37	·661	5	11·5	46	182·7	48	7·02	43
44	35	597	8	17·82	21	·668	10	11·52	21	184·8	3	7·04	44
45	36	600	17	17·92	58	·673	12	11·52	83	185 4	47	7·17	45
46	19	601	68	18·23	17	·679	23	11·52	3	185·8	26	7·25	46
47	6	608	51	18·3	35	·694	65	11·55	36	185·9	57	7·35	47
48	39	616	26	18·43	20	·698	57	11·58	35	189·2	37	7·45	48
49	37	628	33	18·47	33	·701	25	11·59	47	189·2	38	7·5	49
50	21	635	24	18·51	28	·7 3	67	11·6	26	191·4	40	7·51	50
51	17	645	23	18·56	32	·703	24	11·62	8	192·2	62	7·55	51

EXPERIMENTS WITH HORIZONTAL FIRE-TUBE BOILER.—
(*Continued.*)

Relative number.	Combustible per hour.		Combustible per sq. ft. G.S. per hour.		Combustible per sq. ft. H. S. per hour.		Equivalent evaporation pound of combustible.		Equivalent evaporation per sq. ft. H. S. per hour.		Equivalent evaporation per sq. ft. H. S. per hour.		Relative number.
	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	
52	51	659	32	18·58	38	·704	11	11·65	37	196·6	32	7·57	52
53	26	663	33	18·59	30	·706	53	11·67	38	198·	33	7·57	53
54	33	665	52	18·03	34	·706	04	11·72	40	198·2	43	7·63	54
55	28	668	30	18·64	40	·71	66	11·75	32	199·8	46	7·64	55
56	32	669	34	18·64	4	·716	60	11·81	33	199·8	34	7·68	56
57	38	669	40	18·74	60	·723	73	11·81	08	201·3	51	7·76	57
58	30	671	4	18·77	1	·730	72	11·96	43	201·4	58	7·88	58
59	34	671	50	19·	3	·746	46	12·05	34	202·7	61	7·91	59
60	52	671	14	19·02	2	·748	48	12·17	10	203·6	35	7·92	60
61	40	675	12	19·1	27	·771	9	12·2	51	205·	42	8·12	61
62	1	699	1	19·43	36	·779	61	12·28	42	214·4	59	8·32	62
63	3	704	3	19·54	29	·812	13	12·29	24	214·6	41	8·47	63
64	2	711	2	19·75	44	·819	62	12·38	14	214·7	1	8·5	64
65	4	718	27	20·35	42	·821	7	12·45	12	219·7	60	8·53	65
66	27	733	29	21·43	49	·821	15	12·53	41	223·7	36	8·69	66
67	29	772	44	21·62	43	·824	70	12·63	1	224·4	2	8·82	67
68	44	778	49	21·66	45	·839	16	12·71	2	232·9	44	9·08	68
69	42	780	42	21·67	41	·847	47	12·75	44	239·8	49	9·16	69
70	49	780	43	21·76	51	·847	59	12·77	49	241·9	45	9·54	70
71	43	783	31	21·92	53	1·33	50	12·92	50	245·1	53	11·	71
72	45	797	45	22·15	54	2·32	18	13·08	31	245·3	54	15·9	72
73	41	805	41	22·37	52	3·15	63	13·6	45	251·9	52	18·5	73

EXPERIMENTS WITH VERTICAL WATER-TUBE BOILER, ARRANGED WITH REFERENCE TO ECONOMY, CAPACITY, ETC.

Relative number.	Combustible per hour.		Combustible per sq. ft. G. S. per hour.		Combustible per sq. ft. H. S. per hour.		Equivalent evaporation per pound of combustible.		Equivalent evaporation per sq. ft. G. S. per hour.		Equivalent evaporation per sq. ft. H. S. per hour.		Relative number.
	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	
1	15	108	13	3·44	40	·081	74	6·13	18	47·1	82	1·13	1
2	82	108	61	4·27	82	·084	70	7·03	75	54·5	15	1·18	2
3	18	134	8	5·94	15	·085	60	7·17	61	61·5	18	1·45	3
4	13	149	52	6·36	25	·091	75	8·14	82	79·6	13	1·58	4
5	14	152	16	6·51	18	·106	59	9·23	52	85·2	14	1·64	5
6	61	167	75	6·7	13	·118	58	9·8	16	87·9	81	1·83	6
7	81	180	7	7·3	14	·128	17	10·54	76	89·3	61	1·9	7
8	11	207	67	7·78	61	·132	26	10·64	7	97·1	11	2·08	8
9	12	228	15	8·	81	·142	103	40·69	74	104·2	79	2·31	9
10	79	246	13	8·27	11	·194	1	10·83	11	103·7	12	2·4	10
11	52	248	69	8·31	12	·180	22	11·	15	110·4	52	2·63	11
12	16	254	9	8·6	79	·194	49	11·01	13	110·8	9	2·71	12
13	9	258	11	8·64	52	·136	21	11·02	9	114·4	16	2·71	13
14	75	261	65	8·65	16	·201	56	11·03	69	114·7	24	2·98	14
15	7	285	50	9·47	9	·204	54	11·15	104	115·1	7	2·99	15
16	24	285	23	9·59	7	·225	101	11·2	67	115·9	23	3·03	16
17	10	292	81	9·96	24	·225	48	11·22	65	117·7	10	3·13	17
18	23	299	87	9·99	10	·231	53	11·23	102	118·3	25	3·38	18
19	67	304	92	10·05	23	·237	41	11·32	6	123·1	69	3·54	19
20	69	324	89	10·08	67	·24	66	11·33	100	124·4	8	3·57	20
21	25	327	104	10·1	69	·257	44	11·39	98	124·6	67	3·58	21
22	65	337	96	10·14	65	·258	104	11·41	23	124·7	65	3·63	22
23	8	348	102	10·15	8	·275	42	11·54	50	126·	19	3·73	23
24	50	369	84	10·26	50	·292	32	11·62	96	126·3	6	3·8	24
25	19	370	19	10·27	19	·296	102	11·64	92	129·	40	3·8	25
26	40	379	94	10·28	87	·308	99	11·69	94	129·5	50	3·89	26
27	87	390	85	10·32	84	·317	55	11·7	19	130·8	87	4·07	27
28	92	392	98	10·32	85	·319	57	11·.6	87	131·9	20	4·13	28
29	89	393	100	10·37	89	·323	97	11·77	89	132·	84	4·16	29
30	104	394	25	10·42	20	·339	4	11·79	81	132·5	85	4·16	30
31	96	395	6	11·51	92	·345	31	11·82	84	134·6	89	4·23	31
32	102	396	51	11·51	6	·355	43	11·85	85	134·9	5	4·37	32
33	84	400	8	11·61	47	·355	3	11·88	25	136·2	92	4·43	33
34	94	401	90	11·72	51	·355	34	11·97	5	141·6	39	4·45	34
35	98	402	5	11·84	5	·365	5	11·99	20	145·2	4	4·57	35
36	85	403	20	11·88	45	·369	36	12·	90	147·4	2	4·59	36
37	100	404	14	12·02	90	·376	45	12·01	4	147·9	72	4·59	37
38	6	414	10	12·17	2	·38	88	12·03	72	148·4	3	4·65	38
39	20	428	24	12·19	94	·381	95	12·05	2	148·8	51	4·69	39
40	51	449	2	12·31	27	·385	98	12·08	3	150·6	83	4·71	40
41	27	450	47	12·45	72	·385	83	12·09	8	150·8	90	4·73	41
42	90	457	27	12·49	4	·387	33	12·1	51	151·8	22	4·75	42
43	5	462	72	12·51	73	·389	2	12·13	83	152·5	94	4·8	43
44	2	480	4	12·55	3	·391	70	12·14	14	153·6	47	4·86	44
45	47	486	93	12·58	83	·391	20	12·16	95	157·2	66	4·93	45
46	72	488	73	12·61	39	·392	91	12·16	47	157·5	27	4·91	46
47	4	489	3	12·65	1	·395	80	12·19	93	158·8	46	4·94	47
48	93	491	12	12·66	96	·407	100	12·19	27	160·	36	4·96	48
49	73	492	83	12·68	45	·408	6	12·29	46	160·	29	5·07	49
50	3	494	76	12·7	35	·412	35	12·3	24	161·	96	5·07	50
51	76	495	39	12·71	29	·426	46	12·39	99	162·6	73	5·1	51
52	83	495	1	12·81	37	·431	78	12·43	79	162·7	17	5·23	52

EXPERIMENTS WITH VERTICAL WATER-TUBE BOILER, ARRANGED WITH REFERENCE TO ECONOMY, CAPACITY, ETC.—(*Continued*).

Relative number.	Combustible per hour.		Combustible per sq. ft. G. S. per hour.		Combustible per sq. ft. H. S. per hour.		Equivalent evaporation per pound of combustible.		Equivalent evaporation per sq. ft. G. S. per hour.		Equivalent evaporation per sq. ft. H. S. per hour.		Relative number.
	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	Exp.	Lbs.	
53	39	496	46	12·94	68	·431	63	12·47	10	164·7	45	5·3	53
54	29	497	95	13·06	93	·432	96	12·5	73	165·1	31	5·39	54
55	1	500	79	13·64	66	·436	62	12·51	39	166·6	1	5·41	55
56	46	505	29	13·81	80	·451	90	12·55	22	166·7	21	5·45	56
57	95	509	99	13·93	98	·452	38	12·57	49	168·3	93	5·46	57
58	36	521	68	13·97	86	·453	86	12·59	12	168·9	98	5·47	58
59	28	530	45	14·32	28	·454	47	12·61	17	170·1	37	5·5	59
60	30	531	36	14·47	30	·455	93	12·63	97	171·1	80	5·53	60
61	99	543	97	14·55	31	·456	79	12·66	45	171·6	26	5·56	61
62	22	545	80	14·62	38	·465	94	12·68	66	172·9	55	5·56	62
63	37	545	85	14·66	22	·468	11	12·7	36	174·	53	5·65	63
64	68	545	28	14·71	35	·468	19	12·7	103	174·4	86	5·68	64
65	66	550	30	14·75	55	·477	40	12·74	31	174·6	59	5·7	65
66	45	558	31	14·78	95	·484	77	12·75	1	175·	32	5·75	66
67	97	568	37	15·14	33	·491	27	12·76	29	175·	35	5·75	67
68	80	570	22	15·15	21	·495	37	12·81	48	175·5	68	5·79	68
69	86	572	49	15·29	32	·496	14	12·84	21	176·6	95	5·82	69
70	31	577	66	15·29	34	·496	29	12·88	101	177·	56	5·84	70
71	38	587	48	15·37	17	·501	92	12·89	80	178·2	38	5·85	71
72	35	591	55	15·44	100	·501	30	12·94	55	180·2	54	5·86	72
73	49	596	33	15·89	53	·504	72	12·95	86	184·	30	5·89	73
74	48	599	101	15·89	43	·511	71	12·98	68	187·6	34	5·91	74
75	55	602	40	15·92	57	·519	8	13·01	56	189·2	33	5·94	75
76	33	620	21	16·05	26	·523	23	13·02	54	189·8	60	5·95	76
77	101	620	34	16·07	42	·524	28	13·02	30	190·9	100	6·01	77
78	21	626	17	16·23	76	·526	73	13·1	34	191·6	44	6·02	78
79	32	626	38	16·31	54	·527	39	13·11	33	192·4	78	6·02	79
80	34	627	103	16·31	44	·528	85	13·11	60	192·9	41	6·03	80
81	17	633	35	16·42	71	·529	25	13·12	37	193·3	42	6·04	81
82	103	636	43	16·56	56	·532	51	13·15	44	194·9	43	6·04	82
83	53	637	57	16·83	41	·534	84	13·16	78	194·9	58	6·08	83
84	43	646	74	16·95	62	·537	89	13·2	26	195·	57	6·12	84
85	57	657	42	16·97	77	·54	87	13·23	41	195·5	28	6·17	85
86	26	661	78	17·05	102	·54	24	13·25	42	195·5	102	6·29	86
87	74	661	54	17·08	70	·555	7	13·27	43	195·9	77	6·32	87
88	42	662	44	17·1	91	·557	12	13·28	58	197·	62	6·71	88
89	78	665	71	17·12	64	·582	50	13·3	53	198·2	70	6·72	89
90	54	666	56	17·22	97	·585	81	13·31	57	198·2	91	6·76	90
91	44	667	41	17·3	63	·602	64	13·32	28	199·8	71	6·86	91
92	71	668	91	17·37	99	·611	9	13·33	59	200·3	97	6·87	92
93	56	671	32	17·39	104	·612	68	13·39	35	201·7	104	6·94	93
94	41	675	62	17·4	88	·614	52	13·4	32	201·8	64	7·12	94
95	91	677	77	17·5	59	·617	13	13·44	40	201·9	99	7·13	95
96	62	679	53	17·69	58	·619	82	13·44	77	204·8	88	7·41	96
97	77	683	70	17·98	101	·767	10	13·55	38	205·4	63	7·48	97
98	70	701	26	18·36	48	·781	16	13·55	91	211·1	101	8·55	98
99	64	736	64	18·87	60	·831	18	13·7	62	217·5	48	8·76	99
100	63	760	63	19·49	103	·867	69	13·75	70	217·8	103	9·28	100
101	59	779	88	19·96	40	·899	15	13·79	71	222·3	49	9·9	101
102	88	779	58	20·06	75	1·45	65	14·06	64	230·6	75	11·8	102
103	58	782	59	21·65	76	2·74	61	14·37	88	240·	76	19·	103
104	60	1050	60	26·93	74	3·66	67	14·86	63	242·5	74	22·5	104

From a careful study of experiments similar to those that have been described, the laws governing the performance of boilers can be deduced with considerable accuracy. In tracing these laws, it is best to lay down the several results, and draw curves through them to give fair representations of the average. An investigation of this kind by C. E. Emery may be found in the "Reports and Awards, Group XX., International Exhibition, 1876," and the following paper, by the late Theron Skeel, published in the *American Machinist* for December, 1877, embodies, in concise form, a number of important rules deduced from such experiments:

"The general principles which apply to the setting of steam boilers to obtain the most steam from the smallest quantity of coal are extremely simple, and are as follows:

I. The highest chimney.
II. The largest grate on which the coal will burn without going out.
III. The largest amount of heating surface.
IV. The smallest opening through the tubes or flues.

"If a man starts out with these principles in view, he may obtain the best results with almost any arrangement of setting, but unfortunately he needs another thing, namely, *a large bank account.* The problem of obtaining equally good results with a smaller investment of money, is the problem which presents itself to the practical man.

"If a builder has unlimited capital at his command, he can scarcely devise any arrangement of boiler setting by means of which he can evaporate more than 12 pounds of water for a pound of anthracite coal after the ashes and cinders are deducted, and when the feed water is fed in boiling hot (212°). On the other hand, the poorest possible arrangement of setting, and the least amount of boiler surface ever used in practice, will almost always show an evaporation of at least 8 pounds of water evaporated by one pound of anthracite coal after the ashes and cinders are deducted and when the feed water is fed in boiling hot (212°). This difference, 50 per cent, is the greatest ever found in practice. Between these two limits boilers generally vary. Any man who has the most wasteful boilers may make them the most economical by reducing the amount of steam he draws from them, and by checking the draught by *closing the ash pit doors*, provided no cold air leaks in through cracks in the brick-work.

"Generally manufacturers do not have any more steam than they need. The cases where they have a *spare* boiler which is not used, being held in reserve in case one of those in use breaks down, are extremely rare; and indeed it is not desirable that there should be a spare boiler, for, if well built and properly cared for, the boilers are not more likely to break down than the other machinery in the mill. Also, if a manufacturer has more boilers than he needs, there can be more steam made than is necessary. In the course of a few years the manufacturer is sure to find that all the boilers are in use, and of course the *extra steam wasted.*

"If the boilers are pressed to their utmost, the engineer will take care to

keep his safety valve tight, his engine in good order, and to reduce all waste of steam to the lowest possible limit. On the other hand, if he has a spare boiler, it is much easier to start a fire under it than search for and repair the leaks. The coal burned under the extra boiler, of course, comes out of the profits of the business.

"I believe it is the best policy for a manufacturer to have boilers capable of burning the least possible amount of coal he can get along with, *and no more*.

"The amount of coal that may be burned depends upon the height of the chimney, the size of the grate, and the area of opening for the passage of gas and smoke through the tubes or flues. Having fixed these three elements, the manufacturer may increase the length of the boilers and obtain any rate of economy he pleases up to about 12 pounds of water evaporated for each pound of anthracite coal burned after the ashes and cinders are deducted, and the feed water fed in boiling hot (212°).

"The idea that a boiler is necessarily wasteful of coal because it is being forced to its utmost capacity is *erroneous*. A boiler of the proper proportions may be working to its best advantage under these circumstances. The points for a manufacturer to consider, then, are:

I. How much steam does he require?
II. How efficient a boiler is he willing to pay for?

"All estimates based on the so-called 'horse-power' of boilers are entirely erroneous. The horse-power of a boiler called '100 horse-power boiler' may vary from 25 horse-power to 250 horse-power, depending upon the height of chimney, size of grate, kind of engine, etc.

"In considering the maximum capacity of any proposed form of boiler, the following elements must be considered:

I. The kind of coal.
II. The temperature of the feed water.
III. The height of the chimney.
IV. The size of the grate.
V. The area of opening through the flues or tubes.
VI. The amount of heating surface.

"In general the temperature of the atmosphere and the pressure of the steam do not affect the result to any considerable degree.

"If the above six elements are given to a skilful engineer, he can predict with great accuracy how much coal can be burned, and how much steam will be furnished by the boilers. It will be seen that the builder may vary any one of those six elements at his pleasure. The variation of any of them affects the result. The effect of the variation of these elements will be considered in order:

I. The kind of coal.
The kinds of coal in general use in the United States are:
The semi-bituminous coal from Maryland,
The anthracite from Pennsylvania,

The bituminous from Pittsburgh,

The bituminous from Ohio.

"When burned in the ordinary furnace, these coals will furnish steam, per pound of coal, in the following proportion:

Semi-bituminous,	110
Anthracite,	100
Pittsburgh,	90
Ohio,	75

"The weights of these coals that may be burned on the same grate, with the same chimney, will vary as follows:

Semi-bituminous,	120
Anthracite,	100
Pittsburgh,	120
Ohio,	200

"It will be seen, then, that to furnish a given amount of steam with these various coals, there will need to be more grate surface for the anthracite than for any other. The proportional amount of grate surface that will be needed to furnish the same amount of steam will be as follows:

"*Table I. Relative amount of grate surface that will be necessary to burn coal enough to furnish the same quantity of steam with various kinds of coal:*

Anthracite,	100
Pittsburgh,	90
Semi-bituminous,	75
Ohio,	67

"This table refers to the average coal of each kind found in practice. Throughout the following paper the coal spoken of will be anthracite coal. The grate must be made smaller for other coals in the proportion to the above figures.

"II. *Temperature of feed water.*—The hotter the feed water the more steam the boiler will furnish in the same time.

"The relative amounts of steam that will be furnished in the same time, and by the same quantity of coal, are given in the following table:

"*Table II. Relative amounts of steam that will be furnished in same time, or by same weights of coal, with various temperatures of feed water:*

Temperature of feed water.	Relative amount of steam.
212°	100
200°	98
180°	96
160°	94
140°	92
120°	90
100°	89
80°	87
60°	86
40°	83

"Throughout the following paper the feed will be supposed to be at 212°. Whenever the feed is less than this, the amount of steam that will be furnished must be reduced in proportion to the above numbers.

"*Table III. Relative amount of coal that can be burned in the same time with chimneys of various heights:*

Height of chimney in feet.	Relative amount of coal.
120	100
100	90
80	80
60	70
40	57
20	40

"It will be seen that after a moderate height of chimney is reached, the effect of any increased height is very small.

"Throughout this paper the chimney will be supposed to be 120 feet high. Whenever the height of chimney is less than this, the amount of coal that can be burned must be reduced in the proportion of the above numbers.

"IV. In general, the larger the grate the better, but there are certain things which limit its size in practice. Ashes and clinkers collect on the grate and have to be removed with a slice bar. When the grate becomes more than six feet deep it is very hard to clean the back end of it. The width of the grate is generally limited by the two brick walls on both sides of the boiler. The largest grate then that is generally obtainable in practice is six feet deep, and from 4 to 8 inches wider than the boiler. I do not think the grate should ever be made smaller than this under ordinary circumstances.

"V. *Opening through the flues or tubes.*—This is one of the most important items in a boiler, and the one which is most generally neglected. With a given grate the opening through the flues or tubes regulates the amount of coal that can be burned, and therefore the amount of steam that can be formed. The opening should be as small as will burn the amount of coal required. The necessary area is given in the following table.

"There is no use of making the opening through the tubes more than $\frac{1}{7}$ of the grate, for the gas will not fill the extra tubes, and they are *entirely useless.*

"*Table IV. Pounds of coal that may be burned per hour on each square foot of grate, when the opening through the flues is the following proportion of the grate:*

Ratio of opening through the flues to grate surface.	Pounds of coals that may be burned on each sq. ft. per hour.
$\frac{1}{40}$	6
$\frac{1}{30}$	8
$\frac{1}{20}$	11
$\frac{1}{15}$	14
$\frac{1}{10}$	17
$\frac{1}{7}$	20

"VI. *Heating surface.*—The amount of heating surface may be with good effect as much as the manufacturer feels able to pay for. When the opening through the tubes is fixed, the only way he can increase the heating surface is to increase the *length* of the tubes or flues. The longer the tubes or flues the better the result. The length of the flue or tubes must be proportional to its diameter; that is to say, a tube 6 inches in diameter must be twice as long as one 3 inches in diameter to produce the same steam from the same coal.

"*Table V. Ratio of length to diameter of tube or flues, in order to evaporate the following weight of water per pound of coal.*

Ratio of length to diameter.	Pounds of water evaporated per pound of coal.
10	5·4
15	6·7
20	7·4
30	8·3
40	9·1
50	9·7
60	10·0

"The foregoing tables give all the necessary information in regard to proportions of boilers.

"*Example.* Suppose a manufacturer wishes boilers which will furnish 5000 pounds of steam with 700 pounds of coal per hour, that his chimney is 60 feet high, and his feed water fed in at 150° (the usual temperature when a high-pressure engine and heater are used). From Table II. we find that this 5000 pounds steam at 150° is equivalent to $5000 + 93 = 5370$, when feed is 212°, and that therefore he will have to evaporate

$$\frac{5370}{700} = 7{\cdot}7$$

pounds of water for each pound of coal.

"From Table V. it appears that to get this evaporation the tubes must be 25 times as long as they are in diameter—that is to say, if they are 6 inches in diameter, they must be

$$25 + 6 = 150'' = 12\tfrac{1}{2} \text{ feet.}$$

If the manufacturer decides to have two boilers each 5 feet in diameter, the grates may be each 5 feet 8 inches by 6 feet = 34 sq. feet, being 68 sq. feet for both *boilers*. The coal then must be burned at the rate of

$$\frac{700}{68} = 10\tfrac{1}{2}$$

pounds per sq. foot of grate per hour.

"From Table III. it appears that when the chimney is only 60 feet high, the consumption is only $\frac{7}{10}$ as much as when the chimney is 120. The opening through the tubes then must be sufficient to burn

$$\frac{10{\cdot}5}{0{\cdot}70} = 15$$

pounds per sq. foot of grate with a chimney 120 feet high.

"From Table IV. it appears that to burn 15 pounds per square foot of grate requires an opening through the tubes of $\frac{1}{14}$ the grate, or in this case,

$$\frac{68}{14} = 4.9 \text{ sq. ft.}$$

The proportions of the boilers would then be as follows:

Grate,	68 sq. ft.
Length,	12·5 "
Opening through tubes,	4·9 "
Diameter of tubes,	6 inches.

Two tubular boilers, each 5 feet diameter, with 15 six-inch tubes each, would give this result."

An account of a comparative trial of the most prominent types of stationary boilers used in Europe was published in the *Bulletin de la Société Industrielle de Mulhouse*, xlv., 241.

The dimensions of the boilers are contained in the accompanying table:

	Elephant boiler.	Fairbairn boiler.	Lancashire boiler.
Length ..... Feet	3 cylinders, 32·79 1 cylinder, 29·53	2 cylinders 25·77 1 cylinder, 22·97	25·75
Diameter..... Feet	3 cylinders, 1·64 1 cylinder, 3·74	2 cylinders 4·1 1 cylinder, 3·74	6·56
Number of flues		2	2
Diameter of flues..... Feet		2·3	2·3
Grate surface..... Square feet	20·05	20·5	20·5
Air space in grates..... Square feet		5·85	5·85
Heating surface { Shell.. Square feet		684·21	279·25
Heating surface { Flues.. Square feet		333·27	333·25
Heating surface { Total.. Square feet	607·6	1017·48	512·5
Water space..... ....Cub. feet	408·16	544·87	413·21
Steam space..... ..Cub. feet	123·04	97·86	225·1
Total weight..... .. .Pounds	31,900·	43,120·	36,520·
Weight per sq. ft. heating surface..... .Pounds	52·5	42·3	59·5
Cost of boiler..... Dollars	2,153·	3,196·	2,741·
Cost of setting..... Dollars	584·	584·	545·
Draught area { First flue....Sq. feet	8·98	8·28	8·28
Draught area { Second flue..Sq feet	5·05	18·1	8·42
Draught area { Third flue...Sq. feet	5·05	14.6	7·65

The trials were made with different varieties of coal, and were from 36 to 72 hours in duration. A table giving the most important results is appended:

SUMMARY OF TRIALS OF ELEPHANT, FAIRBAIRN, AND LANCASHIRE BOILERS.

Number for reference.	Pounds of combustible per hour.			Pounds of water evaporated from and at 212°			Ratio.		Per cent of refuse.	Temperature of uptake, Fahrenheit degrees.	Remarks.	Number for reference.
	Total.	Per sq. foot of grate surface.	Per sq. foot of heating surface.	Per pound of combustible.	Per sq. foot of grate surface per hour.	Per sq. foot of heating surface per hour.	Heating to grate surface.	Draught area to grate surface.				
1	335	16·36	·549	10·13	166·	5·57	29·8	·404	11·8	555	Ronchamp coal	1
2	178	8·68	·291	10·9	94 6	3·17		·41	14·7	406	" "	2
3	335	16·32	·548	10 26	167·4	5·62		·373	14·4	595	" " Lancashire boiler.	3
4	298	14·54	·488	8·65	125·7	4 22			9·9	555	Saarbrück "	4
1	347	17·27	·57	10·02	173·1	5·71	30·3	·443	14·1	562	Ronchamp coal	1
2	136	9 26	·306	10·76	99·5	3·28		·252	14·	425	" " Elephant boiler.	2
3	309	15·39	·508	8·66	133·2	4·4			9·1	544	Saarbrück "	3
1	320	15·59	·315	11·2	174·6	3·53	49·5	·404	13·8	421	Ronchamp coal	1
2	186	9·07	·183	10·52	95 4	1·93		·883	13·4	338	" " Fairbairn boiler.	2
3	297	14·5	·293	9·27	134·4	2·72		·712	10·6	402	Saarbrück "	3

In addition to the references to boiler experiments contained in the foregoing pages, mention may be made of "Experimental Researches in Steam Engineering," by B. F. Isherwood, 2 vols., Philadelphia, 1863, 1865. This work contains records of numerous experiments, with an unusually complete account of apparatus, full tables of results, and elaborate discussions.]

CHAPTER IV.

THE STEAM ENGINE.

§ 439. **Steam Engines.**—Steam engines (Fr. machines à vapeur; Ger. Dampfmaschinen) are machines which are put in motion directly or indirectly by the force of steam. The steam acts indirectly, when by its condensation an almost empty space or partial vacuum is produced, so that the atmosphere can perform work, such as pressing a piston into this space. It acts directly when by its expansive force it puts a piston in motion, or when by its *vis viva* it performs work, such as putting a wheel in rotation. Machines in which the steam acts indirectly are called atmospheric engines (Fr. machines atmospheriques; Ger. atmosphärische Dampfmaschinen), and are but rarely used, wherefore we shall, in what follows, speak only of the steam engine proper, and then, indeed, only of the *piston* engine.

Steam engines, like water-pressure engines (see § 297), may be either single or double-acting. In the first class, the steam drives the piston only in one direction, and it is moved in the opposite direction by a counterweight. In the second class, the steam acts first on one and then on the other side of the piston. The first cause an up and down or reciprocating motion, and are therefore used for pumps, steam hammers, etc. Double-acting engines, on the other hand, are applied in all cases where it is required to produce a rotary motion.

With reference to the intensity of the steam pressure, steam engines may be divided into

(1) Low-pressure,
(2) Middle-pressure,
(3) High-pressure

engines. In low-pressure engines (Fr. machines à basse pression ; Ger. Niederdruckdampfmaschinen), the pressure of the steam exceeds the atmospheric pressure at most by only one half ; in the middle-pressure engine (Fr. machine à moyenne pression ; Ger. Mitteldruckdampfmaschinen), the pressure of the steam is from two to four atmospheres ; and in the high-pressure engine (Fr. machine à haute pression ; Ger. Hochdruckdampfmaschinen), the steam pressure may be five or more atmospheres.

REMARK.—The first steam engine, Savery's, had no piston, and served only for indirectly raising the water, so that it was similar to a pump in construction. The steam being condensed, the water rushed in to fill the vacuum. This water was again forced out by the entrance of fresh steam, which was again condensed, and so on. Newcomen modified it to an atmospheric or indirect-acting engine with piston, and Watt later changed it to the steam engine proper. The English regard the Marquis of Worcester as the inventor of the steam engine ; Arago, however, endeavors to prove that this honor belongs to Papin. For the history of the steam engine, we can refer to "Annuaire du bureau des longitudes, pour l'année 1837 et 1838." The first contains the history of the steam engine and the second the life of Watt, both by Arago. These, as well as many other articles from the "Annuaire," have been translated into German by Remy and Krieb. Further, Stuart's "Histoire de la machine à feu ;" the second volume (article "Steam") of Robison's "System of Mechanical Philosophy ;" Lardner's "Lectures on the Steam Engine ;" Bourne's "Treatise on the Steam Engine, etc." Also a "Treatise on the Steam Engine," by Russel. See also the author's treatise, "Die Fortschritte des Dampfmaschinenwesens in den letzten hundert Jahren," Freiberg, 1866.

§ **440.**—In the steam engine proper, the steam, after performing its work, is either discharged into the air or it is condensed by cold water. We have, therefore, the steam engine without condensation and with condensation, or condensing and non-condensing steam engines.

The force with which the piston moves is, as in water-pressure engines, the difference of the pressures upon either side. In non-condensing engines, the steam acts upon one

side and the atmosphere upon the other, and, therefore, the effective force is less than the steam tension. In condensing engines, on the other hand, the pressure of the steam upon one side of the piston is opposed only by the feeble back pressure of the imperfect vacuum, and hence the effective force is but little less (about $\frac{1}{10}$ atmosphere) than the steam tension. We see therefore that, other things being equal, condensing engines have a greater efficiency than non-condensing, and also that the advantage of condensation is of less account in high-pressure engines, while low-pressure engines cannot run at all without condensation. In a high-pressure engine of 6 atmospheres steam tension, only $\frac{1}{6}$ of the force is lost by discharging the steam into the air, while in a middle-pressure engine working steam of 3 atmospheres, this loss is $\frac{1}{3}$, and in a low-pressure engine of $\frac{4}{3}$ atmospheres the loss is $1 : \frac{4}{3} = \frac{3}{4}$, so that in such case only $\frac{1}{4}$ of the total force is effective. By condensing the steam so that the back pressure is only $\frac{1}{10}$ atmosphere, this loss is only $\frac{3}{40}$, and hence there is remaining $\frac{37}{40}$ or 0·925 of the total force of the steam as effective. In the first case, on the other hand, by condensing, the effective force would be $\frac{59}{60}$ of the whole, while without condensation it is already $\frac{50}{60}$, so that the gain by condensation is relatively small and of less account.

Although, then, in high-pressure engines, condensation is advantageous, yet it is rarely used, because the condensation water, which is at least 20 times as much as the feed, cannot always be easily obtained, or only at considerable expense, so that there is no actual economy, and because, moreover, non-condensing machines are simpler in construction than condensing.

Finally, we have to distinguish engines with and without *expansion.* Engines without expansion, or non-expansion engines (Fr. machines sans détente; Ger. Dampfmaschinen ohne Expansion), allow the steam to enter the cylinder uninterruptedly during the entire stroke of the piston, so that the steam pressure is constant; while engines with expansion, or expansion engines (Fr. ma-

chines à détente; Ger. Dampfmaschinen mit Expansion), shut off the steam from the cylinder before the completion of the stroke, so that the steam expands in volume and decreases in tension more and more during the latter portion of the stroke. The work which the steam performs during expansion is lost in the non-expanding engine, and, therefore, a greater efficiency is to be expected from machines working expansively than from those working non-expansively.

We can distinguish also stationary and portable engines, or engines which are fixed and those which are placed upon wheels so as to be transported from place to place. A special form of portable engine is the locomotive and steamship, which are designed to move themselves and any attached vehicles. These will be treated of in their appropriate place.

§ **441.** — **Steam Cylinder.** — The principal parts of a steam engine are

(1) The steam cylinder,

(2) The piston and rod,

(3) The valve gear.

The steam cylinder (Fr. cylindre à vapeur; Ger. Dampfcylinder) is a cast-iron tube, accurately bored, which confines the steam while performing its work. It is closed at top and bottom, and has at each end openings for the entrance and exit of the steam. The length should have a convenient ratio to the diameter. Ordinarily the length is 2 to $2\frac{1}{2}$ times the diameter, but in machines which make a great number of strokes per minute, as locomotives and marine engines, this ratio is, however, still smaller.

That there may be as little loss of heat as possible by cooling in the cylinder, the cylinder length should have a certain ratio to the diameter. The cooling of the steam is greater the greater the product of the extent of cooling surface and the time of cooling. In a cylinder, the cooling

surface consists of two circular areas and a cylindrical surface. If the diameter of the cylinder is d, and the time in which the piston passes through its double stroke s is t, we have as the measure of the cooling of the two circular areas,

$$O_1 = 2\,\frac{\pi\,d^2}{4}\cdot t = \frac{\pi}{2}\,d^2\,t.$$

If we assume that the piston in every unit of time $\frac{t}{n}$ passes through the distance $\frac{s}{n}$, we have for the cooling of the cylindrical surfaces, $\pi\,d\,\frac{s}{n}$, $\pi\,d\,\frac{2\,s}{n}$, $\pi\,d\,\frac{3\,s}{n}$, $\pi\,d\,\frac{n\,s}{n}$,

$$O_2 = \frac{\pi\,d\,s}{n}\cdot\frac{t}{n} + \frac{2\,\pi\,d\,s}{n}\cdot\frac{t}{n} + \frac{3\,\pi\,d\,s}{n}\cdot\frac{t}{n} + \ldots + \frac{n\,\pi\,d\,s}{n}\cdot\frac{t}{n}$$

$$= \frac{\pi\,d\,s\,t}{n^2}\,(1+2+3+\ldots+n) = \frac{\pi\,d\,s\,t}{n^2}\cdot\frac{n^2}{2} = \frac{\pi}{2}\,d\,s\,t;$$

hence the measure of the cooling of the whole cylinder during the entire time of motion of piston is

$$O = O_1 + O_2 = \frac{\pi}{2}\,d^2\,t + \frac{\pi}{2}\,d\,s\,t = \left(2\,\frac{\pi\,d^2}{4} + \pi\,d\,\frac{s}{2}\right)t,$$

or equal to the product of the time and the surface of the cylinder whose length is the stroke.

In order that the cooling may be as small as possible, then, not only the time but also the surface of this cylinder must be as small as possible. But we know that of all cylinders, that one has the least surface for a given area whose length is equal to its diameter. Hence the cooling will be a minimum when the length $\frac{s}{2}$ is equal to the diameter, or when the stroke of piston equals the diameter. The length of cylinder should, however, exceed the stroke by a little more than the thickness of the piston.

In order to retard the cooling of the steam as much as possible, the cylinder should be enveloped by poor heat conductors such as wood or felt, or air or steam. Its surface should also be smooth and polished, as in such case the heat radiation is less. Very often a steam envelope is used by furnishing the cylinder with an iron jacket, the interior of which is filled with steam. Here, however, we may have three cases; the steam filling the jacket may be at rest, or it may flow through either before or after its operation in the cylinder. The last method, although rarely used, appears to be the most advantageous, because the heat of the escaping steam is thus utilized. The circumstance that in this case the steam envelope has less heat than the steam in the cylinder, and, therefore, abstracts heat from it, while, in the other case, it imparts heat to it, is no objection to this method of protection, because the cooling increases with the difference of temperature; and this is certainly less for a cylinder enveloped in steam than for an unjacketed cylinder.

Since steam is condensed in the jacket, it must have below a cock for drawing off the water.

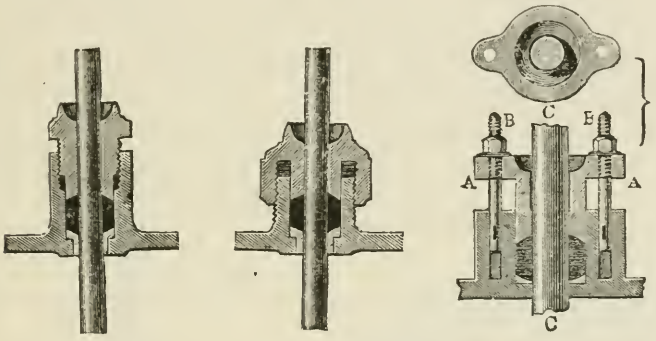
The thickness of the cylinder may be calculated like that of steam pipes generally. On account of the incessant friction and the necessary stiffness, however, it is never less than $\frac{5}{6}$ inch, and is, hence, for the diameter d and steam tension $(p + 1)$ in atmospheres,

$$e = 0{\cdot}005\, p\, d + \tfrac{5}{6} \text{ inches.}$$

§ 442.—**Stuffing Boxes.**—The top and bottom of the steam cylinder are united, by screws and putty, steam-tight to the cylinder jacket. In the centre of the cover is the stuffing box (Fr. boite à garniture; Ger. Stopfbüchse), through which the piston rod passes. The stuffing box (compare § 301) is in general packed with hemp soaked in oil and tallow, though more recently metal rings, placed one over the other, have been used, each consisting of three sectors and pressed against the piston rod by iron

springs inserted between the inner circumference of the stuffing box and the outer circumference of the rings. The packing is pressed down by a cover or gland which is either screwed directly into the stuffing box or joined to it by two or three pinching screws. Stuffing boxes of the first kind are shown in Figs. 686 and 687. Fig. 688 shows

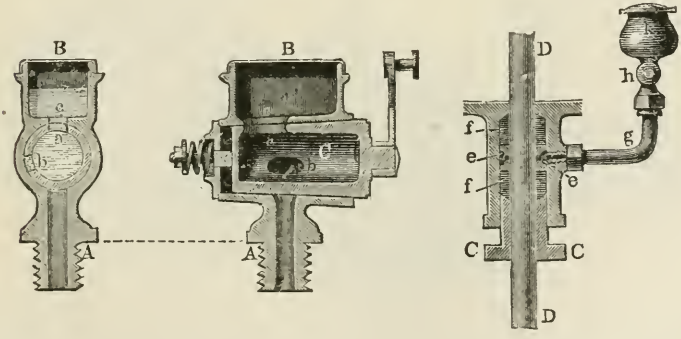
Fig. 686. Fig. 687. Fig. 688.



a stuffing box *A A* with screws *B B*, by means of which the gland can be pressed down upon the packing.

Both the cylinder and stuffing-box covers have a cavity for the reception of oil, and when hemp packing is used

Fig. 689. Fig. 690.



for the piston, one or more oil funnels are set in the cylinder cover. The arrangement of such an oil funnel is shown in Fig. 689. The apparatus is screwed at *A* into

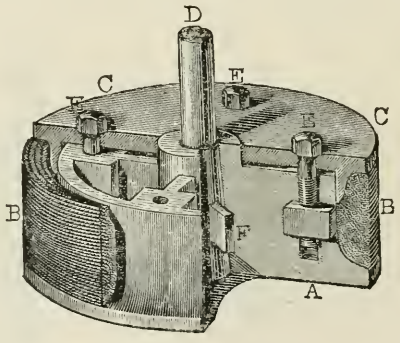
the cylinder cover; *B* is the oil cup, and *C* is a cock with two holes *a* and *b*. If the hole *b* is below, the oil flows through *A* into the cylinder; but if *a* is on top and directly below *c*, the oil flows from the cup into the cock.

Sometimes, though rarely, the piston rod passes through the bottom of the cylinder. This is to be avoided so far as possible, because the stuffing box in this position does not retain the lubricating material well, and also ceases soon to be steam-tight owing to the grinding of sediment deposited upon the bottom. Fig. 690 shows a form of box which may be used in such a position. *A B* is the stuffing box, *C C* the gland, *D D* the piston rod, *c c* a brass plate with an external groove and six to eight radial holes, *f* the packing, *g* a copper pipe communicating with the groove, *k* the oil cup, and *h* a cock, which is only opened when the machine is at rest.

The cylinder should finally be fixed firmly to a solid foundation by means of a strong sole plate, to which it is screwed.

§ **443.—Steam Piston.**—The steam pressure acts at first upon the steam piston (Fr. piston à vapeur; Ger. Dampf-

Fig. 691.



kolben), which moves back and forth in the cylinder (compare § 300), and is then transferred to the piston rod. The piston consists of a short cylinder very exactly fitted to the steam cylinder, and consists essentially of three parts

—the piston stock, the packing, and the cover or *junk ring*. In the centre of the piston stock is a conical cavity for the reception of the piston rod, the end of which is made to fit. The piston stock and junk ring are made of cast-iron, and the packing is either hemp packing (Fr. garniture de chanvre; Ger. Hanfliderung) or metallic (Fr. garniture métallique; Ger. Metall-liderung).

The arrangement of a piston with hemp packing is shown in Fig. 691. *A A* is the piston stock, *B B* the packing of hemp strands, *C C* the junk ring, which can be screwed down upon the packing by means of screws *E E* which screw into the stock, and *D* is the piston rod, which

Fig. 692.

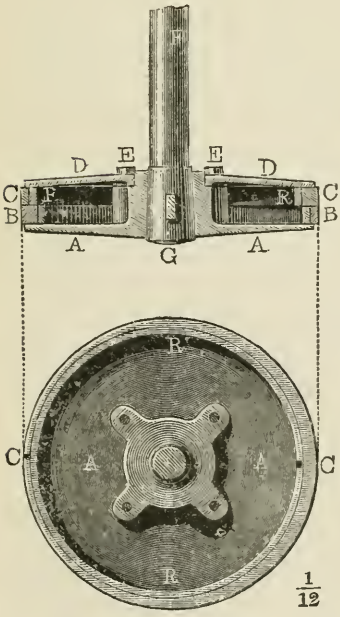


Fig. 693.

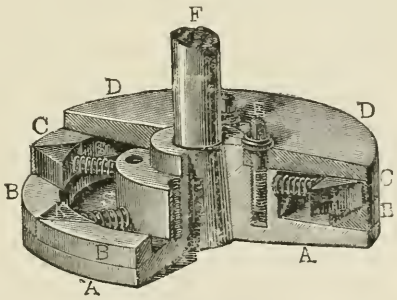
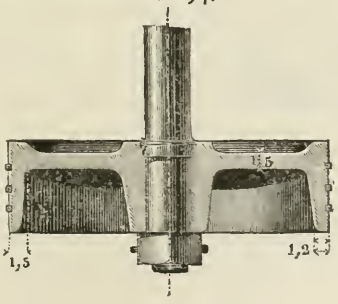


Fig. 694.



is further fastened by means of the splint or *cotter F*. Hemp packing cannot be used in high-pressure engines, as it is too quickly injured by the hot steam and great friction. Instead of it, metal packing is used, which is not only more durable but causes less friction. There are a

great number of metal packings, but essentially they consist of exactly turned metal rings, which are forced outwards by springs against the inner surface of the cylinder. In Figs. 692 and 693, we have represented two of the best forms of this packing. In both figures, *A A* is the stock or body of the piston, *D D* the junk rings, *F G* the piston-rod end, and *E, E* the screws by which the junk ring is screwed down. The packing consists of two superimposed metal rings *B B* and *C C*, made elastic by hammering, and cut through so that they may spring against the cylinder. In Fig. 692 each of these rings is cut through at its weakest point, and is pressed outwards by a steel ring *R*, which is also cut. In Fig. 693, the rings are cut at the broadest part, and wedges *K, K* inserted, which are pressed out by spiral springs *S, S*. The piston *A A*, Fig. 694, of Ramsbottom, is very simple. The packing here consists of three or five elastic steel or brass rings. In order that they may spring out and grip the cylinder, they are bent, before insertion, to a circle whose diameter exceeds that of the cylinder by one tenth.

The steam piston of Kraus has a packing consisting of two double rings—the inner one of wrought-iron and the outer one of a composition of 80 parts tin, 10 parts antimony, and 10 parts copper, called Babbitt's metal. These rings are pressed out by the steam, and thus form a self-acting packing. A tongue *Z* is let in at the joint in order to make a tighter fit.

FIG. 695.

§ 444.—**Piston Rod.**—In the piston rod (Fr. tige de piston; Ger. Kolbenstange) and piston, two dimensions are of especial importance—viz., the ratio of the piston, or packing height to the diameter of the piston, and the ratio between the diameter of the piston rod and the diameter of the cylinder. Since neither the inner surface of the cylinder nor that of the packing is perfectly smooth, the packing can only be perfectly tight when it has a certain extent; and, on the other hand, this breadth cannot be excessive, because then the friction is increased (§ 320). For

perfect tightness, it is also necessary that the piston surface shall not be inclined to the cylinder axis. But this may result from an eccentric position of the rod and from an unequal distribution of the friction upon the circumference of the piston, if the packing is not deep, and there is, therefore, a certain ratio which must be preserved between the breadth of packing and the cylinder diameter. Tredgold has sought to show theoretically that this ratio must be equal to the coefficient of friction, but the foundation of this proof is too uncertain to be relied upon, and recourse must be had to relations established by experience. It has thus been found that for hemp packing this ratio is $\frac{1}{3}$ to $\frac{1}{6}$, and for metal packing, $\frac{1}{6}$ to $\frac{1}{9}$—the greater value being taken for small and the less value for larger pistons.

The piston rod, which is, in general, of steel or wrought-iron, must possess sufficient strength in order to transmit the piston pressure without permanent or noticeable change of shape. The formulæ for the determination of its dimensions may be found from the theory of elasticity, but we must distinguish whether, as in single-acting machines, the piston rod is subject only to a tensile strain, or whether, as in double-acting engines, it is alternately extended and compressed. If p is the difference in atmospheres of the steam pressure upon both sides of the piston, and d the diameter of the piston, we have for the force acting upon it,

$$P = \frac{\pi d^2}{4} \cdot 14{\cdot}7\, p \text{ lbs.}$$

If, however, d_1 is the diameter of the piston rod and T the modulus of proof strength, we have for the strength of the piston rod,

$$P = \frac{\pi d_1^2}{4} T.$$

Putting these expressions equal to each other, we have for a piston rod *in extension*,

$$d_1^2\, T = 14{\cdot}7\, d^2\, p,$$

or

$$d_1 = d\sqrt{\frac{14{\cdot}7\, p}{T}}.$$

If we take for T one half of the value given for wrought-iron in Vol. I., § 212, or $T = 9000$, we have for the determination of the diameter of piston rod in single-acting engines,

$$d_1 = \frac{d}{24{\cdot}7}\sqrt{p} = 0{\cdot}04\, d\sqrt{p},$$

or, if we express p, not in atmospheres, but in pounds per square inch,

$$d_1 = 0{\cdot}01\, d\sqrt{p}. \quad \text{(See § 301.)}$$

In determining the diameter of the piston rod of double-acting engines, we can use two formulæ, according as we consider the resistance to compression or to bending. On account of the length of the piston rod, we should consider the first (Vol. I., § 211), but since, by a moderate eccentricity in a force upon a cylindrical piston rod, the strength is considerably decreased (Vol. I., § 269), and this can be easily caused by an inaccurate union of the piston with the rod, it is better to apply the formula for the last, and, therefore, to use a less value for T. For these reasons, the diameter of a wrought-iron piston rod for double-acting machines is given by the empirical formula,

$$d_1 = 0{\cdot}08\, d\,(\sqrt{p} + 0{\cdot}25) \text{ inches},$$

where p is the effective pressure in atmospheres.

The piston in large steam engines, especially marine, has *two* piston rods.

EXAMPLE.—What diameter must the wrought-iron piston rod of a double-acting engine have when working steam of 5 atmospheres without condensation—*i. e.*, 4 atmospheres effective pressure—and with a diameter of cylinder of 24 inches? According to the last formula, the diameter is

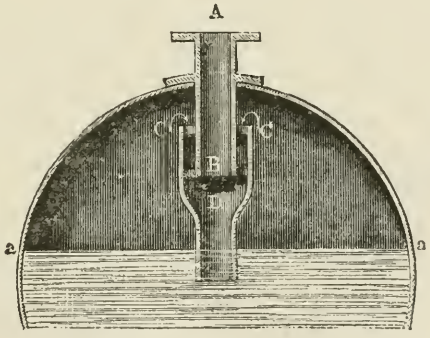
$$d_1 = 0{\cdot}08\, d\,(\sqrt{5-1} + 0{\cdot}25) = 0{\cdot}08 \times 2{\cdot}25\, d = 0{\cdot}18\, d$$

$$= 0{\cdot}18 \times 24 = 4{\cdot}32 \text{ inches.}$$

§ **445.—Steam Pipe.**—The steam is conducted by the steam pipe (Fr. tuyau à vapeur; Ger. Dampfrohr) from the boiler to the steam box (Fr. boîte à vapeur; Ger. Dampfkammer), or to that space where the regular distribution of the steam is caused by the valve gear. In the steam pipe is the steam valve (Fr. valve regulatrice; Ger. Admissionsklappe)—*i. e.*, a throttle valve—by means of which the entrance and, therefore, the pressure of the steam can be regulated.

As to the steam pipe, it should enter the boiler at that point where the most rapid generation of steam takes place, and should rise from this point in order that as little water as possible may be carried away with the steam, and that this water may flow back into the boiler. Fig. 696

FIG. 696.

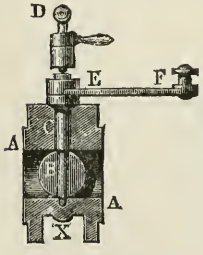


shows a good arrangement by which dry steam is secured. To the steam pipe *A B* a larger pipe *C C D* is fitted which reaches down below the water level. The steam entering

at *C C* passes downwards, and thus is made to part with its water before it enters the steam pipe *A B*.

In order that the resistance to motion in the steam pipe may be as small as possible, the steam pipe should not be unnecessarily long, all sudden changes of direction and cross-section should be avoided, and it should have a sufficient diameter. In order, however, to diminish as much as may be the loss of heat, the cooling surface should be small, and hence the steam pipe short and narrow and covered with some non-conducting material, or by a polished metal surface. We see, therefore, that the steam pipe must fulfil very different conditions from ordinary pipes for conduction of water. While the pipes, for instance, of water-pressure engines are to be made wide in order that they may offer little resistance, the steam pipe should have a medium width only, in order that the cooling may not be too great, so that the sum of the losses of work due to pneumatic resistances and cooling may be a minimum. The determination of the minimum is, however, too complicated to be given here. We can only state here that the diameter of this pipe should be in general $\frac{1}{5}$ of the diameter of steam piston, and, therefore, $\frac{1}{25}$ of its area. The velocity of the steam is then 25 times as great as that of the piston, or, since this is generally from 3 to 5 ft., 75 to 125 ft. The losses of work due to this great steam velocity we shall discuss hereafter; but it may be remarked here that it is better to make the steam pipe rather wide than narrow, especially in machines of high pressure and great piston speed.

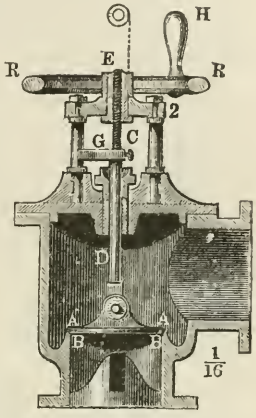
Fig. 697.



The arrangement of the regulating or steam throttle valve is shown in Fig. 697. *A A* is a portion of the steam pipe, *B* the valve, *C X* its axis, *D* a set screw, and *E F* the lever for moving the valve. By this valve, the steam is not entirely shut off. In order to effect this in high-pressure engines, a special stop valve is used. In low-pressure engines, such a valve is less necessary,

since by the condensation such machines are brought to rest. The arrangement of the stop valve is shown in Fig 698. The valve *A A* here has a stem *C D*, which, by the screw at *E*, can be lifted by turning the wheel *E* from the valve seat *B B*.

Fig. 698.



§ **446.—Valve Gear.**—The steam which is led into the steam chest enters the cylinder by special channels or passages, and is discharged by them into the atmosphere or condenser. The regular entrance and exit of the steam is caused by the regulating valve (Fr. régulateur; Ger. Steuerung). Here also, as in the water-pressure engines, which so closely resemble the steam engine, we may distinguish the inner and outer regulator. The inner regulator is in the interior of the steam chest, and consists of cocks, pistons, slides, or valves, which alternately open and shut the steam passages. We shall, in what follows, treat more fully of these important and manifold parts of the steam engine.

The piston regulator is but seldom used in steam engines. As it has already been described in the water-pressure engine, it needs no further notice here.

The cock regulator is also rare and only used in small, high-pressure engines. The cocks wear out quickly, require great force to turn them, and give a small steam quantity. In the older steam engines, cock regulators were used, especially the four-way cock (Fr. robinet à quatre voies; Ger. Vierweghahne), the application of which to piston machines has been already noticed in § 297.

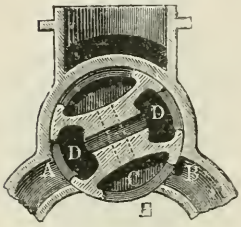
Maudslay has made use of peculiar cock regulators for small steam engines, as has also Cavé for oscillating engines. (See "Récueil des machines, etc., par le Blanc.")

The most ordinary and the best regulators are the slide-valve (Fr. tiroir; Ger. Schiebventil) and valve regulators. These are flat and hollow valves. Rotating slide valves

(Fr. tiroir à rotation; Ger. Drehschieber) are between ordinary valves and cocks.

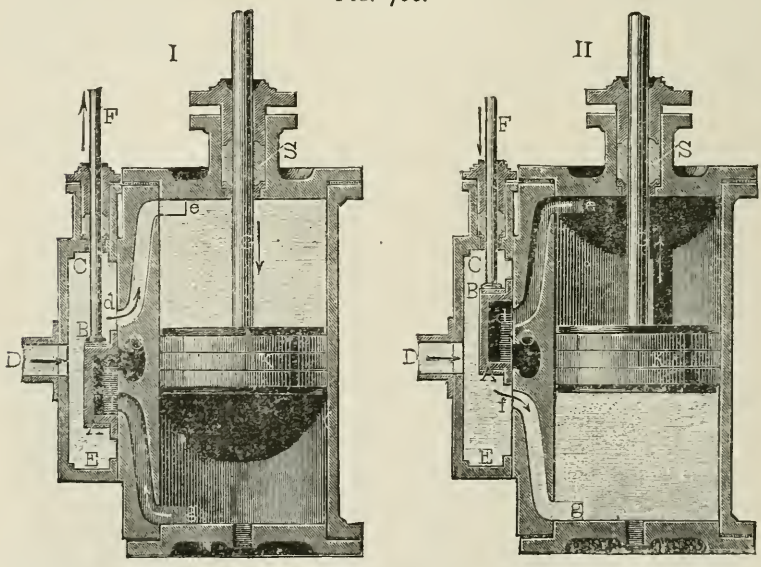
The rotating slide valves of Wilson and also of Corliss are not essentially different from cocks. Schwarzkopf's is similar to that of Schitko's—already described in water-pressure engines, § 303. The cross-section is shown in Fig. 699. The steam, passing through *D*, *D*, enters alternately by *A* and *B*, before and behind the piston, according to the position of the slide, while the exhaust steam, by either *A* or *B*, escapes through *C*. In order to balance the pressure of the valve against the sides of its box, we have the channel *D D*, so that the steam presses just as much upon one side as upon the other.

FIG. 699.



§ **447.—Slide Valves.** — The long D and the three-ported valves are those most generally used in the steam

FIG. 700.

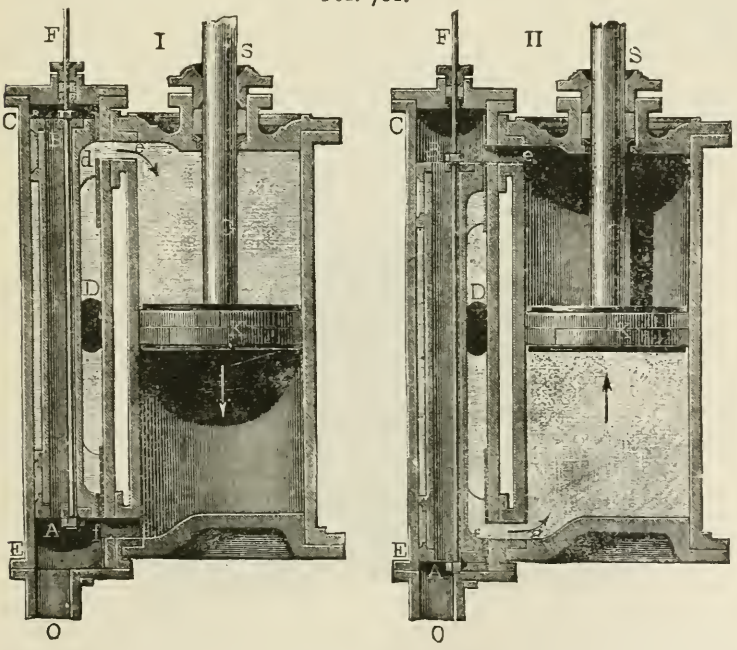


engine. The latter resemble somewhat the letter C, whence we may call it the C valve. Such a slide valve

is shown in Fig. 700, I and II. AB is the slide which is confined by the steam chest CDE, and moved by the valve rod BF. The steam passing through D enters, in I, through de above the piston K and drives it down, while in II it enters through fg below the piston and drives it up. In the first case, the exhaust steam passes through gf and the exhaust pipe at O, either into the air or the condenser; in the second case, it passes through cd to O.

In large machines, the filling of the passages de and fg at every stroke causes too much loss of steam, and hence the long D slide is used. Fig. 701, I and II, shows such a

FIG. 701.

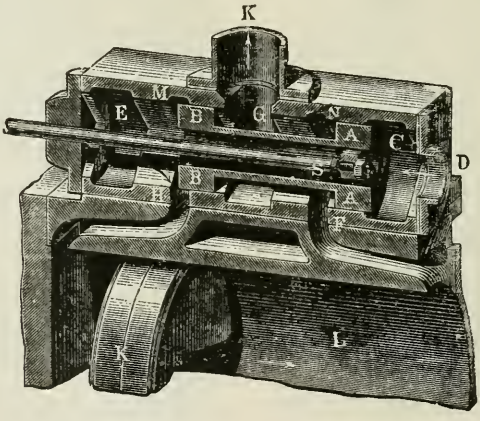


slide valve. The steam enters through D, the interior of the slide $A\,d$, and from it passes, according to its position, above or below the piston by de or fg. Upon the back of the slide there is a pipe $A\,B$, open at both ends, of semi-circular cross-section, and fitted steam-tight at A and B to

the steam chest. We see at once how the waste steam, during the rise of the piston, passes through *c d*, *B A*, and, finally, through the exhaust pipe at *O*, to the condenser, while on the down stroke it passes through *g f* direct to *O*.

The last slide has the further advantage over the first of being surrounded by the entering steam, and there is therefore no one-sided pressure, so that there is less friction than in the simple *C* slide. This friction in large, high-pressure engines may require an expenditure of several horse-powers. In recent times, therefore, we have even short slides for high-pressure engines, similar to the long Watt slide, so that the steam may not press upon one side only. The arrangement of such an equilibrium valve (Fr. tiroir équilibré; Ger. Entlastungsschieber) by Jobin ("Bulletin de la Société d'Encouragement," T. V., 1858), applied to a steam engine with horizontal cylinder, is shown in Fig. 702. The steam chest *C E* resembles the regulator

Fig. 702.

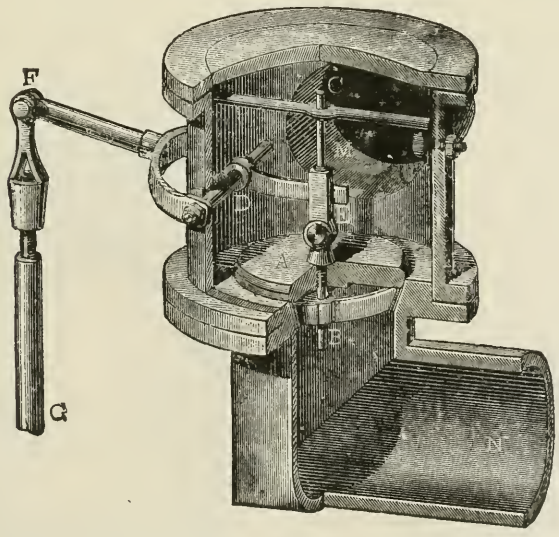


cylinder of a water-pressure engine, and the steam slide *A B* is in like manner essentially a union of two pistons *A A* and *B B* with a hollow piston rod *A B*. The steam, entering at *D*, the steam chest, not only fills the space *C* and *E* at both ends of the slide, but also the inner space *S*, and presses the slide therefore equally in all directions. The

steam, leaving the cylinder L through the passage K, surrounds the central portion $A\ B$ of the slide, and therefore causes no side pressure. Since the steam chest is widened at the points MH and NF, where the steam passages open, there is no side pressure even when one of these passages is shut off.

§ **448.—Valve Regulators.**—Valve regulators are applied to large engines, especially to those which are single-acting, since in such cases the slide would be too long in order to shut off with sufficient exactness, and the opening and shutting of the steam passages would take place too slowly. The valves used are either conical (see Vol. I.,

FIG. 703.



§ 445) or pipe valves. The latter differ from the first in having that part movable which in the conical valve is fixed, and that part slides which in the conical valve forms the seat. Both kinds are either single or double, and the latter are used especially in large machines, because they are more easily moved. The valves, moreover, are moved either by rods or levers. Fig. 703 shows a simple conical

valve with lever motion. A is the valve, $B\,C$ the valve stem, B and C the guides, D the axis of revolution which passes through the valve box, $D\,E$ a lever arm within and $D\,F$ a lever arm without the valve box. The first is attached to the valve stem at E, and the other is moved by the rod $F\,G$. If this rod is pulled, the lever combination turns at D, the valve A is raised, and communication opened between M and N. In Fig. 704 we have a pipe valve with rod motion. Here the valve plate A is fixed, and the box $B\,B$ is moved by a rod $C\,D$ passing through a stuffing box C. In the figure, B rests upon A, and the communication between M and N is closed, but if $B\,B$ is raised by means of $C\,D$, M and N are put in communication, and the steam can pass from M through B and under B to N. This valve, first used by Hornblower, has the great advantage that it is easier moved than the disc valve, because the cross-section is a circular ring instead of a full circle.

Fig. 704.

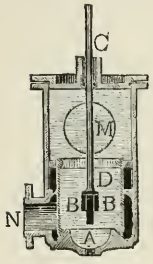
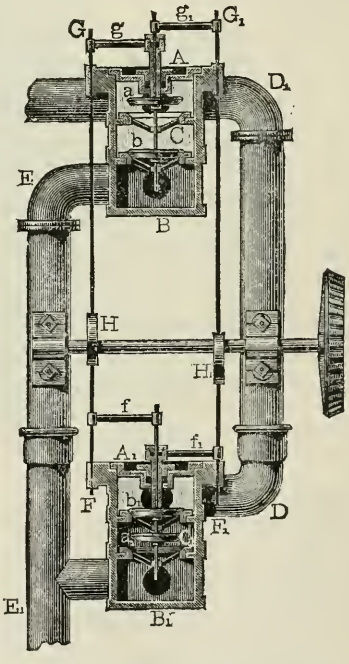


Fig. 705.

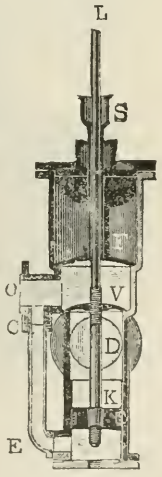


In order from one point to move two valves by means of stems, the stem of one valve is made hollow, and the stem of the other is passed through it. In this way we obtain the concentric valve of Murdoch. Fig. 705 shows an arrangement of this kind. The steam enters two chambers $A\,B$ and $A_1\,B_1$. Both are divided into three parts by two valve seats, the upper ones communicating by the steam pipe $D\,D_1$, the centre ones

with the cylinder, and the lower ones with the exhaust pipe. $F G$ and $F_1 G_1$ are two rods moved by eccentrics $H_1 H_1$ (see § 454), or some other means, which, by means of arms f, g, f_1, g_1, move the stems of the four valves a_1, a, b_1, b. We see from the figure that the stems of a and b_1 are hollow, and those of b and a_1 go through them. If $F G$ is raised, the valves a and a_1 are raised, and the steam passes through $D D_1$ into C, and so into the cylinder above, while below the steam escapes from the cylinder through C_1 to the discharge pipe $E E_1$. If, on the other hand, $F_1 G_1$ rises, and $F G$ falls, b and b_1 are opened, a and a_1 closed, and fresh steam enters through C_1 below the piston, while the exhaust steam escapes through C and $E E_1$.

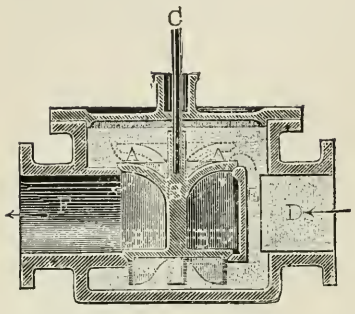
§ **449.—Steam Valve.**—The force necessary to lift a simple valve is the product of the steam pressure p and the valve surface F. Since in high-pressure engines F and p are large, this force is considerable. We have already pointed out that pipe valves, since they have a less section, cause less loss of work than conical valves, and must now call attention to the fact that, by the use of a counter piston or counter valve, this loss of work can be considerably diminished. Fig. 706 shows a conical valve with counter piston. V is the valve, K the counter piston, and $C E$ a pipe, which unites the pipe O communicating with the cylinder with the space below the counter piston. The steam presses the valve down and the piston up with about the same force, and hence the force necessary for lifting the valve is only that necessary for overcoming the friction. A second valve F, whose stem $F S$ surrounds the stem of V and K, is lifted in order to discharge the waste steam.

Fig. 706.

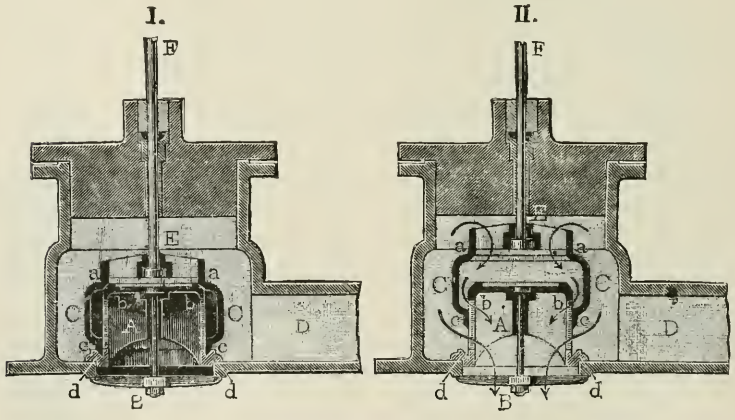


The purpose is more effectually accomplished by the double or lantern valve shown in Fig. 707. Here *A A* is one and *B B* the other valve plate; *S C* the stem, by means of which the valve is raised. The steam entering at *D* surrounds both valves and their seats on all sides, and presses as much up as down. But little force, therefore, is necessary to raise the valve, but as soon as this is done, the steam can enter through both the ring-shaped orifices between the valves and their seats, and thus enter the pipe *F*.

Fig. 707.



Finally, we may have double-pipe or bell valves, as shown in Fig. 708, I. and II. Here the valve seats *b b* and *d d* are fixed, and the box *c c* is moved by means of the



stem *E F*. If the valve is closed as in I., the ground faces *a a* and *c c* of the valve are in contact with the ground faces *b b* and *d d* of the seat. The steam, entering at *D*, presses then the entire bell-shaped valve about as much up as down, and the force necessary for lifting it is but small. When completely lifted (see II.), the steam can enter, be-

tween *a* and *b* and *c* and *d*, the space *A*, and from there passes through *B* to where it is needed.

The construction of such a bell valve is shown in Fig. 709. From the plate *G*, we have the wings *ff*, which serve as guides for the bell *C C* and arms *e*, *e*, which unite the rod *E F* with the bell.

Pipe valves may also be double-seated (see Reuleaux,

Fig. 709.

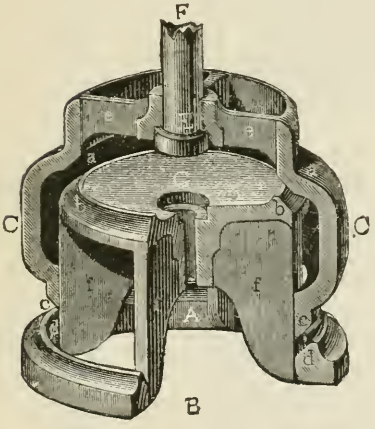
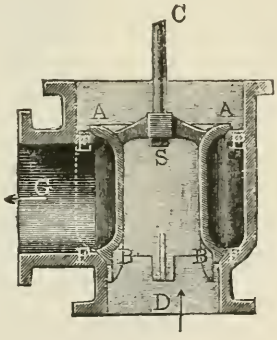


Fig. 710.

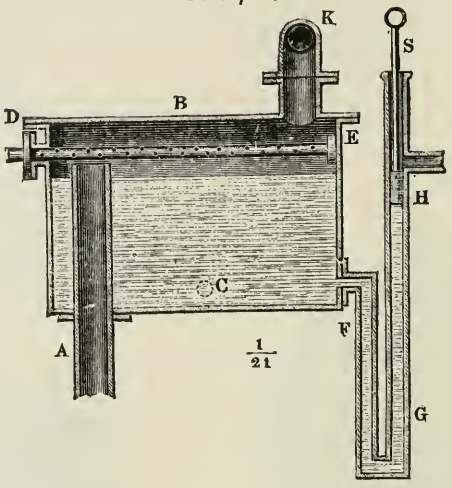


"Ein neues Doppelsitzventil," in the "Schweiz. polytechn. Zeitschrift," 1856). Such a valve is shown in Fig. 710. Here the pipe *A B B A*, which is lifted by the rod *S C*, is widened at both ends and turned off so that it sits upon the valve seats at *E F F E*. As shown in the figure, the steam entering at *D* is shut off from the pipe *G*, but, if the valve is raised, the steam can pass into *G* through *A A* and *E E* as well as through *B B* and *F F*. The force with which the steam presses the valve on its seat is, of course, equal to the difference between the cross-sections *A A* and *B B*.

§ **450.—Condenser.**—In the non-condensing engine, the steam, after acting in the cylinder, is discharged into the air or under water; in the condensing engine, however, it passes into the condenser (Fr. condenseur; Ger. Kühl-

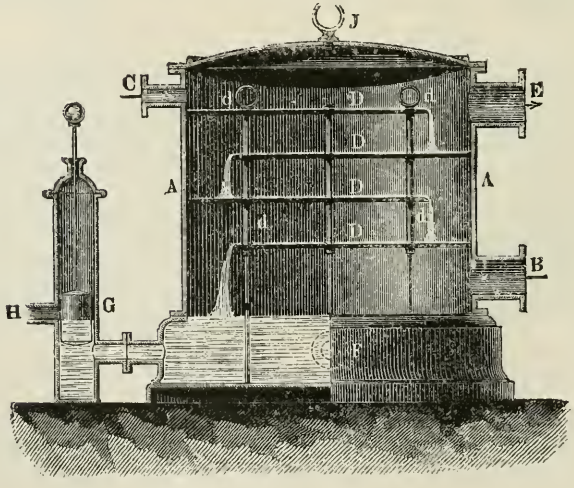
gefäss). In the first case, it is well to pass the steam through the feed water. An apparatus for this purpose is

Fig. 711.



shown in Fig. 711. *A* is the discharge pipe, which conducts the waste steam into the reservoir *B C*, and *D E* the

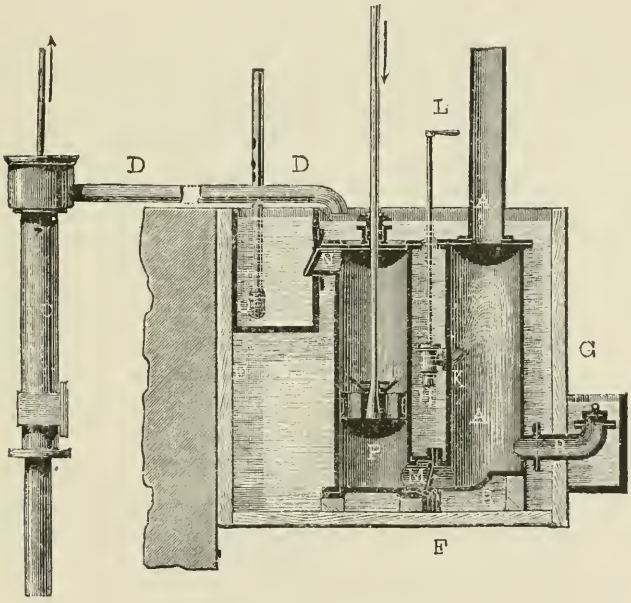
Fig. 712.



discharge pipe of the cold-water pipe, which is pierced with a number of small holes through which the water is

forced in jets into *B C*. This water is warmed by the steam, and most of it passes through *C* to the feed pump, which forces it into the boiler. The surplus water flows through the pipe *F G H*, which contains a float *S*, and the surplus steam escapes through *K*. A better apparatus is shown in Fig. 712. The feed-water enters the heater *A B C* through a pipe at *C*, spreads out in layers over the plates

FIG. 712.



D, *D* . . . and falls from one plate to the next lower, so that its temperature is raised at least 70° by the steam which enters at *B* and departs at *E*.

The condenser, which is designed to condense the greater part of the waste steam, is a cast-iron vessel *A B*, Fig. 713, surrounded externally by cold water, and into which a stream of cold water, called the injection water (Fr. eau d'injection; Ger. Einspritzwasser) is forced in a fine spray. The cold water is pumped into the external tank *E F G* by the cold-water pump (Fr. pompe d'eau

froide ; Ger. Kaltwasserpumpe) by means of the pipe *D D*. In this tank is an apparatus *H*, by which the injection water is led into the condenser. The tank water enters this apparatus from below, and flows through *H K*, which is fitted with a rose, in a number of fine jets, with great velocity, since there is but $\frac{1}{10}$ to $\frac{1}{8}$ of an atmosphere pressure in the condenser. To regulate the amount of injection water, we have a valve or cock, worked by means of the handle *L* and the rod *L H*. In connection with the condenser is the air pump (Fr. pompe à air; Ger. Luftpumpe), which pumps out of the condenser the injection water, the air brought in with the injection water, the uncondensed steam, and the water arising from the condensed steam. It is an ordinary suction pump, with hollow piston *P*, suction valve *M*, and pressure valve *N*. The warm water flows through *N* into the hot-water well *N O*, from which a portion is drawn by the feed pump by means of the pipe seen at *O*, and forced into the boiler. Finally, we have a short pipe *R* connected with the condenser, having a valve opening upwards. This is the blow-through valve (Fr. soupape à souffler; Ger. Ausblaseklappe). It serves to carry off the air which collects in the condenser after the engine has been some time at rest.

To obtain more perfect condensation, in recent times, double-acting air and hot-water pumps have been used.

A short barometer, connected with the condenser, serves to show the pressure within.

Besides the above injection condenser of Watt, there is the surface condenser of Hall. In this, the steam passes through a system of pipes, which are surrounded by cold water. The great amount of cooling surface required by such an arrangement has prevented its more general use. The salt contained in sea water renders it necessary, in marine engines, to blow off from time to time a portion of the boiler water, so that there is considerable loss of heat from this cause. For this reason, the surface condenser, which renders this blowing off unnecessary, is of special importance in marine engines.

[In modern marine engine practice, the increased pressure of steam carried on nearly all ocean steamers has rendered the use of surface condensers a matter of necessity; so that jet condensers are rarely to be found on such vessels. The difficulties of keeping the earlier forms of surface condensers tight, on account of the expansion of the tubes, is overcome in modern practice by arranging the tubes so that they are free to contract and expand, one or both ends passing through stuffing-boxes which are made tight by glands or grummets composed of wood or compressed paper. The

SURFACE CONDENSERS.

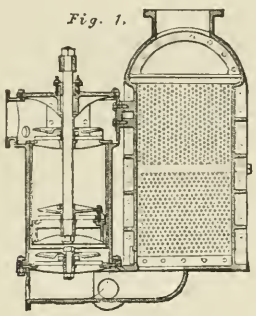


FIG. 713*a*.

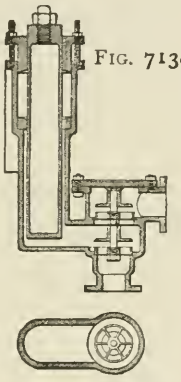
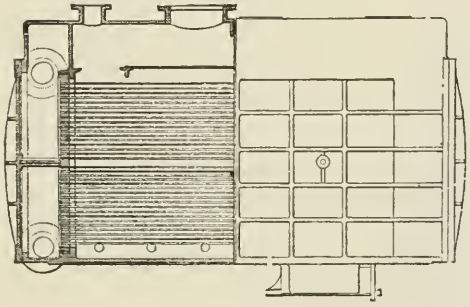


FIG. 713*c*.

FIG. 713*b*.



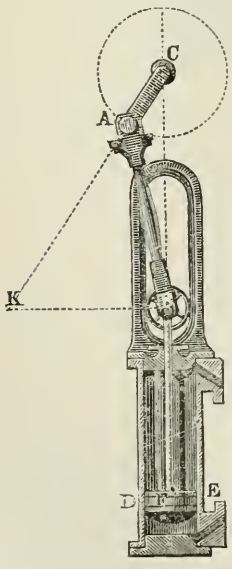
general arrangement of a surface condenser, such as is used in connection with marine engines, is shown in the accompanying figures. It consists, as will be seen by reference to Figs. 713*a*, 713*b*, of a number of tubes within a box or case. The exhaust steam passes around the tubes, and the water of condensation is removed by the air pump shown in Fig. 713*a*. Cold water is pumped through the tubes to condense the steam by a circulating pump, Fig. 713*c*. In practice there are, of course, many modifications of this gen-

eral principle. Sometimes the exhaust steam passes through the tubes, and the condensing water circulates around them: and independent air and circulating pumps, driven by small engines, are often employed.

It is very common, in proportioning surface condensers, to make the condensing surface $\frac{2}{3}$ of the boiler heating surface, and there are a number of engines with surface condensers giving good results, in which the condensing surface in square feet is from $2\frac{1}{2}$ to 3 times the indicated horse-power. Experiments on the amount of condensation that can be effected by thin tubes cooled by the circulation of water give from 50 to 120 pounds of steam condensed per hour, by each square foot of tube surface; but these experiments were made with clean tubes, and under conditions much more favorable than those which generally obtain in practice. (See *Engineering*, xx. 449.) Considerable information on the construction of surface condensers, together with reports of their performance, may be found in the "Transactions of the Society of Engineers," London, 1863, page 96, and "Transactions of the Society of Engineers in Scotland," iv. 69, 90, 124, v. 70, 124, xi. 107.]

§ **451.**—In the ordinary piston steam engine, only a rectilinear reciprocating motion is directly caused. When, now, the working machinery which is set in motion by the engine must also have a rectilinear reciprocating motion, the connection of the two may be direct or by means of a lever. But if the working machinery, as is generally the case, has a continuous circular motion, then special *intermediate* machinery (see § 108) is needed, which shall transform the rectilinear motion of the engine into the rotary motion of the working machinery. In general, this intermediate machinery consists:

FIG. 714.



1. Of a crank (Fr. manivelle; Ger. Kurbel) (see § 142).

2. Of a connecting rod (Fr. bielle; Ger. Kurbelstange).

3. Of a fly-wheel (Fr. volant; Ger. Schwungrad).

The crank CA, Fig. 714, forms a part of the shaft C, and is joined to the piston rod BF by means of the connecting rod AB. In order that the cross-

head B may not be drawn to one side by the connecting rod, it is made to move between guides, and in order that the crank shaft C may not experience sudden changes of velocity owing to the variable action of the connecting rod, a fly-wheel (see § 111) is attached to it. The most usual piston engines are stationary. Portable steam engines find special application in agriculture.

§ **452.**—The various vertical piston steam engines may be grouped under the following classes:

I. According to the number of cylinders:
   1. Single cylinder.
   2. Double-cylinder engines.

II. With reference to the construction of cylinders:
   1. Fixed cylinder.
   2. Movable cylinder.

   In the first case, the cylinders are,
   (*a*) Vertical.
   (*b*) Horizontal.
   (*c*) Inclined.

   In the second case, they are,
   (*a*) Oscillating.
   (*b*) Rotary.

III. With reference to the action of the steam:
   1. Single acting.
   2. Double acting.

IV. With reference to the transmission of the steam power:
   1. Direct acting.
   2. Indirect acting.

   And in the latter case either,
   (*a*) With balance lever.
   (*b*) Without balance lever or working beam.

Besides piston engines, we have also rotary engines, in which the steam acts upon the paddles of a wheel enclosed in a vessel and sets it in rotation. Such direct-acting rotary engines have had, however, no general acceptance. (See "Verhandlungen des Vereins für Gewerbefleiss in Preussen," Jahrgang 1838.) The water-wheel represented in Fig. 580 may also be used as such an engine. In England, Bishop's disc engine has had the greatest success of any. (See "The Steam Engine, etc., by Tredgold," Vol. III., by J. Weale, 1853; also, "Traité des machines à vapeur, etc., par C. E. Jullien," Sect. I.)

Locomotive engines are used only for the transportation of loads, and are either land locomotives or marine engines.

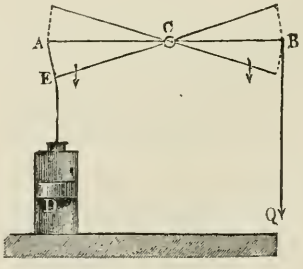
§ **453.**—Many of the above classes are illustrated in the following sketches:

Fig. 715 represents a single- and direct-acting engine. The load, as, for example, a pump at *Q*, is directly attached to the piston rod *D E*.

Fig. 716 is a sketch of a single-acting machine with

Fig. 715.

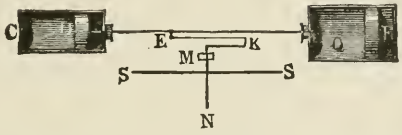
Fig. 716.



working beam. *A C B* is the balance beam turning about *C*, *D E* the piston rod, *A E* the connecting rod, and *B Q* the balance rod to which the counterpoise is attached.

Fig. 717 is a horizontal double- and direct-acting steam engine. The steam piston D moves here, by means of the

Fig. 717.



prolonged piston rod DF, another piston F of a pump or blowing engine. To prevent irregularities of motion, a crank rod EK and crank MK impart motion to a fly-wheel SS on the shaft MN. Fig. 718 shows a double-acting engine with working beam and rotary motion. It is therefore indirect acting. MK is the crank, BK the connecting rod for the fly-wheel SS.

Fig. 718.

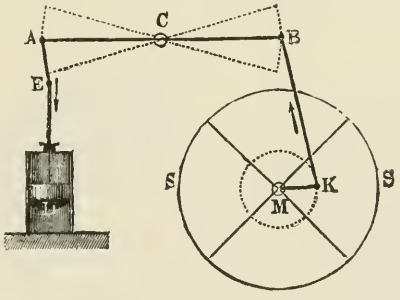


Fig. 719.

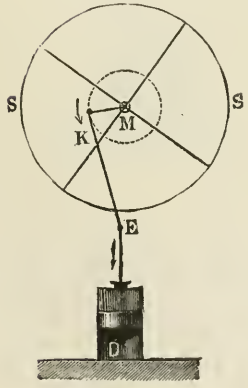


Fig. 720.

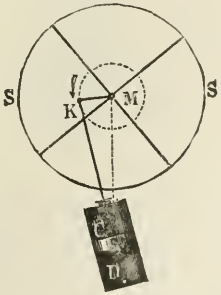


Fig. 719 is an indirect-acting engine, double acting, without balance beam. Fig. 720 the same, without connecting rod, and the cylinder therefore oscillating.

In order that the piston rod in Figs. 718, 719, and 720 may move in a vertical line, there are at E guides; and, in Fig. 720, these guides are attached to the cylinder so that the piston rod may oscillate with it. If the distance CM of the axis of oscillation C from M is less than the length MK of the crank arm, the oscillating motion of the cylinder is transformed into a rotary motion about M.

Fig. 721 is the sketch of a *double-acting* steam engine like that in Fig. 719, except that here, to economize space, the crank is not fastened to the end of a massive connecting rod, but at the middle of a hollow piston rod EF. Such a construction is called a *trunk engine*.

Fig. 722 is a two-cylinder, double-acting engine without walking beam, by Maudslay. Here both piston rods BD,

Fig. 721.

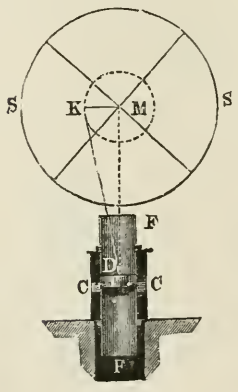
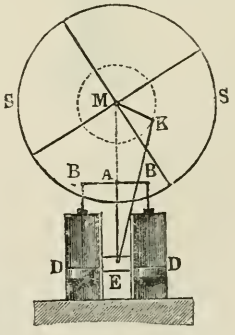


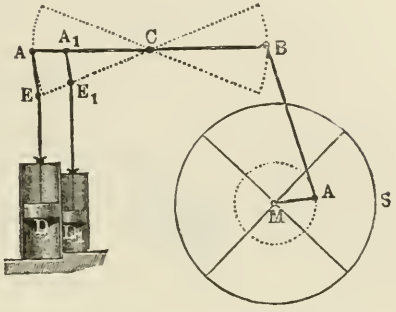
Fig. 722.



BD are connected by a cross-head BAB, and this again with another cross-head E by a third rod AE, which runs in vertical guides between the two cylinders, and is also connected with the connecting rod KE.

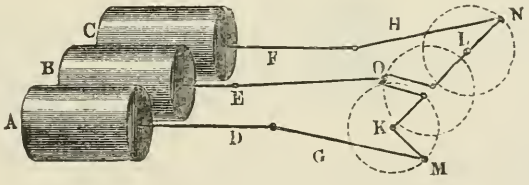
In Fig. 723 is a sketch of a so-called Woolf engine with two cylinders, whose pistons move up and down at the same time, and are connected by the rods $D\,E$, $D_1\,E_1$ to a walking beam $A\,C\,B$. The steam which puts the large piston D in motion has previously acted in the smaller cylinder D_1. Such engines are called *compound* engines.

Fig. 723.



In modern times, Woolf engines are made, especially for the French Navy, with three horizontal cylinders $A\,B\,C$, Fig. 724, of which only the middle one B is fed

Fig. 724.

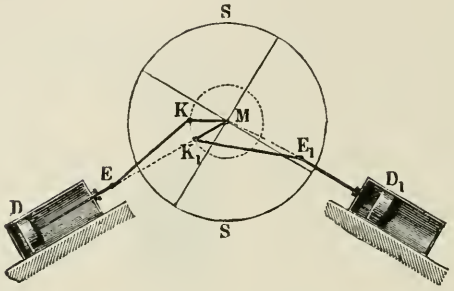


with fresh steam, while the others A and C are operated by steam working expansively. The three piston rods D, E, F of this machine are connected by the rods G, H and $E\,O$ with the three cranks of the shaft $K\,L$. The outer crank pins M and N are 90° apart, while the middle one O makes an angle with the first $= \pm 135°$.

Finally, Fig. 725 represents an engine with two inclined cylinders. The connection of the piston rods $D\,E$, $D_1\,E_1$ with the cranks $M\,K\,M\,K_1$ is exactly the same as in the engine in Fig. 719. The angle $K\,M\,K_1$ between the cranks is equal to the angle $D\,M\,D_1$ between the two piston rods minus 90°. If the cylinders, as in marine engines, lie upon the same side, the angle $D\,M\,D_1 = 0$ de-

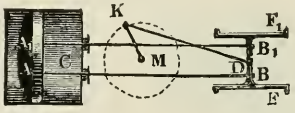
grees, and, therefore, the angle between the crank pins is 90°.

FIG. 725.



A horizontal marine engine with two long piston rods $A\,B$, $A_1\,B_1$ is shown in Fig. 726. To economize space, the

FIG. 726.



crank shaft M and the connecting rod $K\,D$ are placed between the cylinder C and the guides $F\,F_1$.

§ **454.—Eccentrics.**—The interior valve gear consisting of the so-called distributers must be operated by the engine itself, and it is necessary, therefore, to connect it with the piston rod, the walking beam, the fly-wheel, or with some other moving part of the machine. The apparatus which effects this, and which is called the outer valve gear, consists essentially of either

(1) A continually revolving eccentric (Fr. excentriques; Ger. excentrischen Scheiben); or

(2) An oscillating lever (Fr. encliquetage; Ger. Hebel); the first being used only for double-acting and the latter

especially in single-acting engines, because these have not a rotary motion.

The eccentric or eccentric cam is found with very different forms—it may be either circular, triangular, or with numerous teeth. The circular eccentric is the simplest form for the outer gear, and is that most generally used; we will explain it, therefore, first.

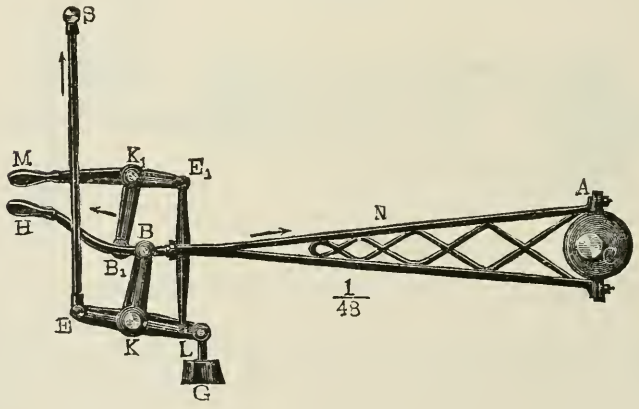
The eccentric consists of a cast-iron cylindrical pulley *A C A*, Fig. 727, revolving about the axis *D*, which is not at the geometrical axis *C*, and is surrounded by a band of brass or wrought-iron which carries the long latticed rod called the eccentric rod. The other end of this rod, furnished with the handle *H*, connects with one arm *K B* of a

FIG. 727.

bell-crank lever, the other arm *K E* of which is fastened to the valve stem *E S*. In order to counterbalance the weight of this latter, a counterweight *G* is hung upon a third arm *K F*. The working of this apparatus is easily explained; the centre *C* of the eccentric describes with each revolution of the fly-wheel, to which the eccentric is usually affixed, a circle, and throws the strap outward in every direction successively a distance equal to its eccentricity *C D*, and, therefore, moves the eccentric rod axially a distance of $2 \times \overline{CD}$ back and forth. The end *B* of the rod of course participates in this motion, and the same is transferred by the bell-crank lever *B K E* to the valve stem *E S*.

In many engines, especially in those which operate elevators in the shafts of mines, it is necessary to be able to reverse the motion at any time, that is to make it revolve in the opposite direction. This will be attained when the regulator is given the opposite position, since then the steam enters the opposite chamber of the steam cylinder. Fig. 728 shows one of the older contrivances for effecting this. Here, besides the lever $E\,K\,B$, a second lever $E_1\,K_1\,B_1$, movable about the axis K_1, and connected with the first by the rod $E_1\,L$, is used. To reverse the engine, it is only necessary, at the middle position of the steam piston, to raise the slot of the eccentric rod from the bolt B of the first lever, and then, by means of the handle M, to move the upper lever so that the bolt B_1 may

FIG. 728.



fall into this slot. By this operation, the steam is admitted to the opposite chamber of the steam cylinder and changes the stroke of both the steam piston and regulator. This reversion may be more simply accomplished by the use of Stephenson's link motion, the arrangement and working of which will be explained more fully hereafter.

§ 455.—**Watt's Steam Engine.**—The application of combined eccentric and slide valve gear is shown by the

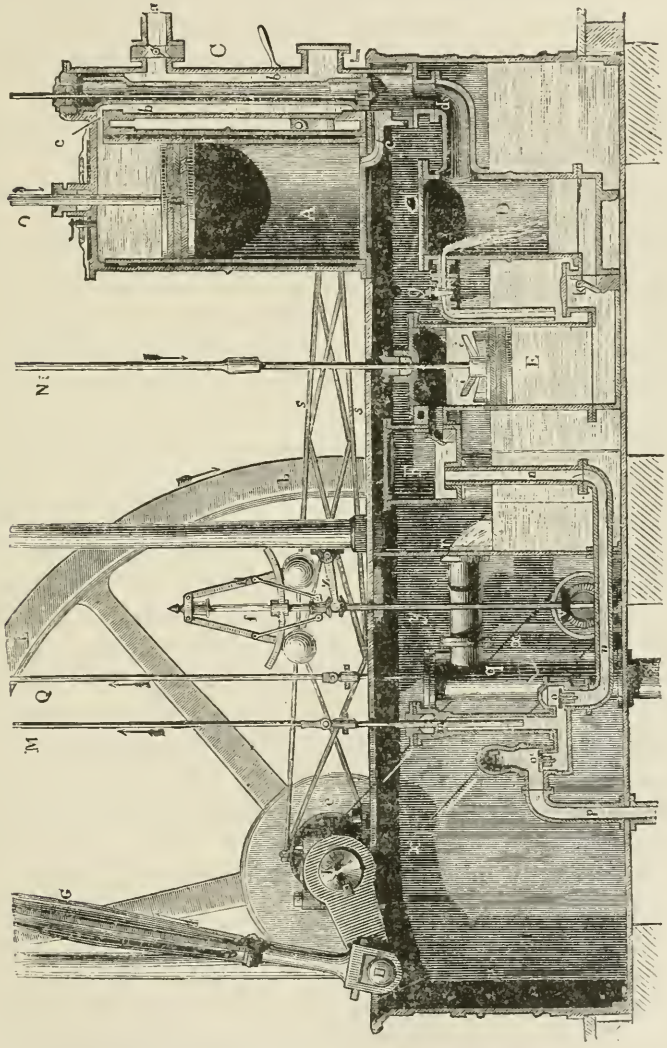


Fig. 729.

representation in Fig. 729 of a Watt low-pressure engine: the figure also gives a clear idea of a complete engine and its essential parts. *A* is the steam or working cylinder, *B*

the steam or driving piston, and *C* the steam box from which the steam from the steam-pipe *a* is admitted, by the hollow slide valve *b b*, through the passages c_1 and c, now below and now above the steam piston *B*, and forces this to ascend and descend. Further, *D* is the condenser and *E* the air pump; to the former is carried, by the pipe *d*, the exhaust steam after its work is done, and the latter removes the air and water to a reservoir *F*, from which the air escapes by holes in the cover, whilst the greater part of the water flows off through a side pipe. A small part of this water of condensation, however, flows through *n n* to the feed pump *m*, and is forced by this through the pipe $o\ o_1\ p$ into the boiler. Behind the feed pump is the cold-water pump *q*, seen in elevation, which continually pumps cold water into the reservoir which surrounds *D* and *E*, through the pipe *q r*. At *O* is seen the driving piston rod, and at *N* the piston rod of the air pump, as well as those of the feed and cold-water pumps at *M* and *Q* respectively—all four being connected with the working beam, not seen in the figure, the two first being also connected with each other by the so-called "parallel motion" of Watt. The oscillating motion imparted to the working beam by the driving piston is transformed into a rotary motion by the connecting rod *G* and the crank *H K*, which motion is made uniform by the fly-wheel *L L*. Upon the shaft of the latter is placed the eccentric cam *e*, which, by means of the rod *s s* and a bell-crank lever (not seen in the figure), moves the slide-valve stem up and down. The arrangement in detail of the regulating apparatus is to be seen from Figs. 701 and 727, and has already been explained.

The apparatus *f* is the so-called *centrifugal governor*, which is made to revolve by the fly-wheel by means of the endless band *x x*, the gearing *v*, and the shaft *y*, and is so connected, by its rod and the lever *z*, with a throttle valve in the steam pipe, that this valve is opened or closed as the two metal balls approach each other or separate. The amount of steam admitted to the cylinder is thus

changed and great variations in the speed of the engine avoided.

The detailed description and theory of this apparatus, as well as of Watt's parallel motion, etc., must be reserved for a special section of the third volume of this work.

§ **456.—Lead of the Slide Valve.**—The ports (Fr. lumières; Ger. Wege), which the steam traverses from the steam box to the cylinder, must have a certain cross-section in order not to cause too great resistance. It is best to make the area of these channels as large as the area of the steam pipe—*i. e.*, one twenty-fifth the area of the steam piston. Sometimes, especially in high-pressure engines, it is even made more, or one twentieth to one fifteenth the piston area. In order to render the force consumed in moving the slide valve as small as possible, it is necessary to make the opening of the steam ports wider or broader than high, because the stroke of the slide valve is then less (compare Vol. II., § 327). Usually, the ratio between the width and the height of this opening is four to one or five to one.

Moreover, the slide valve causes a special contraction, especially when it is operated by an ordinary eccentric, since the openings of the steam passages are not opened and closed suddenly, but gradually. That the steam may work most uniformly and the engine most advantageously, it is necessary that the slide valve begin to open the steam passages before the piston has finished the last part of its stroke, because then the entering steam can exert its full force at the commencement of the opposite stroke. For similar reasons, it is advantageous to shut off the steam and open the exhaust ports behind the piston before the end of the stroke. This earlier opening of the steam ports is effected by a given ratio between the dimensions of the slide valve and those of the ports, as well as by a certain placing of the eccentric on the crank shaft, and is called the *lead* (Fr. avance; Ger. Voreilen) of the slide valve. The *lead*, then, is the amount by which the steam port is

opened when the piston is just beginning its stroke. Experiment shows that the earlier opening of the exhaust ports is especially of advantage, and in the best engines the lead of the valve upon the side of the exhaust, or the *inside lead*, is $\frac{1}{25}$ to $\frac{1}{15}$—*i. e.*, the slide valve at the lowest or highest position of the piston has made an opening whose height is $\frac{1}{25}$ to $\frac{1}{15}$ of the whole throw of the slide valve. The *outside lead* of the slide valve or the lead on the steam side, on the other hand, is much smaller, and is often only $\frac{1}{100}$ of the whole throw of the valve.

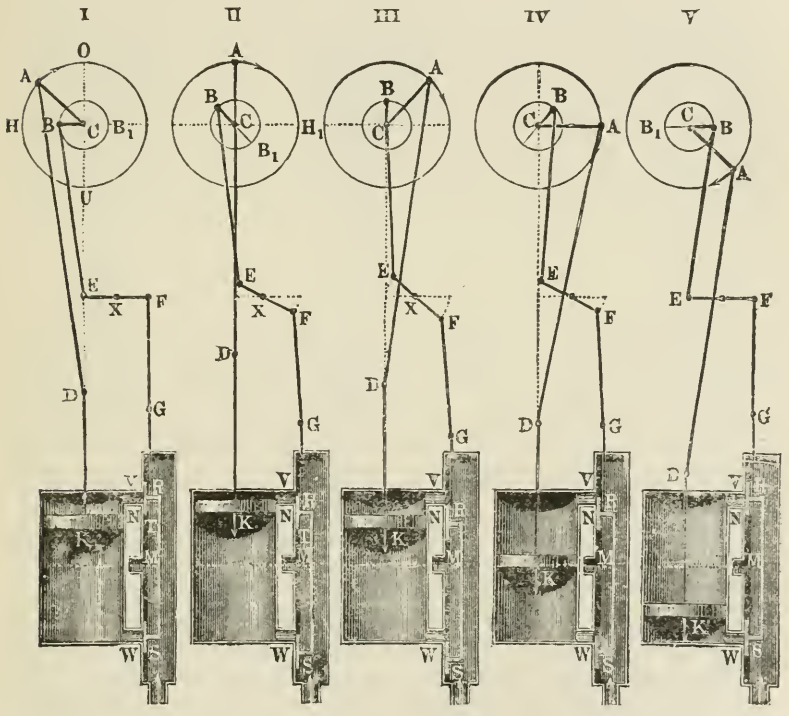
§ **457.—Setting the Slide Valve.**—The way in which the slide valve opens and closes the steam ports in its various positions is shown in Fig. 729*a*, (I., II., III., IV., V.). *V*, *W*, and *M* are the three steam passages; *V* leads above, *W* below the piston, and *M* into the open air. The steam surrounds the slide valve upon the outside and enters the cylinder through *V* or *W* as the valve is moved down or up. This arrangement is followed in high-pressure engines, whereas in the Watt low-pressure engine the steam is admitted through *M* and first surrounds the valve on the outside after acting on the piston. We will consider, however, only the first method of steam distribution.

The middle position of the valve is shown in I. and V.; in this position the steam is neither admitted to nor released from the cylinder. If the valve is moved downward so that the position II. is attained, the admission and release ports are first opened, and when the valve reaches the position III. both passages are fully opened. If the valve now again ascends to IV., the closing of both passages commences, and when the valve reaches the position V. both are entirely closed and the steam is cut off, as in I., from both admission and release. If the valve continue to ascend, it will reach a position (not shown in the figure) which will permit the release of the steam behind the piston, and the admission of steam in front; later, at the highest position of the slide valve, both ports will be completely open, and the descent of the valve will again

gradually close them, until finally the valve reaches the position I., which is the beginning of a new stroke.

If, now, the slide valve is to have any lead, *i. e.*, if the steam passages are to be opened at all, at the highest and lowest positions of the piston, the eccentric must not throw the slide valve into the positions I. and VI. (not shown),

FIG. 729*a*.



when the piston is in those positions. The middle position of the slide valve must, therefore, be reached somewhat before the highest and lowest position of the piston. But this highest and lowest position of the piston will not then correspond to the middle position of the slide valve, and, finally, the steam will be cut off from sides of the piston a certain time before the end of the stroke. During this period of cut-off, the steam must expand on one side of the

piston and be compressed on the other side, whereby a loss of force, although at the same time a saving of steam, is occasioned. It is easy to be seen that, by changing the width $R\,T$ of the valve face, the so-called *lap* (Fr. recouvrement; Ger. Deckung) of the valve, the time of admission, cut-off, and release of the steam may be varied. If the *outer lap* be diminished, or the width $R\,T$ of the face be lessened by taking away from it at R, the time of admission of the steam through V or W is increased; if the *inner lap* be diminished, or the face of the valve be lessened by taking away at T, the release of the steam through M will be effected sooner and continue longer. If, on the contrary, the width of the face of the valve, and therefore the lap, is increased, the opposite effect will be had on the admission, cut-off, etc. of the steam.

§ **458.—Law of Motion of the Crank.**—In order, now, to show how, by the proper placing of the eccentric with reference to the crank, the above-described positions of the slide valve may be effected, it is necessary to study first the general conditions of the motion of this part of the engine.

Imagine the crank pin P to be a point, and assume that it revolves at a distance $C\,A = C\,B = r$ from the axis C, Fig. 730. If the crank pin A has travelled the angular distance $A\,C\,P = \beta$ from the highest or so-called *dead point* A to the point P, the connecting rod $A\,D$ whose length $= l$ will have reached the position $P\,Q$, and the space travelled by its end in the direction of the centre line $C\,D$ is

$$\overline{D\,Q} = \overline{A\,N} + \overline{N\,Q} - \overline{A\,D},$$

i. e.,

$$s = r - r\,cos.\,\beta + \sqrt{l^2 - r^2\,(sin.\,\beta)^2} - l$$

$$= r\,(1 - cos.\,\beta) - l\left[1 - \sqrt{1 - \left(\frac{r\,sin.\,\beta}{l}\right)^2}\right],$$

or, since the length l of the connecting rod is at least five times the radius r of the crank-pin circle, approximately,

$$s = r\,(1 - cos.\,\beta) - \frac{r^2\,(sin.\,\beta)^2}{2l},$$

wherefore we will assume

$$s = r\,(1 - cos.\,\beta).$$

Fig. 730.

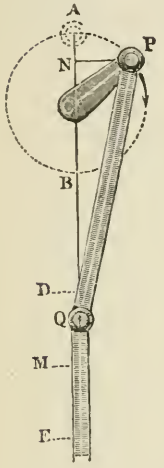


Fig. 731.

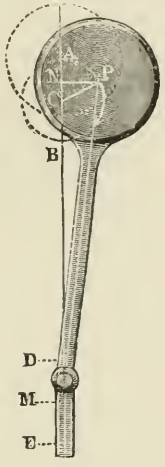
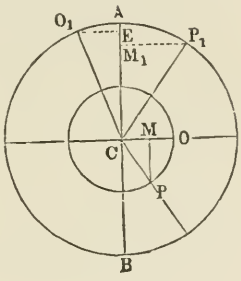


Fig. 732.



The end D of the rod would really only describe the space given by the last expression in the case of an infinitely long connecting rod.

The crank pin is, in point of fact, cylindrical; this, however, has no effect upon the conditions of motion, since the centre of the hole in the connecting rod coincides with the axis of the crank pin; this point, then, has the same motion as though it were connected directly with the axis P. The relation is not altered even when the crank pin has a greater diameter than the crank-pin circle, as shown in Fig. 731. Since in this case the crank becomes an eccentric cam, it follows that the formula

$$s = r\,(1 - cos.\,\beta)$$

applies to the eccentric also when the length $\overline{DA}$ of its rod exceeds many times its eccentricity $r = \overline{CA}$.

§ **459.—Slide-Valve Curves.**—At the middle position of the slide valve, the centre of the eccentric, in accordance with the foregoing, must be at the middle O, Fig. 732, but the axis of the crank pin O_1 must be a certain angular distance $O_1 C A = \alpha$ from the dead point A, because, at this position of the slide valve, the stroke of the steam piston will not have been quite finished. If the shaft upon which both crank and eccentric are placed revolves through the angle $O C P = O_1 C P_1 = \beta$, the eccentric throws the slide valve a distance

$$\overline{M P} = y = r \sin. \beta,$$

while the steam piston first finishes the remainder,

$$\overline{E A} = r_1 (1 - \cos. \alpha),$$

of its ascent $2 r_1$, and afterwards traverses the distance

$$\overline{A M_1} = r_1 [1 - \cos. (\beta - \alpha)]$$

of its return stroke, so that it will stand at the distance

$$\overline{C M_1} = x = r_1 \cos. (\beta - \alpha)$$

from its middle position. If we substitute in the formulæ,

$$x = r_1 \cos. (\beta - \alpha)$$

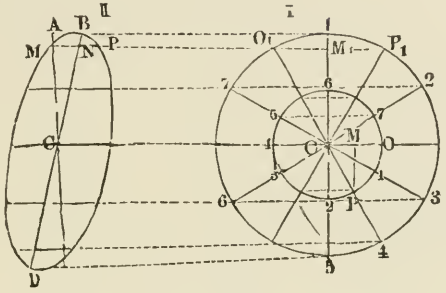
and

$$y = r \sin. \beta$$

for β all values from 0° to 360°, we obtain all the possible positions of the slide valve in reference to the steam piston, and to represent these more clearly we can construct, with the paths x and y as co-ordinates, a curve or the so-called diagram for the slide valve. The manner in which these curves are drawn is shown by Fig. 733, I. and II. In I. the larger circle is the crank circle, and the smaller one the eccentric circle; II. shows the curve constructed with the x's and y's. Equal numbers on the two circles desig-

nate corresponding positions of the crank and the eccentric: if the latter stands at O, 1, 2, etc., the former is at O_1, 1, 2,

FIG. 733.



etc.; if the eccentric has travelled from O to P and thrown the eccentric

$$\overline{MP} = y = r \sin. \beta$$

from mid-gear, the crank pin has likewise revolved from O_1 to P_1, and the piston is

$$\overline{CM_1} = x = r_1 \cos. (\beta - \alpha)$$

from its middle position. If we lay off in II., $CM = x$ as an abscissa, and $MP = y$ as an ordinate, we obtain one point P of the desired curve. If we put $\beta = \alpha$, we obtain the ordinates $CA = r_1$ and $AB = r \sin. \alpha$ of the point B through which one axis of the curve may be drawn; and if we take this as one axis of co-ordinates, we obtain a very simple formula for the curve. We have for the angle $BCA = \delta$, which this axis makes with the old axis, the expression,

$$\tan. \delta = \frac{AB}{CA} = \frac{r}{r_1} \sin. \alpha,$$

therefore, the new abscissa is

$$CN = x_1 = \frac{CM}{\cos. \delta} = \frac{x}{\cos. \delta} = \frac{r_1 \cos. (\beta - \alpha)}{\cos. \delta},$$

and the new ordinate,

$$NP = MP - MN;$$

i. e.,

$$y_1 = y - x \tan. \delta = r \sin. \beta - r \cos. (\beta - \alpha) \sin. \alpha,$$

$$= r [\sin. (\beta - \alpha + \alpha) - \cos. (\beta - \alpha) \sin. \alpha] = r \sin. (\beta - \alpha) \cos. \alpha;$$

since, now

$$[\sin. (\beta - \alpha)]^2 + [\cos. (\beta - \alpha)]^2 = 1,$$

it follows that in this case

$$\left(\frac{y_1}{r \cos. \alpha}\right)^2 + \left(\frac{x_1 \cos. \delta}{r_1}\right)^2 = 1.$$

If we put $\frac{r_1}{\cos. \delta} = a$ and $r \cos. \alpha = b$, we obtain the well-known formula of the ellipse,

$$\left(\frac{x_1}{a}\right)^2 + \left(\frac{y_1}{b}\right)^2 = 1.$$

The motion curve is, therefore, an ellipse whose semi-axes are

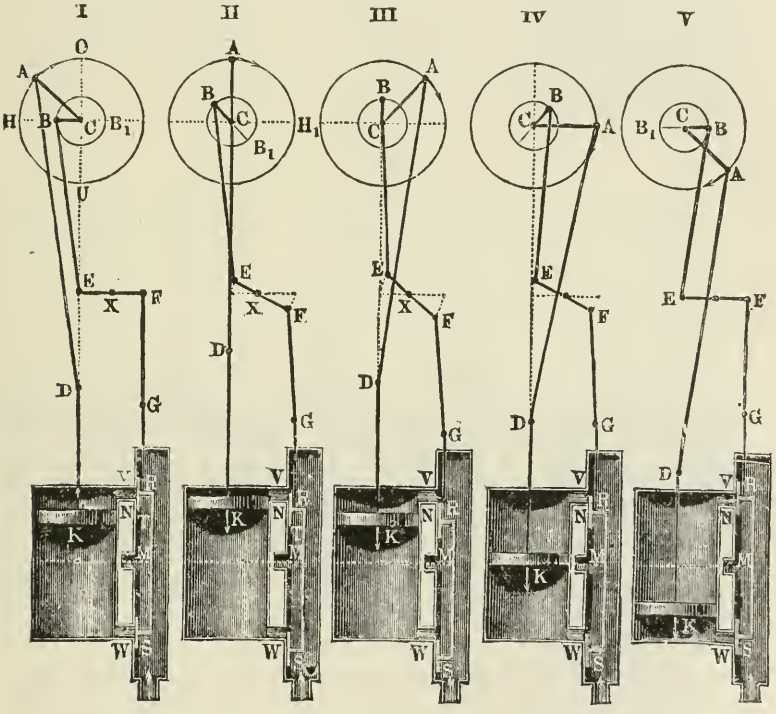
$$a = \frac{r_1}{\cos. \delta} \quad \text{and} \quad b = r \cos. \alpha.$$

§ **460.–Eccentric Gear.**—The manner in which the slide valve is moved by the eccentric is shown in Fig. 734, I., II., III., IV., V.

The piston K, by means of the piston rod KD and the connecting rod DA, sets in motion the crank CA. Upon the axis C the eccentric is fastened, which thus constitutes a second crank BC. The slide RS, the motion of which has already been considered (§ 457), is connected with an equal-armed lever EF by means of the rod FG, this lever being, in turn, connected by the rod BE to the centre B of the eccentric disc. Thus the motion of the slide is the

same in amount, but opposite to the direction it would have if it were directly connected at E with the eccentric rod, and, therefore, has precisely the same motion as though moved directly by a crank CB_1. If, now, the central angle ACB_1 between the crank and eccentricity

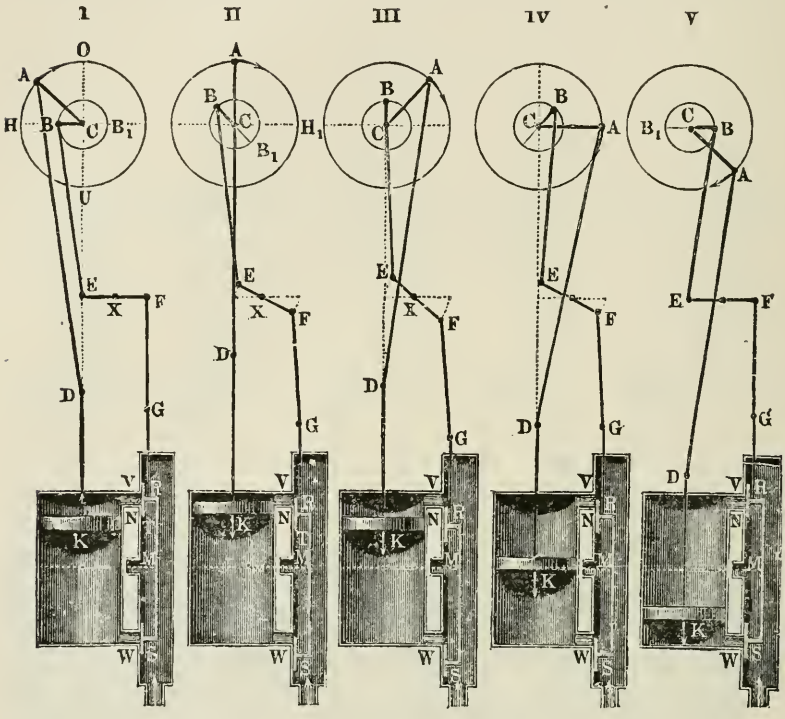
FIG. 734.



were $= 90°$, the slide RS would be in its central position at the times when the piston is at the end of its stroke, or when the crank is on its dead points. In order, therefore, that the entrance port may be a little open when the piston arrives at the end of its stroke (Fig. 735, II.), the angle ACB_1 must exceed 90° by an angle $ACO = \alpha = H_1CB_1$. This angle is called the angular advance of the eccentric. The amount by which the port is thus opened at the end

of the stroke is called the *lead* of the valve, and depends upon the angle of advance. The radius CB is called the

FIG. 735.

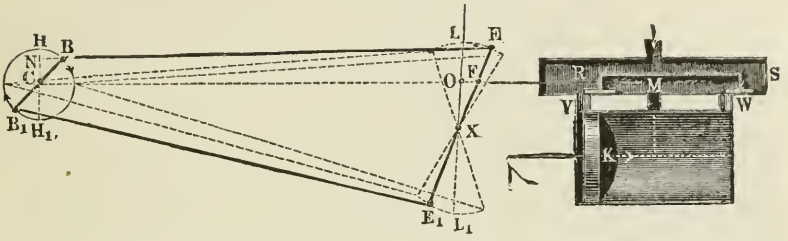


eccentricity. The angle of advance then is the angle made by the eccentricity with the vertical to the direction of the valve face, when the crank is at one of its dead points.

§ 461.—**Double Eccentric with Link.**—In order to change the travel of the valve, and thus obtain a greater or less time for steam admission and cut-off, it is only necessary to change the point of rotation X of the lever EF, and thus convert it into one of unequal arms. A still better way is by the application of the double eccentric

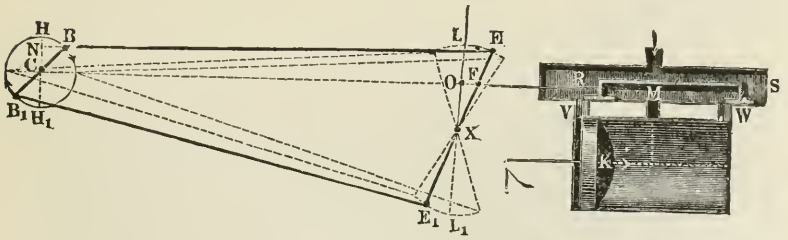
as shown in Fig. 736. CB and CB_1 are the eccentricities of two eccentrics keyed to the same shaft C, and both are joined by the rods BE and B_1E_1 with the lever EE_1, whose

FIG. 736.



fulcrum X may be raised or lowered at pleasure. This lever carries the end F of the valve stem FR in grooves, so that by the motion of the lever the end F moves simply back and forth in the direction FR. If the lengths of $BE = B_1E_1$ are very large with respect to CB and XE, we can assume that the ends E and E_1 move in the direction of CF through the same distance as B and B_1. Since then the motion of E_1 is opposite to that of E, the middle point X of the lever will preserve its position, and the

FIG. 737.



motion of any point F will be less than that of E in the ratio of XF to XE. If, therefore, s is the distance $NB = LE$, which the valve travels when moved directly by the eccentric B, we shall have for the distance OF,

$$OF = \frac{XF}{XE} \cdot LE,$$

or

$$s_1 = \frac{y}{c} s,$$

if we denote the distance XF by y, XE by c, and OF by s_1. Since, by raising and lowering the centre X, the distance XF can be changed at pleasure, the travel of the valve can be made any amount between s and $-s$. If the centre X is raised up to the level of the valve stem, $y = 0$ and the travel of the valve becomes zero, or it remains at rest. If the centre X is raised still further, the valve moves in the opposite direction. If the ends E or E_1 are brought into the line of the valve stem, the valve is moved directly by one or the other eccentric. It is thus easy to see how, by this arrangement, any desired expansion may be effected, or the machine brought to rest (§ 454). This arrangement is known as Stephenson's link-motion (Fr. coulisse de Stephenson; Ger. Stephenson'sche Coulissensteuerung). The complete theory will be given in the third volume of this work (see also the Treatise of Professor Zeuner, "Ueber die Schiebersteuerungen," Leipzig, 1874, IV. Aufl. English translation by Moritz Müller, London, 1869, E. & F. N. Spon. Also, "Die Schiebersteuerungen bei Dampfmaschinen, von T. Hentschel," Leipzig, 1859).

[The method developed by Prof. Zeuner, in the work referred to above, for designing valve-gear and link-motion, is much used, on account of its simplicity and accuracy, and some notes are added to show its application.

In Fig. 737*a* AB represents, on any convenient scale, the stroke of the engine. DC is drawn perpendicular to AB, and EG and EF represent the positions of the crank at the instants of admission and cut-off, respectively. HE is drawn so as to bisect the angle GEF, and is produced to M. $EH = EM =$ half-travel of valve, and circles are drawn with these lines as diameters. EQ is the position of the crank when release commences, and ER the position of the crank when release commences, the angles QEM, REM being equal. In a figure so constructed, Prof. Zeuner has demonstrated that

$$R = \frac{EL}{EH} = \frac{EK}{EH} = \text{ratio of steam lap to half-travel,}$$

$$r = \frac{EO}{EM} = \frac{EP}{EM} = \text{ratio of exhaust lap to half-travel,}$$

and $HED =$ angular advance of eccentric.

Now let

S = travel of valve,
p = maximum port opening,
L = steam lap,
l = exhaust lap.

Then,

$$R = \frac{L}{.5\,S}$$

$$L = \frac{R}{2} \times S$$

$$r = \frac{l}{.5\,S}$$

$$l = \frac{r}{2} \times S$$

$$S = 2\,p + 2\,L = 2\,p + R \times S$$

$$S = \frac{2p}{1-R}$$

Drawing the radii $I\,K$, $N\,O$,

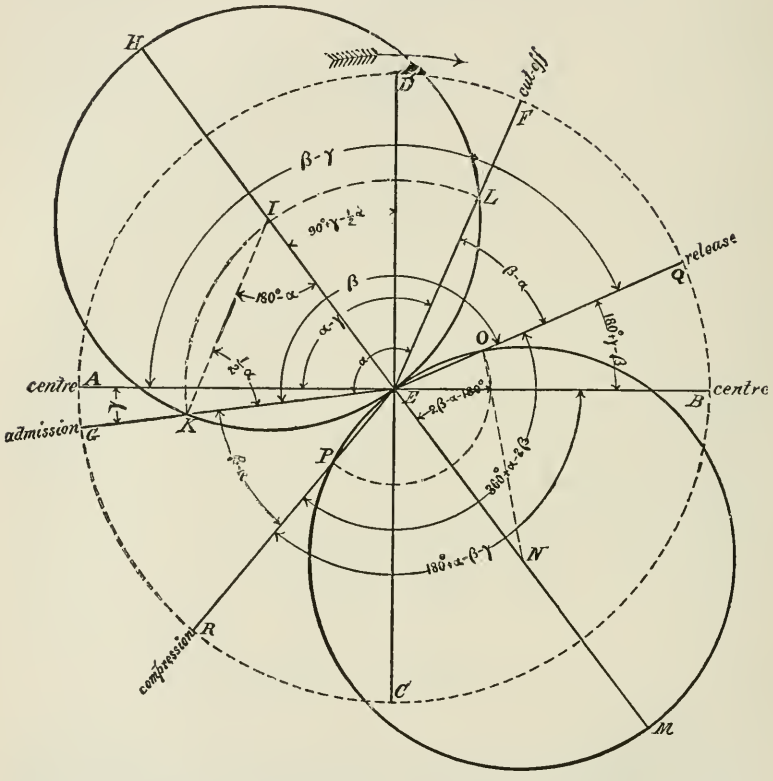
$$R = \tfrac{1}{2}\frac{E\,K}{E\,I} = \tfrac{1}{2}\frac{Sin\ (180^\circ - \alpha)}{Sin\ \tfrac{1}{2}\,\alpha}$$

$$r = \tfrac{1}{2}\frac{E\,O}{E\,N} = \tfrac{1}{2}\frac{Sin\ (180^\circ - Q\,E\,R)}{Sin\ \tfrac{1}{2}\,Q\,E\,R}.$$

From the foregoing, it is evident that if the angle $G\,E\,F$ is given, together with the width of port opening, the travel of the valve and steam lap can readily be determined, and that the exhaust lap can be found, if the angle $Q\,E\,R$ is known. Ordinarily, in designing a valve, the positions of the piston at the instants of admission, cut-off, and release are given; and from these, when the length of the connecting-rod is known, the angles α, B, and γ can be calculated. A reference to Fig. 737*a*, and the preceding formulas, shows that these are the only angles required for determining the lap and travel of the valve, and the angular advance of the eccentric. A table calculated by the aid of these formulas is appended, which gives the lap and travel when the required angles are known; but as the calculation of these angles from given piston positions is somewhat tedious, a second table is added, showing the crank angles for various piston positions, and for connecting-rods of such lengths as commonly occur in practice.

The latter table is calculated by the formulas that follow, which may be used to check or extend this table.

FIG. 737*a*.



Let $c = \dfrac{\text{length of connecting-rod}}{\text{length of stroke}}$

$r = \dfrac{\text{distance from end of stroke to given position of piston}}{\text{length of stroke}}$

c = distance from centre of cross-head to centre of shaft.

C = angle of crank.

(NOTE.—The forward stroke in direct-acting engines is that in which the cross-head moves from the end of the guides nearest the cylinder, and this is the return stroke, in the case of back-acting engines.)

For the forward stroke,

$$a = \sqrt{c^2 + 0{\cdot}25 - c \times \cos.\left(180^\circ - C - \sin.^{-1}\frac{\sin. C}{2c}\right)}$$

$$r = c + 0{\cdot}5 - a.$$

For the return stroke,

$$a = \sqrt{c^2 + 0{\cdot}25 - c \times \cos.\left(C - \sin.^{-1}\frac{\sin. C}{2c}\right)}$$

$$r = a + 0{\cdot}5 - c.$$

Lap and Travel of Slide-Valve.

Angle between positions of crank at points of admission and cut-off, or release and compression.	Ratio of lap to travel of valve.	Ratio of travel of valve to width of port opening.	Angle between positions of crank at points of admission and cut-off, or release and compression.	Ratio of lap to travel of valve.	Ratio of travel of valve to width of port opening.	Angle between positions of crank at points of admission and cut-off, or release and compression.	Ratio of lap to travel of valve.	Ratio of travel of valve to width of port opening.
30°	·483	58·7	81°	·380	8·35	131°	·207	3·42
31°	·482	55·0	82°	·377	8·15	132°	·203	3·37
32°	·481	51·6	83°	·375	7·97	133°	·199	3·33
33°	·479	48·6	84°	·372	7·79	134°	·195	3·28
34°	·478	45·8	85°	·369	7·61	135°	·191	3·24
35°	·477	43·2	86°	·366	7·45	136°	·187	3·20
36°	·476	40·9	87°	·363	7·28	137°	·183	3·16
37°	·474	38·7	88°	·360	7·13	138°	·179	3·12
38°	·473	36·7	89°	·357	6·98	139°	·175	3·08
39°	·471	34·9	90°	·354	6·83	140°	·171	3·04
40°	·470	33·2	91°	·351	6·69	141°	·167	3·00
41°	·468	31·6	92°	·347	6·55	142°	·163	2·97
42°	·467	30·1	93°	·344	6·42	143°	·159	2·93
43°	·465	28·7	94°	·341	6·29	144°	·155	2·89
44°	·464	27·5	95°	·338	6·17	145°	·150	2·86
45°	·462	26·3	96°	335	6·05	146°	·146	2·83
46°	·460	25·2	97°	·331	5·93	147°	·142	2·79
47°	·459	24·1	98°	·328	5·82	148°	·138	2·76
48°	·457	23·1	99°	·325	5·71	149°	·134	2·73
49°	·455	22·2	100°	·321	5·60	150°	·129	2·70
50°	·453	21·3	101°	·318	5·50	151°	·125	2·67
51°	·451	20·5	102°	·315	5·40	152°	·121	2·64
52°	·449	19·8	103°	·311	5·30	153°	·117	2·61
53°	·448	19·0	104°	·308	5·20	154°	·113	2·58
54°	·446	18.4	105°	·304	5·11	155°	·108	2·55
55°	·444	17·7	106°	·301	5·02	156°	·104	2·53
56°	·442	17·1	107°	·297	4·94	157°	·100	2·50
57°	·439	16·5	108°	·294	4·85	158°	·095	2·47
58°	·437	16·0	109°	·290	4·77	159°	·091	2·45
59°	·435	15·4	110°	·287	4 69	160°	·087	2·42
60°	·433	14·9	111°	·283	4·61	161°	·083	2·40
61°	·431	14·5	112°	·280	4·54	162°	·078	2·37
62°	·429	14·0	113°	·276	4·46	163°	·074	2·35
63°	·426	13·6	114°	·272	4·39	164°	·070	2·32
64°	·424	13·2	115°	·269	4·32	165°	·065	2·30
65°	·422	12·8	116°	·265	4 26	166°	·061	2·28
66°	·419	12·4	117°	·261	4·19	167°	·057	2·26
67°	·417	12·0	118°	·258	4·12	168°	·052	2·23
68°	·415	11·7	119°	·254	4·06	169°	·048	2·21
69°	·412	11·4	120°	·250	4·00	170°	·044	2·19
70°	·410	11·1	121°	·246	3·94	171°	·039	2·17
71°	·407	10·8	122°	·242	3·88	172°	·035	2·15
72°	·405	10·5	123°	·239	3·83	173°	·031	2·13
73°	·402	10·2	124°	·235	3·77	174°	·026	2·11
74°	·399	9·93	125°	·231	3·72	175°	·022	2·09
75°	·397	9·68	126°	·227	3.66	176°	·017	2·07
76°	·394	9·44	127°	·223	3·61	177°	·013	2·05
77°	·391	9·20	128°	·219	3·56	178°	·009	2·04
78°	·389	8·97	129°	·215	3·51	179°	·004	2·02
79°	·386	8·76	130°	·211	3·46	180°	·000	2·00
80°	·383	8·55						

Crank Angles corresponding to Piston Positions for Connecting-Rods of Various Lengths.

Fraction of stroke completed.	Crank angles, in degrees, for connecting-rods, with ratio of length to stroke, of														Fraction of stroke completed.
	2		2¼		2½		2¾		3		3¼		3½		
	Forward stroke.	Return stroke.	Forward stroke.	Return stroke.	Forward stroke.	Return stroke.	Forward stroke.	Return stroke.	Forward stroke.	Return stroke.	Forward stroke.	Return stroke.	Forward stroke.	Return stroke.	
·01	10·3	13·2	10·4	13·0	10·5	12·8	10·6	12·7	10·6	12·6	10·7	12·5	10·7	12·4	·01
·02	14·6	18·7	14·7	18·4	14·9	18·1	15·0	17·9	15·1	17·8	15·1	17 7	15·2	17·5	·02
·03	17·9	22·9	18·1	22·5	18·2	22·2	18·4	22·0	18·5	21·8	18·6	21·6	18·7	21·5	·03
·04	20·7	26·5	20·9	26·0	21·1	25·7	21·3	25·4	21·4	25·2	21·5	25·0	21·6	24·9	·04
·05	23·2	29·6	23·4	29·1	23·6	28·7	23·8	28·4	24·0	28·2	24·1	28·0	24·2	27·8	·05
·10	33·1	41·9	33·5	41·3	33·8	40·8	34·0	40·4	34·3	40·1	34·4	39·8	34·6	39 6	·10
·15	41·0	51·5	41·5	50·7	41·9	50·2	42·2	49·7	42·4	49·3	42·7	49·0	42·9	48·7	·15
·20	48·0	59·6	48·5	58·8	48·9	58·2	49·3	57·7	49·6	57·3	49·8	56·9	50·1	56·6	·20
·25	54·3	66·9	54·9	66·0	55·4	65·4	55·8	64·8	56·1	64·4	56·4	64·1	56·6	63·7	·25
·30	60·3	73·5	60·9	72·7	61·5	72·0	61·9	71·5	62·2	71·0	62·6	70·7	62·8	70·3	·30
·35	66·1	79·8	66·7	79·0	67·3	78·3	67·8	77·7	68·1	77·3	68·5	76·9	68·8	76·6	·35
·40	71·7	85·8	72·4	84·9	73·0	84·3	73·5	83·7	73·9	83·3	74·2	82·9	74·5	82·6	·40
·45	77·2	91·5	78·0	90·7	78·6	90·1	79·1	89·5	79·6	89·1	79·9	88·7	80·2	88·4	·45
·50	82·3	97·2	83·6	96·4	84·3	95·7	84·8	95·2	85·2	94·8	85·6	94·4	85·9	94·1	·50
·55	88·5	102·8	89·3	102·0	89·9	101·4	90·5	100·9	90·9	100·4	91·3	100·1	91·6	99·8	·55
·60	94·2	108·3	95·1	107·6	95·7	107·0	96·3	106·5	96·7	106·1	97·1	105·8	97·4	105·5	·60
·65	100·2	113·9	101·1	113·3	101·7	112·7	102·3	112·2	102·7	111·9	103·1	111·5	103·4	111·2	·65
·70	106·5	119·7	107·3	119·1	108·0	118·5	103·5	118·1	109·0	117·8	109·3	117·4	109·7	117·2	·70
·75	113·1	125·7	114·0	125·1	114·6	124·6	115·2	124·2	115·6	123·9	115·9	123·6	116·3	123·4	·75
·80	120·4	132·0	121·2	131·5	121·8	131·1	122·3	130·7	122·7	130·4	123·1	130·2	123·4	129·9	·80
·85	128·5	139·0	129·3	138·5	129·8	138·1	130·3	137·8	130·7	137·6	131·0	137·3	131·3	137·1	·85
·90	138·1	146·9	138·7	146·5	139·2	146·2	139·6	146·0	139·9	145·7	140·2	145·6	140·4	145·4	·90
·95	150·4	156·8	150·9	156·6	151·3	156·4	151·6	156·2	151·8	156·0	152·0	155·9	152·2	155·8	·95
·96	153·5	159·3	154·0	159·1	154·3	158·9	154·6	158·7	154·8	158·6	155·0	158·5	155·1	158·4	·96
·97	157·1	162·1	157·5	161·9	157·8	161·8	158·0	161·6	158·2	161·5	158·4	161·4	158·5	161·3	·97
·98	161·3	165·4	161·6	165·3	161·9	165·1	162·1	165·0	162·2	164·9	162·3	164·9	162·5	164·8	·98
·99	166·8	169·7	167·0	169·6	167·2	169·5	167·3	169·4	167·4	169·4	167·5	169·3	167 6	169·3	·99
1·00	180·0	180·0	180·0	180·0	180·0	180·0	180·0	180·0	180·0	180·0	180·0	180·0	180·0	180·0	1·00

Crank Angles corresponding to Piston Positions for Connecting-Rods of Various Lengths—(*Continued*).

Fraction of stroke completed.	Crank angles, in degrees, for connecting-rods, with ratio of length to stroke, of														Fraction of stroke completed.
	3½		4		4¼		4½		4¾		5		∞		
	Forward stroke.	Return stroke.	Forward stroke.	Return stroke.	Forward stroke.	Return stroke.	Forward stroke.	Return stroke.	Forward stroke.	Return stroke.	Forward stroke.	Return stroke.	Forward stroke.	Return stroke.	
·01	10·8	12·3	10·8	12·3	10·9	12·2	10·9	12·2	10·9	12·1	10·9	12·1	11·5	Crank angles for return stroke are the same as those for corresponding piston positions in forward stroke.	·01
·02	15·3	17·4	15·3	17·4	15·4	17·3	15·4	17·2	15·5	17·2	15·5	17·1	16·3		·02
·03	18·8	21·4	18·8	21·3	18·9	21·2	18·9	21·1	19·0	21·1	19·0	21·0	19·9		·03
·04	21·7	24·7	21·8	24·6	21·9	24·5	21·9	24·4	22·0	24·3	22·0	24·3	23·1		·04
·05	24·3	27·7	24·4	27·5	24·5	27·4	24·6	27·3	24·6	27·3	24·7	27·2	25·8		·05
·10	34·7	39·4	34·9	39·2	35·0	39·0	35·1	38·9	35·2	38·8	35·2	38·7	36·9		·10
·15	43·0	48·5	43·2	48·3	43·3	48·1	43·4	48·0	43·5	47·9	43·6	47·7	45·6		·15
·20	50·2	56·4	50·4	56·2	50·6	56·0	50·7	55·8	50·8	55·7	50·9	55·5	53·1		·20
·25	56·8	63·5	57·0	63·3	57·2	63·1	57·4	62·9	57·5	62·7	57·6	62·6	60·0		·25
·30	63·1	70·1	63·3	69·8	63·4	69·6	63·6	69·4	63·7	69·3	63·9	69·1	66·4		·30
·35	69·0	76·3	69·2	76·1	69·4	75·8	69·6	75·7	69·7	75·5	69·9	75·3	72·5		·35
·40	74·8	82·3	75·0	82·0	75·2	81·8	75·4	81·6	75·6	81·5	75·7	81·3	78·5		·40
·45	80·5	88·1	80·7	87·9	80·9	87·6	81·1	87·5	81·3	87·3	81·4	87·1	84·3		·45
·50	86·2	93·8	86·4	93·6	86·6	93·4	86·8	93·2	87·0	93·0	87·1	92·9	90·0		·50
·55	91·9	99·5	92·1	99·3	92·4	99·1	92·5	98·9	92·7	98·7	92·9	98·6	95·7		·55
·60	97·7	105·2	98·0	105·0	98·2	104·8	98·4	104·6	98·5	104·4	98·7	104·3	101·5		·60
·65	103·7	111·0	103·9	110·8	104·2	110·6	104·3	110·4	104·5	110·3	104 7	110·1	107·5		·65
·70	109·9	116·9	110·2	116·7	110·4	116·6	110·6	116·4	110·7	116·3	110·9	116·1	113·6		·70
·75	116·5	123·2	116·7	123·0	116·9	122·8	117·1	122·6	117·3	122·5	117·4	122·4	120·0		·75
·80	123·6	129·8	123·8	129·6	124·0	129·4	124·2	129·3	124·3	129·2	124·5	129·1	126·9		·80
·85	131·5	137·0	131·7	136·8	131·9	136·7	132·0	136·6	132·1	136·5	132·3	136·4	134·4		·85
·90	140·6	145·3	140·8	145·1	141·0	145·0	141·1	144·9	141·2	144·8	141·3	144·8	143 1		·90
·95	152·3	155 7	152·5	155·6	152·6	155·5	152·7	155·4	152·7	155·4	152·8	155·3	154·2		·95
·96	155·3	158·3	155·4	158·2	155·5	158·1	155·6	158·1	155·7	158·0	155·7	158·0	156·9		·96
·97	158·6	161·2	158·7	161·2	158·8	161·1	158·9	161·1	158·9	161·0	159·0	161·0	160·1		·97
·98	162·6	164·7	162.6	164·7	162·7	164·6	162·8	164·6	162·8	164·5	162·9	164·5	163·7		·98
·99	167·7	169·2	167·7	169·2	167·8	169·1	167·8	169·1	167·9	169·1	167 9	169·1	168·5		·99
1·00	180·0	180·0	180·0	180·0	180·0	180·0	180·0	180·0	180·0	180·0	180·0	180·0	180·0		1·00

The diagram, Fig. 737*a*, shows the action of one end of the valve, and the other end, being rigidly attached to it, moves in the same relative manner—that is, if the cut-off, for instance, takes place at a crank angle of 150° on the forward stroke, it will occur at the same angle on the return stroke. On account of the angularity of a connecting-rod of finite length, corresponding piston positions on forward and return strokes respectively give different crank angles; so that if the angle at which the cut-off takes place is the same for forward and return strokes, the point of stroke at which the cut-off occurs will be different in the two cases. For example, if the connecting-rod is three times the length of stroke, and the cut-off angle is 130°, the table of crank angles shows that the point of cut-oft is about 0·85 for the forward stroke, and 0·8 for the return. The cut-off can, however, be equalized for both strokes, by several adjustments, to be explained.

In the example that follows, illustrating the use of the diagram and tables, some of the data and methods are taken from "Link and Valve Motions," by W. S. Auchincloss, New York, 1869, a work of great interest and value.

A valve is to be designed for an engine having a cylinder 20 inches in diameter, a stroke of 3 feet, and making 80 revolutions a minute. The valve is to cut off the steam at $\frac{3}{4}$ of the stroke, and to release the steam when the piston has completed $\frac{95}{100}$ of the stroke. The length of the connecting-rod is $2\frac{3}{4}$ times the stroke.

In solving this problem, it is necessary first to determine the area of the port. The following table gives the minimum port area allowable in good practice:

Speed of piston in feet per minute.....	100	200	300	400	500	600				
Ratio of port area to piston area.....	·02	·04	·06	·07	·09	·1				
Speed of piston in feet per minute.	700	800	900	1000	1100	1200	1300	1400	1500	
Ratio of port area to piston area.	·12	·14	·15	·17	·19	·2	·22	·24	·25	

The area of the piston in the present example being 314·16 inches, and the piston speed 480 feet per minute, the port area is

$$314{\cdot}16 \times 0{\cdot}09 = 28{\cdot}27 \text{ square inches};$$

and if the length of the port is made equal to the diameter of the cylinder, the length will be

$$\frac{28{\cdot}27}{20} = 1{\cdot}4, \text{ or say } 1\tfrac{1}{2} \text{ inches.}$$

The crank angles corresponding to $\frac{3}{4}$ stroke are, 115° for forward and 124° for return stroke. To determine the lap and travel, take the angle for return stroke, as in this way it is possible to equalize the cut-off. On the supposition that the lead angle is 3°,

$$\alpha = 127^\circ,\ \gamma = 3^\circ,\ \beta = 159^\circ,$$

and the angular advance of the eccentric is

$$90^\circ + 3^\circ - \frac{127^\circ}{2} = 29{\cdot}5^\circ.$$

From the table for lap and travel, the travel of valve is

$$1{\cdot}5 \times 3{\cdot}61 = 5{\cdot}415 \text{ inches.}$$

Steam lap,

$$5{\cdot}415 \times 0{\cdot}223 = 1{\cdot}21 \text{ inches.}$$

Exhaust lap,

$$5{\cdot}415 \times 0{\cdot}091 = 0{\cdot}49 \text{ inches.}$$

With the valve thus designed, the cut-off occurs at a crank angle of 124° in each stroke, while it should occur at 115° in the forward stroke. To effect this, since the difference in the crank angles corresponding to $\frac{3}{4}$ stroke, forward and return, is 9°. add one half this difference, $4\frac{1}{2}^\circ$, to the angular advance, making the corrected angular advance, 34°. Then by lengthening

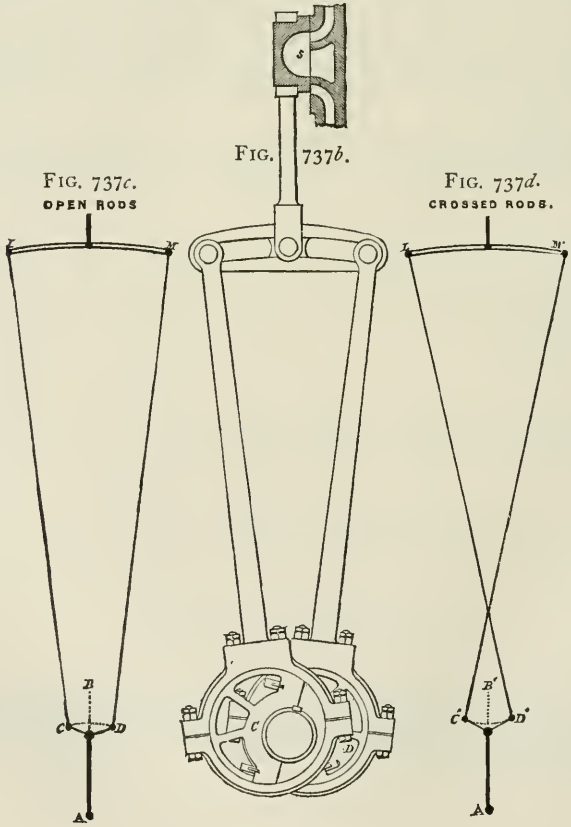


FIG. 737*b*.

FIG. 737*c*.
OPEN RODS

FIG. 737*d*.
CROSSED RODS.

the eccentric rod, so that the lead angle for the return stroke is increased by the difference in crank angles, from 4° to 13°, it is evident that the cut-off will take place at a crank angle of 115° in the forward stroke, and 124° in the return, the change merely producing a slight inequality in the amount of steam lead for the forward and return strokes.

Prof. Zeuner's diagram can also be employed to simplify the design of link motion, by finding a single imaginary eccentric which shall produce the same effect as the two eccentrics with the link connection. The two eccentrics C, D, C', D', Figs. 737*b*, 737*c*, 737*d*, are supposed to be of equal stroke, and to make equal angles with the position of the crank, $B\,A$, $B'\,A'$, the only difference in the two arrangements shown in Figs. 737*c* and 737*d* being, that in the first case the eccentric rods are open, and in the second they are crossed.

The effect of open rods is to increase the steam lead as the link approaches a central position, and the contrary effect is produced when the rods are crossed.

To find the effect produced by changing the position of the link, when open eccentric rods are used, as in Fig. 737*c*, draw a circle with $A\,B$ as a diameter, Fig. 737*e*, to represent the path of the crank, and draw $O\,C$, $O\,D$, to represent the positions of the eccentric radii, when the crank is on the centre. Then lay off the link and eccentric rods to scale, as in Fig. 737*c*, and from L and M as centres, with radii, $L\,C$, $M\,D$, describe arcs cutting $A\,B$, Fig. 737*e*, at the point c. Draw a circular arc through the three points, C, c, and D. Straight lines drawn from O to this curve, as $O\,c$, $O\,c'$, $O\,c''$, $O\,c'''$, represent the positions and radii of imaginary eccentrics, which produce the same effect as the two eccentrics with link connection, when the link is in positions corresponding to c, c', c'', and c'''. The central circle is the steam lap circle, and by drawing circles on the several eccentric radii, the action of the valve for different positions of the link can readily be determined. In the case of crossed eccentric rods, as in Fig. 737*d*, the construction is almost the same as before, as will be evident by inspecting Fig. 737*f*. Here the point c, in the curve $C\,c\,D$, is found, as before, by the intersection on $A\,B$ of arcs described with radii, $L'\,D'$, $M'\,C'$, Fig. 737*d*, from L' and M' as centres—Fig. 737*d* being drawn on Fig. 737*f*, to the same scale.

Fig. 737*e*.

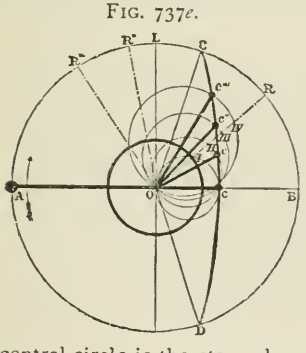
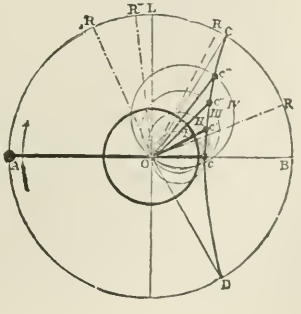


Fig. 737*f*.



The preceding principles are not only applicable to the designing of slide valves, but can also be used in adjusting valves that have been improperly designed or set. It is to be remembered, however, that a valve cannot be accurately set without some experimental tests. It can be designed and the position of the eccentric can be determined with great precision by the foregoing principles ; but the final adjustment of the length of the eccentric rod depends in some measure upon elements which can only be determined by trial—the amount of lost motion in the connections, and the effect of a change of temperature. Whenever it is practicable, after a valve has been set as accurately as is possible, while cold, indicator diagrams should be taken from the engine, and the length of the eccentric rod adjusted, if necessary, to correct slight irregularities. It rarely happens, when this is done, that some improvement is not effected.]

462.—Valve Gear with Eccentrics. — Valves may also be moved by eccentrics, although, in such case, an arrangement of levers answers the purpose better, because a quicker opening and closing of the valve is thus attained. In single-acting engines, and generally in all engines in which there is no rotation, such an arrangement can of course be used.

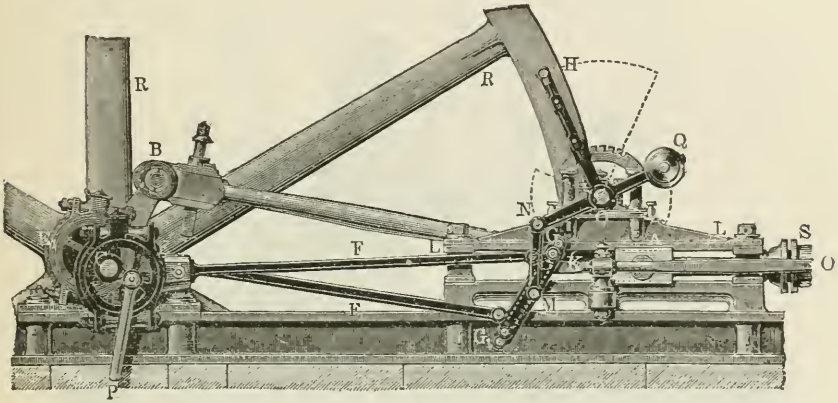
A valve gear with eccentrics has been already described (§ 448), and is represented in Fig. 705. The valve rods $F\,G$ and $F_1\,G_1$ are moved up and down by two eccentrics H and H_1 keyed to a horizontal shaft, which is made to turn by means of a toothed wheel set in motion by the machine. In the following Figs. 738 and 739, we have represented the horizontal steam engine of Révollier (see Armengaud, "Publication Industr.," II. Vol., as also the "Civilingenieur," Bd. 4). The valve gear with eccentric motion is here a more perfect one.

Fig. 738 gives only an elevation of the exterior gear, together with those parts by which the rectilinear motion of the piston is transformed into rotary motion. A is the cross-head of the piston rod, moving between the guides $L\,L$, S being the stuffing box through which the piston rod passes to the cylinder, which is not shown in the figure; $A\,B$ is the connecting rod, and $B\,C$ the crank, by means of which the shaft C and fly-wheel $R\,R$ are made to revolve. Upon this shaft are two eccentrics E and E_1, of which the first also moves the connecting rod P of the feed pump,

and both are attached by the rods F and F_1 to the link $G\,G_1$, in which slides the end of the rod $K\,O$, which moves the valves.

The link is fastened at its centre M to a lever $N\,Q$,

FIG. 738.



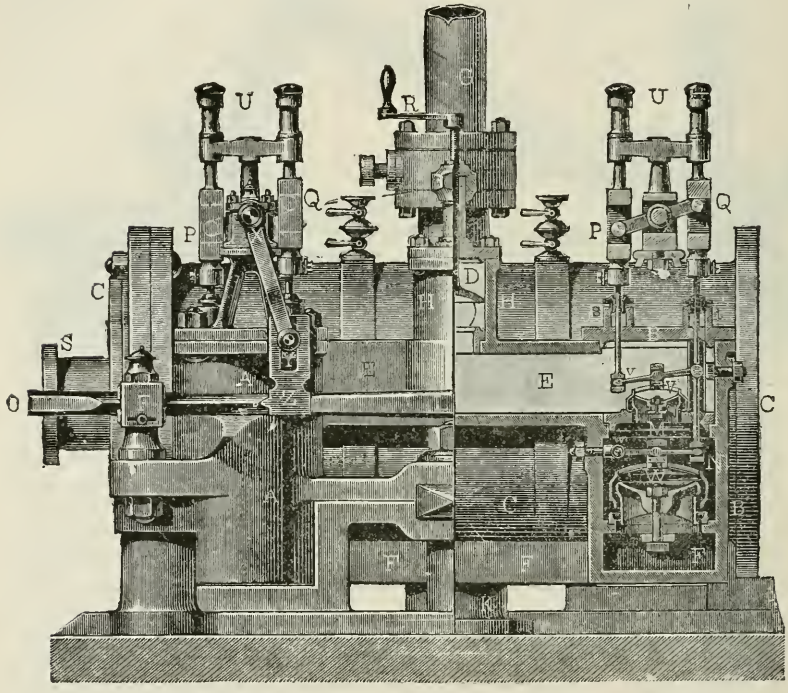
which turns out O and carries the counterpoise Q. By means of the handle $O\,H$, the link may be raised or lowered at will, and so placed that the end K of the valve rod may have any desired position between the ends G and G_1.

In Fig. 739 we have the valve gear proper, half in elevation and half in section. $C\,C\,C$ is the cylinder with the stuffing box S already indicated in the preceding figure, and $O\,L$ is the valve rod which passes through a guide F, the end of K of which moves in the link when the link is raised or lowered. The steam cylinder $C\,C\,C$ is one piece with the two cylindrical valve chests $A\,A$ and $B\,B$, and the two steam passages $E\,E$ and $F\,F$. One of these passages communicates at H with the steam pipe G, and the other at K with the discharge pipe. The entrance of steam is regulated by the crank R and the valve D, and when admitted it fills not only $E\,E$, but also the upper spaces of the valve chambers $A\,A$ and $B\,B$. In each of these chambers are two valves, a small or admission valve V and a larger eduction valve W. When the first is opened, the steam enters the space M, and from there passes by N to the

cylinder. When the second is opened, the steam escapes through *N* and *W* to *F* and *K*.

The valves *V* and *W* are attached to the levers *v* and *w*, and these again to the vertical rods which enter the steam chambers through stuffing boxes at *s* and *t*. These rods

Fig. 739.

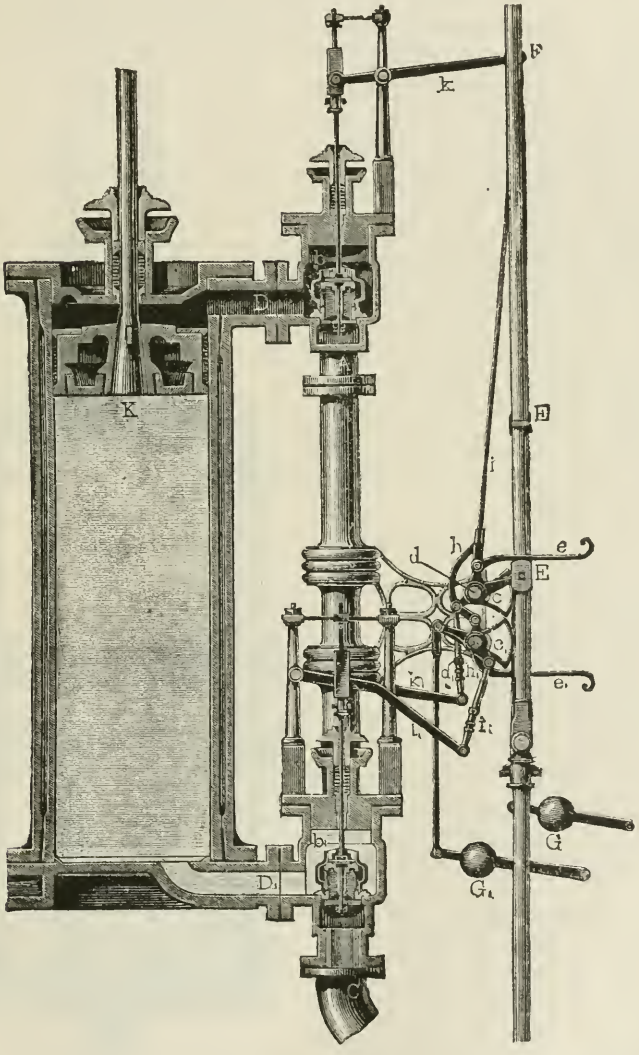


are split at *P* and *Q*, and their upper ends enter boxes at *U* containing springs. The raising and closing of the valves is effected by the lever *P Q*, which is connected by the arm *X Y* and the lug *Y Z* to the rod *O L*. The ends of this lever *P Q* move in the slits and set the valves in motion only towards the end of the stroke of *K O*. The boxes at *U* not only serve as guides, but also, by means of the adjustable springs they contain, accelerate the fall of the valve as well as diminish the impact when rising.

The manner of action is now easily seen from the figure.

§ **463.—Valve Gear with Tappets.** — The manner in which the valves of a steam engine are operated by the

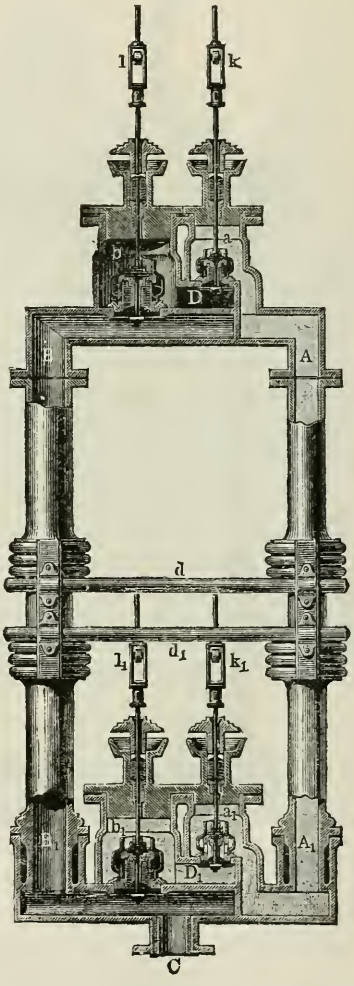
Fig. 740.



tappet arrangement already described in § 309, is shown in Figs. 740 and 741, which represent a double-acting Cornish

steam engine. We see from Fig. 741, that we have a pair of small valves a and a_1 and a pair of larger ones, b and b_1, the one for admission the other for exit of the steam. The first pair communicates with the steam pipes D and D_1 leading to the cylinder from below, the second from above The steam is brought in by the pipe $A\,A_1$, and discharged by $B\,B_1$, or by it led to the condenser. We see at once that by opening the valves a and b_1, fresh steam passes through a and D and presses the piston K down, while the waste steam below K passes through D_1 and b_1, and passes by the pipe $B\,B_1\,C$ to the condenser. If, on the other hand, the valves a_1 and b are opened and a and b_1 closed, steam enters through a_1 and D_1 below the piston and forces it up, while the exhaust steam passes through D and b and $B\,B_1\,C$ to the condenser. The two upper valves a and b are attached to two-armed levers k and l, the two lower to one-armed levers k_1 and l_1, and these levers are united by rods h, i, h_1, i_1 with the two shafts d and d_1, h and i_1 to d_1, and h_1 and i to d. These shafts also have the long levers e and e_1, which are struck and moved up or down by the tappets E and E_1, which are fixed to the piston rod $E\,F$ of the air pump. The action of the gear is now easily seen. In the

Fig. 741.

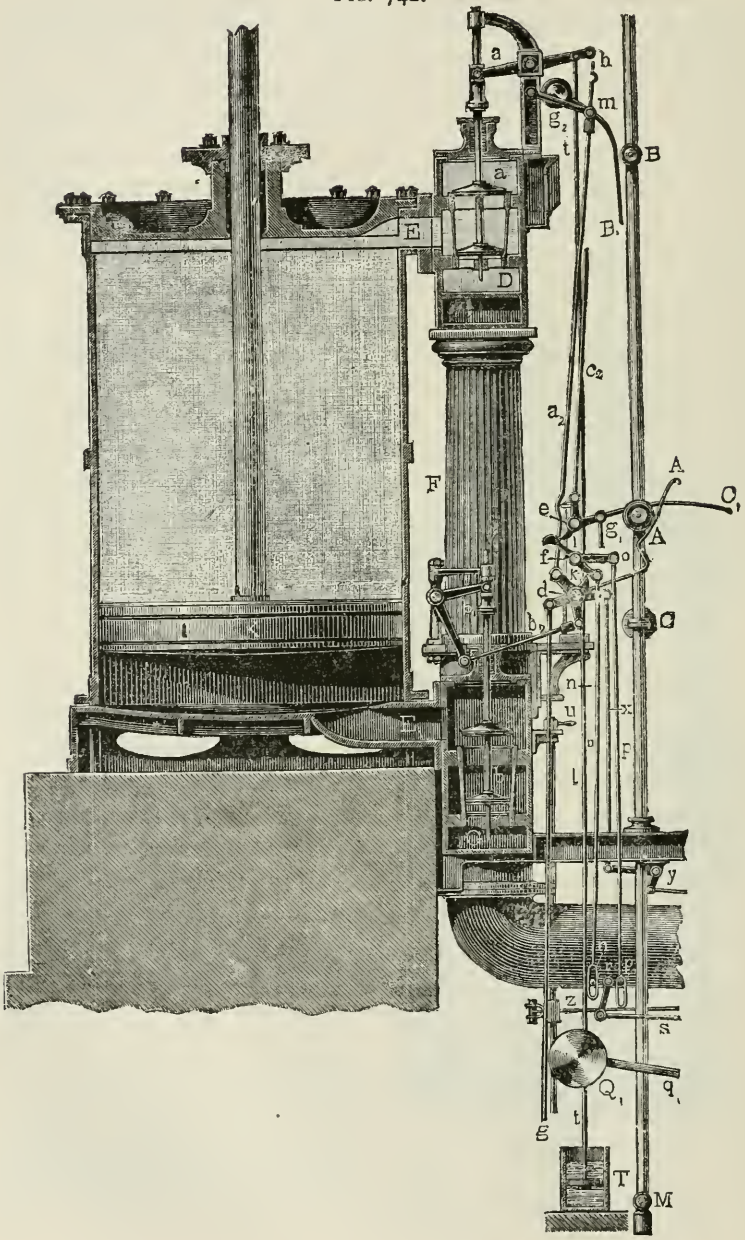


position shown in the figures, the piston K is at the top of its stroke. The tappet E has raised the lever e and turned the shaft d through a certain angle from right to left. By this the weight G is raised, h_1 and therefore a_1, by means of h_1, as also b, by means of i, are closed, the sector c raised, and hence the sector c_1 set free. The weight G_1 then falls, turns the shaft d_1 from right to left, and opens a by means of h, as also b_1 by means of i_1. The steam below the piston K flows now through b_1 to C and the condenser, and fresh steam enters through D, driving K downwards, until, towards the end of the down-stroke, the tappet E_1 strikes the lever e_1 and turns the shaft d_1 through a certain angle from left to right. The weight G_1 is thus raised again, the valve a is closed by the rod h, and b_1 by i_1, and the sector c_1 is turned so far that the sector c is free. The weight G then falls, and thus opens a by means of h_1, and b by means of i, so that now steam enters through a_1 and D_1 below the piston, and escapes through b above it. This process is repeated during every stroke of the piston.

§ 464.—**Single-Acting Steam Engine.**—If the entrance of steam is checked before the end of the stroke, so that the steam acts expansively during the rest of the stroke, a special cut-off valve is necessary, moved by a special lever arrangement, or by special mechanism we must effect not only the simultaneous opening of the admission and eduction valves, but also cause the first to shut before the second. How this can be effected in a single-acting engine is shown by Figs. 742 and 743, which represent a pumping engine by Hick and Bolton.

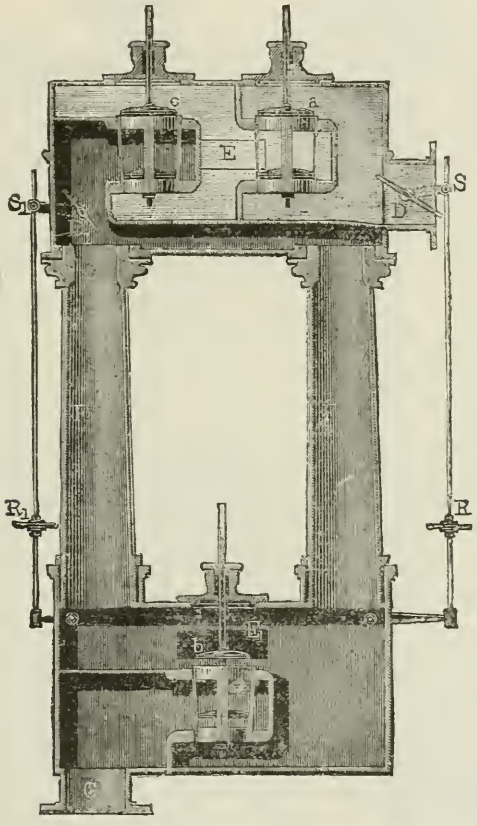
The machine has three double valves, a, b, c. The first is the admission valve (Fr. soupape d'admission; Ger. Einlassventil). When it is open, the steam passes by D through E to the cylinder and drives the piston down. The valve b is the eduction valve (Fr. soupape d'émission; Ger. Auslassventil). When it is open, the steam passes through G to the condenser. The valve c, in the same

FIG. 742.



chamber with a, opens when the piston K is raised by a counterweight, in order that the steam above it may pass by $E F E_1$ around beneath the piston. Since thus there is

FIG. 743.



on both sides of the piston about the same steam pressure, this valve is called the equilibrium valve (Fr. soupape d'équilibre; Ger. Gleichgewichtsventil). The opening and closing of these three valves during a complete stroke must take place in the following order.

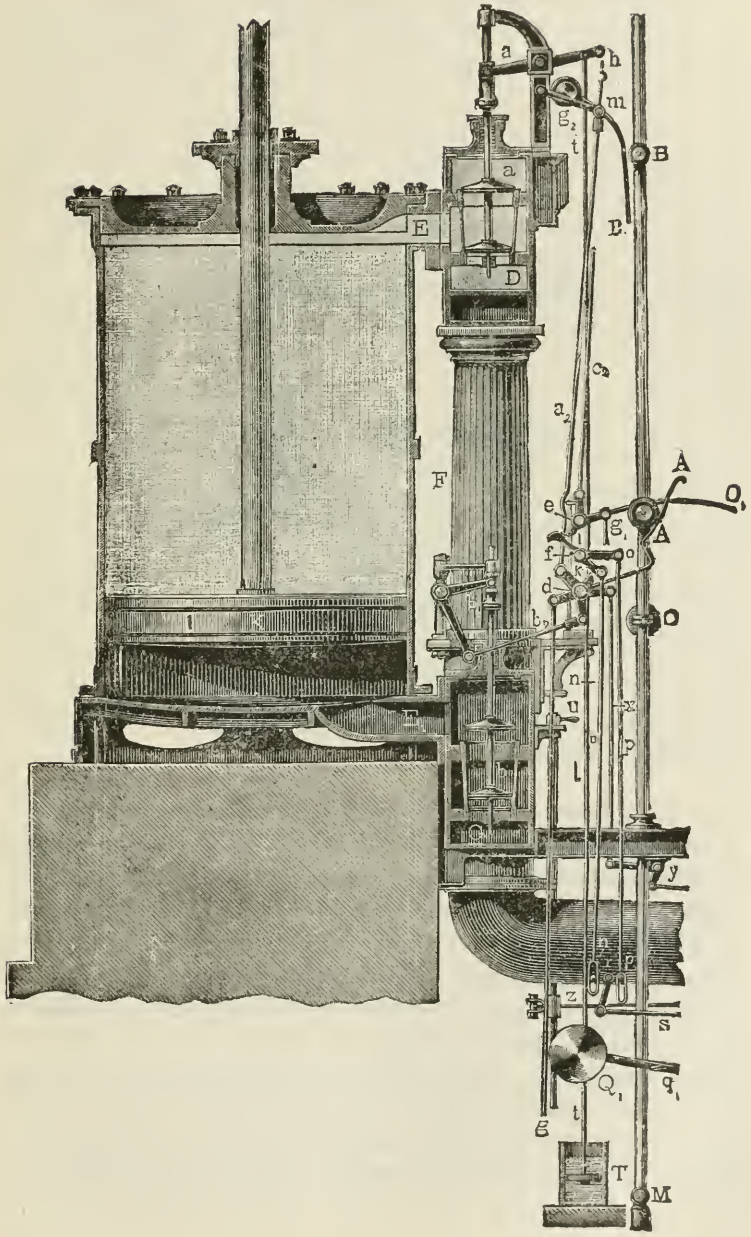
At first, let the piston K be above and all three valves closed. At the beginning of the stroke, a and b are simul-

taneously opened, and steam enters to drive K down, while the waste steam below K passes through E_1 and G to the condenser. When the piston K has made a part of its stroke, the valve a is closed, steam no longer enters above, and the enclosed steam now acts expansively for the rest of the stroke, as shown in the figure. When K reaches the bottom, b also is closed and c is opened; the piston rises by the action of the counterweight, and upon its return stroke drives the steam which has passed around under it through b to the condenser. At the end of the up-stroke, c is closed and a new stroke commences as above.

For the motion of the valves, we have the mechanism shown in Fig. 744, which is similar to that represented in Figs. 559 and 740. Here d and e are the shafts furnished with levers and ratchets, and f is movable about an intermediate shaft, and alternately acted upon by the teeth k and l. The stem of the admission valve a is joined, by a straight lever a_1 and a rod a_2, to one arm of the shaft d, and the stem of the eduction valve b, by an angle lever b_1 and a rod b_2, to another arm on this shaft, while the equilibrium valve c (not shown in the figure), by means of stem, lever and a rod c_2, is joined to an arm on the shaft e. Upon both shafts d and e, we have also rods g and g_1 hung from appropriate arms, which carry counterweights, which, by the release of the catches at k and l, turn d from right to left, and e from left to right, and thus open either a and b or c. The closing of the valves is effected by the rod $B\,C\,M$, which moves up and down with the piston and carries the tappets A, B, and C, which strike the tappet bars A_1, B_1, and C_1, of which A_1 moves the shaft d, C_1 the shaft e, while B_1 is attached as shown to the end m of the rod a_2 of the admission valve a. This last bar carries the counterweight g_2 and an arm $m\,h$, which engages, by means of a hook, with the end of the lever a_1 of the valve a.

Finally, we have to remark, that each of the two arms $f\,k$ and $f\,l$ may be turned about f, the one by means of the rod n and the other by means of the arm $f\,o$ and the rod p.

FIG. 744.



The action of the mechanism is now as follows:

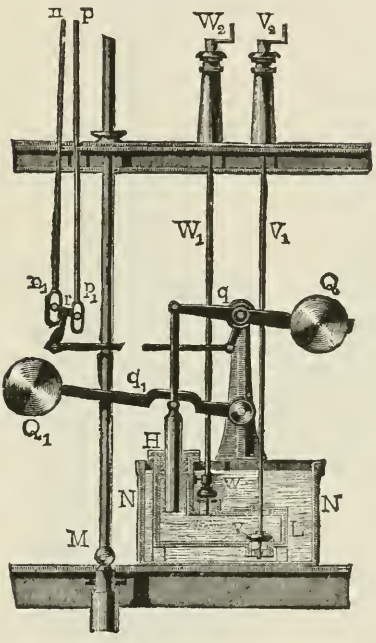
At first let the piston be above and all three valves closed. If, now, the arm $f k$ is moved upwards by means of the rod n, the catch at k is disengaged, the weight g falls, and valves a and b are opened. The steam now enters through E and drives the piston downwards, while below the piston the steam passes through E_1 G to the condenser. After the piston has passed through a certain distance, the tappet B strikes the bar B_1, presses it down and disengages the hook at h, and the admission valve a at once closes. During the remaining stroke of the piston, the steam therefore acts expansively. Towards the end of the stroke, the bar A_1 is struck by the tappet A, and the weight g is raised again, thus closing the eduction valve also and hooking again h in a_1.

If, now, the piston is to rise, the rod p must be raised, thus disengaging l, and then the weight g_1 falls and, by means of c_2, opens the equilibrium valve c. The counterpoise now draws the piston up, while at the same time the steam passes by $E F E_1$ around below the piston. Finally, when the piston arrives at the top, C strikes C_1 and g_1 is raised again, closing the equilibrium valve, while n is again raised, thus opening a and b, and the same cycle of operations again takes place.

§ **465.—Cataract.**—In the single-acting steam engine, we have still other arrangements for the regulation of its motion. To regulate the velocity, a stop valve is placed in the steam pipe which is turned by hand. In order to regulate the piston stroke, we can raise or lower either the position of the admission valve or change the position of the tappets. In order, finally, to regulate the time of the entire stroke, the *cataract* is used, an apparatus by means of which a pause of any duration may be caused at the end of the stroke. The arrangement of the cataract varies. That belonging to the machine represented in Figs. 743

and 744 is shown in Fig. 745. *H L* is a water pump with plunger piston *H* and two valves *V* and *W*, the first open-

FIG. 745.



ing inwards and the second outwards. The play of these valves can be regulated by the rods V_1 and W_1 and the cranks V_2 and W_2. The entire body of the pump is enclosed in a chest *N N* which is full of water. Upon drawing up the piston *H*, water enters through *V* from the chest, while during its descent water passes out through *W* into the chest. To thus raise and lower the piston, we have two levers *q* and q_1 with counterweights *Q* and Q_1, of which the first has a third arm, which, by means of the rod *s*, acts upon the three-armed lever *r*, whose two side arms move the rods *n* and *p* (Fig. 744).

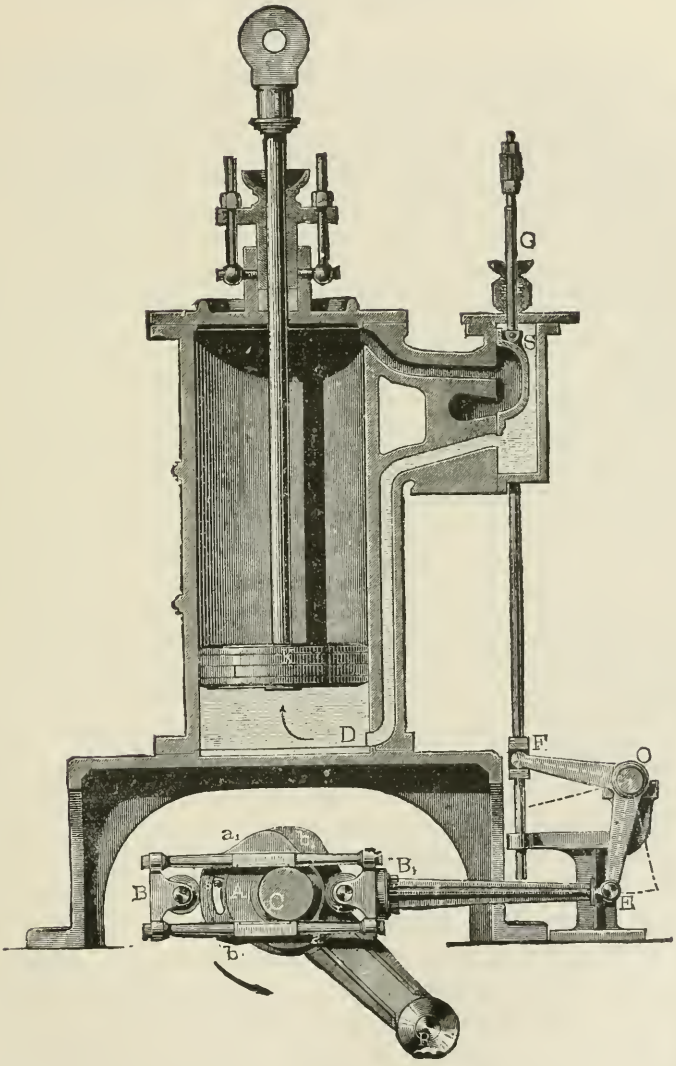
The action of the cataract is as follows: During the rise of the piston (Fig. 744), a fourth tappet *M* strikes the

lever q_1 and raises the weight Q_1, so that the weight Q acts and raises the piston H of the cataract, more or less slowly, according to the lift of the suction valve V. As this weight Q thus sinks, it raises by means of $r\,s$ the rod $n\,n_1$, so that k is disengaged (Fig. 744), and a stroke commences. By return of the steam piston, M falls below q_1 and the weight Q_1 now presses the piston H downwards more or less slowly, according to the lift of the valve W, and thus the rod $p\,p_1$ is finally lifted and the equilibrium valve raised. Since the motion of the piston H depends upon the amount of play of the valves V and W, we may regulate, by means of the apparatus V_1 V_2 and W_1 W_2, the length of pause before the fall or before the rise of the steam piston.

Fig. 744 shows also the following auxiliary apparatus. First, we have from the lever a_1 of the admission valve, a rod $t\,t$ with a plate T, which moves in a vessel filled with water and breaks the fall of the admission valve. (See the moderator in § 134.) Further, upon the arm $d\,k$ is a rod x, which, by means of an angle lever y, etc., sets in motion a valve which regulates the entrance of the injection water to the condenser. By the sinking of the weight g, therefore, at the beginning of the piston descent, x is moved up and the valve in the injection pipe opened, while towards the end of the piston descent x is forced down by the tappet A_1, the valve in the injection pipe closed, and therefore no injection water flows into the condenser during the upward stroke. Finally, the amount of injection water is regulated by a special cock which is moved by a handle u with lever and rod attachment z.

§ **466.—Slide Valve.**—We have thus far only noticed the regulation of steam engines by the simple slide, and we have therefore still to describe *expansion* slides—*i. e.*, those slides by means of which the steam is cut off during the motion of the piston, and thus forced to work expansively. In general, we have four methods—viz.:

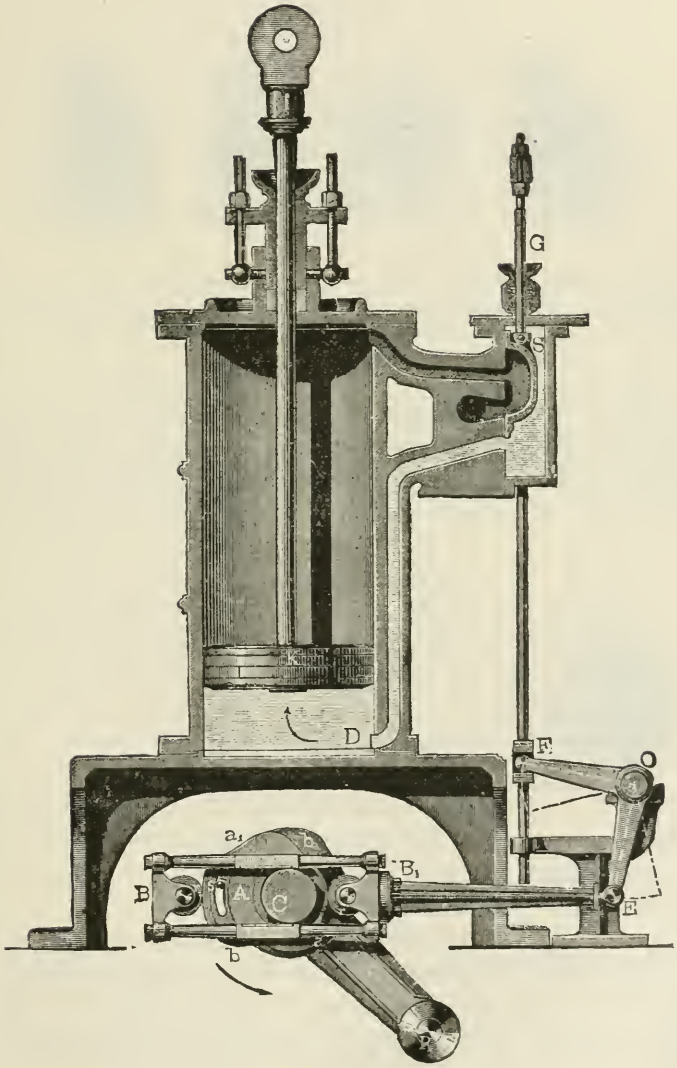
Fig. 746.



1. By means of a single slide.
2. By two separate slides.
3. By two slides, one over the other.
4. By a slide and a valve.

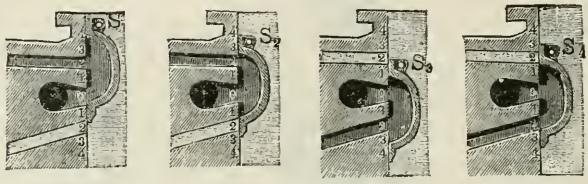
We have already seen (§ 457) that a single-slide valve set in motion by an eccentric can be made to cut off the steam before the end of the stroke, it being necessary only to give it a certain lap, so that when in its central position its ends extend beyond the steam ports. If, then, the eccentric is so placed with reference to the crank that the port is uncovered a little just before the beginning of a stroke, then the steam will be cut off before the end of the stroke, and the steam must act expansively during the remainder of the distance. This end is more completely attained by the use of a toothed eccentric or cam. The construction, arrangement, and mode of action of a valve gear with such an eccentric may be seen from Fig. 746, which represents a machine by Saulnier the elder. D is the cylinder and C the shaft, which, by means of the crank CR, etc., is set in motion by the piston rod KL. S is the slide valve, A the eccentric, BB_1 a box frame furnished with friction rollers and enclosing the shaft C and the eccentric, BE the eccentric rod, and FG the valve rod joined to BE by the angle lever EOF. The eccentric has the shape $abca_1b_1$. In the position shown, the slide has the position S_1, Fig. 748. If, by further revolution of the eccentric, the point a comes to the roller r, the point E moves to the right and the slide moves to the position S_2. If, still revolving, b comes to r, the eccentric rod moves still further to the right, the slide falls still lower, and takes the position S_3. Later, a comes to r_1 on the left, the eccentric rod moves to the left, and the slide moves upwards to S_4. Finally, b_3 comes to r_1, the eccentric rod moves still further to the left, and the slide comes to its first position S_1. In order that the slide may open and close the steam ports at the right time, its interior length must be four times, its outer length six times, and its travel three times the breadth of opening

Fig. 747.



of the steam ports. Also when the piston is at the middle of its stroke, it must pass through one third of its travel,

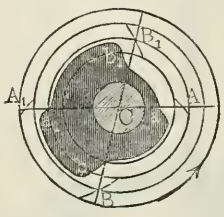
Fig. 748.



and at the end of the stroke through the other two thirds, so that b on the eccentric should be as high again as a.

§467.—**Eccentric for Variable Expansion.**—The construction of the eccentric is seen in Fig. 749. Two diameters $A\,A_1$ and $B\,B_1$ divide the eccentric into four equal or unequal parts, and upon each of these lines we have a step.

Fig. 749.

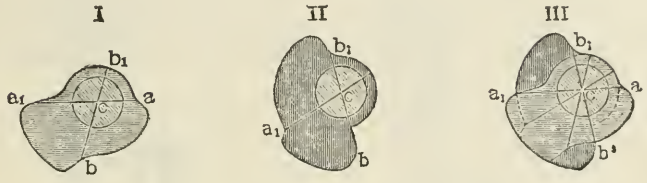


A and B are the rising and A_1 and B_1 the falling steps; A and A_1 have the single height and B and B_1 the double. In order that the eccentric may not jam between the rollers, the steps must be so formed that all diameters which join opposite points shall be equal to the distance between the rollers. Since, finally, the eccentric is grasped by friction rollers, we must describe at a distance from the curve $A\,B\,A_1\,B_1$ equal to the radius of the rollers, the curve $a\,b\,a_1\,b_1$, and form the eccentric to this curve.

The degree of expansion can also be very easily changed, if the eccentric is formed of two plates, such as shown in I. and II., Fig. 750, one being turned through a certain angle with respect to the other, and fastened by means of a screw s (Fig. 747). The plate I. lacks the step b and the plate II. the step a. If both are fastened together, they form a complete eccentric, as shown in Fig. 749, which cuts

off at about one third of the stroke. If, however, we turn I. through a certain angle before laying it on II., the cen-

FIG. 750.



tral angle between a, b, a_1 and b_1 is changed, as, for instance, the central angle $a\,b_1$ and $a_1\,b$ greater, and $a_1\,b_1$ and $a\,b$ smaller, so that the steam is cut off later, or at one half of the stroke, instead of one third. Moreover, the central angle $a\,C\,b_1 = a_1\,C\,b = \beta$, which corresponds to a certain expansion, can be easily calculated. The distance passed over by the piston corresponding to the angle of rotation β, is (§ 458)

$$s = r\,(1 - cos.\,\beta),$$

and hence the ratio to the entire travel of piston $2\,r$ is

$$\frac{s}{2\,r} = \frac{1 - cos.\,\beta}{2}.$$

If we put this $= \frac{1}{n}$, we have

$$cos.\,\beta = 1 - \frac{2}{n}.$$

Thus, for example, if the steam is cut off at $\frac{1}{3}$ of the stroke, we have

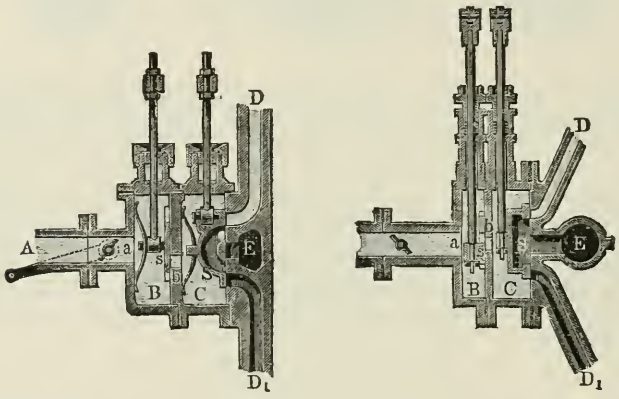
$$cos.\,\beta = 1 - \tfrac{2}{3} = \tfrac{1}{3},$$

or

$$\beta = a\,C\,b_1 = 70\tfrac{1}{2}^\circ.$$

§ **468.—Expansion Slide.**—Two methods are in use for producing expansion by means of a special expansion slide; the slide may be either a simple plate or a plate pierced with holes—in the first case shutting off the steam and in the second letting it pass. Fig. 751 shows the first and Fig. 752 the second kind. The steam passes through the pipe *a* at first in the steam chest *B*, from whence, by the opening *b*, it

751. FIG. 752.



passes into the second steam chest *C*, and thence, by the passages *D* and D_1, to the cylinder. *S* is the ordinary slide, *E* the exhaust pipe, and *s* the expansion slide which opens and closes the port *b*. In Fig. 751, this consists of a solid plate, and in Fig. 752 of a plate with an opening.

The solid slide may move either upon one or upon both sides of the steam port. Fig. 753 shows the first case. The slide *A B* passes in front of the port *D*, and must make for every piston stroke two strokes, one forward and one back, while the distributing slide makes but one. It is therefore necessary to move the expansion slide either by a circular eccentric, which in the same time makes twice as many rotations as the eccentric for the distributing slide, or to move it by an elliptical eccentric, or directly by means of lugs on the crank shaft.

In order to regulate the expansion, it is only necessary

to change the length of the slide rod by means of a simple screw. By lengthening this rod, $A\,B$ takes the position

Fig. 753.

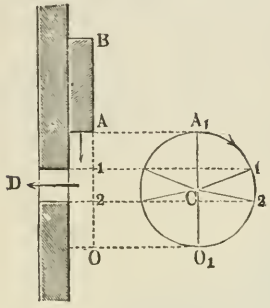
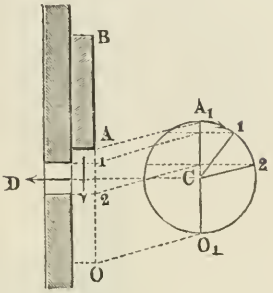
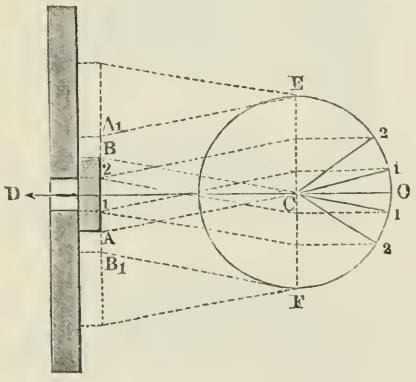


Fig. 754.



shown in Fig. 754, and the travel $s_1 = 2\overline{O} + \overline{O2}$ is greater than in the first position. If the expansion slide $A\,B$, Fig. 755, cuts off at each end A and B, the expansion can only be

Fig. 755.



altered by changing the travel of the slide. Here expansion occurs while the slide moves the distance $s = \overline{A\,1} + \overline{2\,B} = 2\,\overline{A\,1}$, and the eccentric through the angle $\beta = 2 < O\,C\,1$. But, for the eccentricity $C\,E = r$, we have

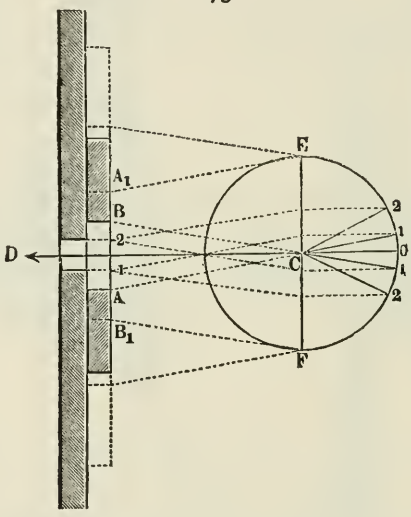
$$sin.\ O\,C\,1 = sin.\ \tfrac{1}{2}\beta = \frac{s}{2\,r}\ ;$$

therefore, the time of expansion, which increases with the angle β, is greater, the less for the same travel s the eccentricity r is.

If, as is generally the case, the slide is attached to the eccentric rod by a lever, the travel can be easily changed by altering the length of the lever arms.

A similar relation holds for the pierced slide $A\,B$, Fig.

FIG. 756.



756. This slide cuts off the steam while it passes through the distance

$$s = \overline{2\,A_1} + \overline{A_1\,2} = \overline{1\,B_1} + \overline{B_1\,1},$$

and when the eccentric has turned through the angle

$$2\,\beta = 2\,\overline{E\,C\,2} = 2\,\overline{F\,C\,2},$$

whence

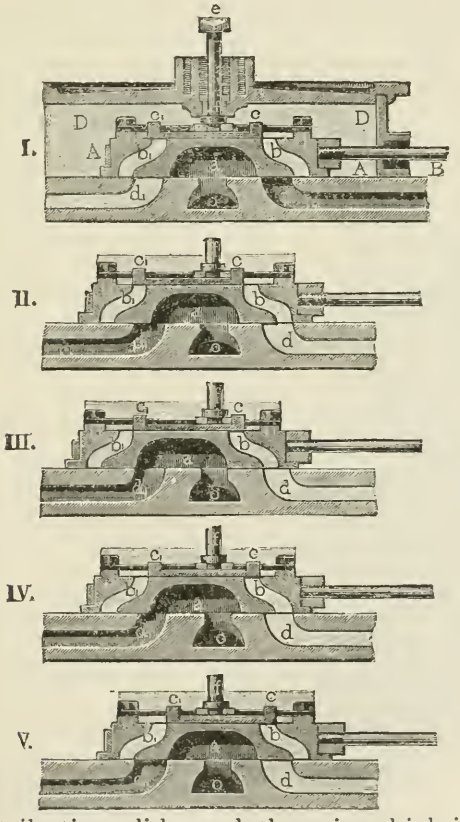
$$cos.\,\beta = \frac{r - \frac{1}{2}s}{r}.$$

Since, now, β increases as $cos.\,\beta$ diminishes, and $cos.\,\beta$ diminishes with r, it follows here also that the time of ex-

pansion which increases with β is greater, the less the eccentricity r or the entire travel $2r$ of the slide. Moreover, the distance s passed over by the slide during expansion depends of course upon the width of the steam port D.

§ **469.—Double Slide.**—The expansion arrangement of two slides, one over the other, admits of various forms. We may distinguish those in which the expansion slide moves

FIG. 757.

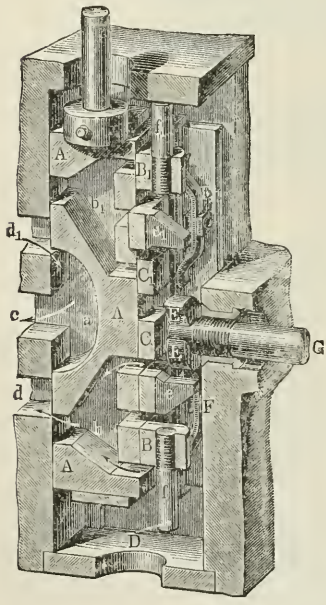


with the distributing slide, and those in which it is moved by a special rod. In Figs. 757 and 758 we have expansion slides of the first kind, and in Figs. 759 and 760 those of

the second kind. The distributing slide *A A* in Fig. 757, I., II., III., IV., has, in addition to the usual exhaust steam space *a*, two passages *b* and b_1, and the steam in *D* passes through these passages to the steam passages *d* and d_1, and thus to each side of the piston. The expansion slide is a flat plate $c\,c_1$ with checks *c* and c_1, and moving in guides. Between these checks there is an elliptical plate *f*, turning upon an axis *c f*, and adjustable by means of a lever. If the slide *A A* is moved back or forth, $c\,c_1$ goes with it only so far as the checks, which strike against *f*, allow it to go, and hence, if the slide moves further along one of the passages *b* or b_1, is closed.

In I., we have the central position of the slide, the piston being at the end of its stroke. Fig. II. shows the next position, where the piston is returning. III. shows the position in which the steam is cut off by the expansion slide, the regulating slide having reached the end of its stroke, and the piston travelling the rest of its way by expansion. In IV., the slide has returned, and in V. it is again in its central position, while the piston has arrived at the other end of its stroke. The same succession now takes place in the return stroke.

Fig. 758.

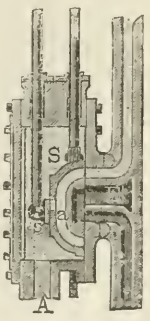


Very similar to the above is the slide valve of Farcot, represented in Fig. 758. Here the back of the slide *A A A* has six rectangular orifices for the passage of the steam which enters at *D*. In other respects, the arrangement is as above. Upon the back of the slide, we have two expansion plates *B C* and $B_1\,C_1$, each of which has two holes, and is pressed by the spring $F F_1$ against the

slide, so that all may move together. This common motion is, however, limited in either direction by the checks e and e_1 and f and f_1, the former striking against E and E_1 and the latter against the ends of the steam chamber. In the position shown in the figure, the piston is at the lower position, and the steam enters through the three lower apertures, passes through d and so below the piston, while the steam above the piston escapes through $d_1 a c$. The slide now rises and takes $B C$ with it, while, on the other hand, $B_1 C_1$ remains stationary because its motion is prevented by f_1. As the slide moves upwards, the check e strikes against E, further motion of $B C$ is thus prevented, and as the slide continues to move upwards the steam is shut off and expansion takes place. Later, the slide moves back, taking with it both expansion slides, so that when the piston reaches the end of its stroke, $A A A$ has again its central position, and at the same time the upper three orifices are opened and fresh steam enters above the piston, while the exhaust steam passes through $d a c$. (See "Principien der Daumensteuerung," by Eyth, in Civilingenieur, Bd. 4.)

§ **470.**—In the system shown in Fig. 759, the expansion slide s, which is moved by a separate eccentric, covers the port a when the distributing slide S is at its highest or lowest position. In Fig. 760, on the other hand, there are two ports a and a_1 in the distributing slide, which are alternately opened and closed by the expansion slide.

Fig. 759.

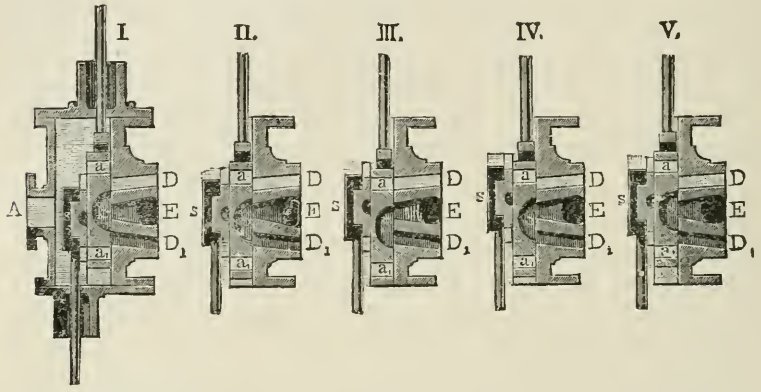


In order to obtain a clearer conception of the action of this arrangement, we have given in Fig. 760 the slide in five successive positions.

In the central position, I., the slide S covers both steam passages and the piston is near the end of its stroke. In II., communication is opened between a and D so that the piston is driven down. In III., D is perfectly

opened, and the steam would enter freely if the aperture *a* were not closed by the expansion slide. The steam, therefore, in this position is cut off and expansion commences.

Fig. 760.



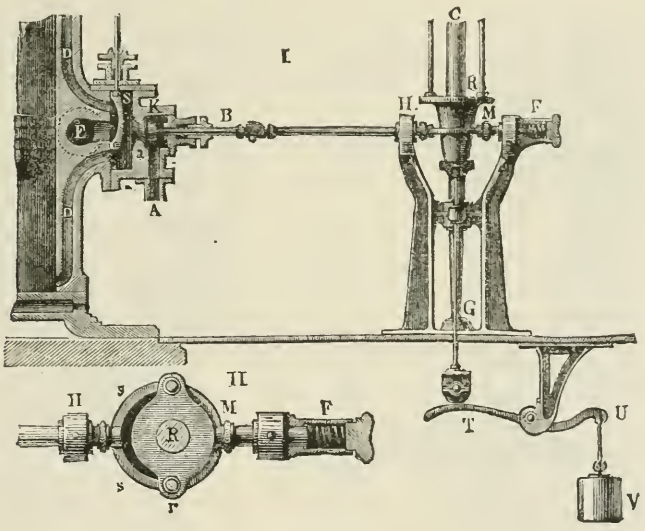
In IV. both slides have moved up so that *a* remains closed, and in V. the slide has moved up still further, but the expansion slide has moved down. The aperture *a* is, therefore, again open, but the steam is still cut off and the slide has again its central position. The piston is now at the bottom of its stroke, and the slide moves up just as before it moved down, opening D_1 to the steam and causing the piston to rise.

It is also easy to see how the eccentrics should be placed with reference to each other and the crank. The eccentric of the slide *S* is at about 90°, that of the expansion slide at about 180° with the crank.

§ 471.—**Meyer's Expansion Slide.**—The valve motion of Meyer, shown in Fig. 761, is peculiar in having a *variable expansion.* The aperture *a* through which the steam entering at *A* is admitted to the steam chest, is closed by a conical plug. The steam is regulated by the slide *S* in the usual manner. The regular opening and closing of the aperture *a* by the plug *K* is effected in the following manner: The rod

BH passes through the ring HM (II.), and butts against a spiral spring F. The ring HM encloses a cone R furnished with two ribs or flanges, and made to revolve by the spindle CG. The spring F shoves the ring in the direction MH, and thus closes a, while the cone R, by means of its ribs r and r_1, moves the ring in the direction HM, and thus draws the plug K out of the aperture a. In the latter case, steam enters, while in the first case it is shut off. If the spindle CG, and therefore R, makes the same number of revolutions as the crank shaft, then for every revolution by means of r

FIG. 761.

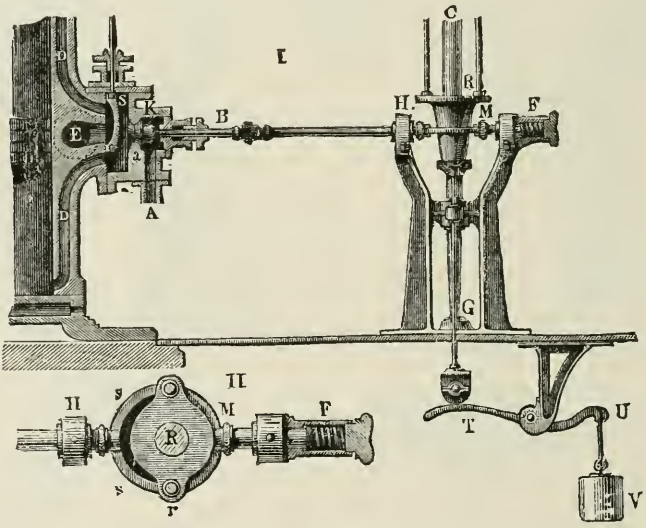


and r_1, the steam will be admitted twice, or once for each stroke of the piston. If R is raised, the rib where it touches the ring has a less elevation, and the time of opening of a will be less, and if R is lowered, r and r_1, where they touch the ring, project out farther, and hence a will be open a longer time, and the admission of steam will be greater. In order to effect this raising and lowering of R in accordance with the need of steam, by means of the machine itself, we may unite R by vertical rods with the governor.

In Fig. 763 we have represented an engine fitted with

Meyer's variable expansion cut-off. *A* is the cylinder, *B* the piston rod, *C D* the connecting rod, *D* the crank, *E F* the shaft, and *G G* the fly-wheel. The rods *B* and *C D* are united by a link *O*, furnished with friction wheels rolling on guides *c*, *c*. The steam is brought on by the steam pipe *a* to the steam chest *b*, and from there passes through *b d* and *b* d_1 alternately above and below the piston, the exhaust steam passing out through the pipe *e*. The expansion valve or plug in *b* is moved, as we have just explained, by a spiral spring *f* and a double-ribbed cone *r*. This cone *r* is

FIG. 762.



movable up and down on the spindle *K L*, which is made to revolve by the gear wheels *H*, *K*. The governor *M M*, when the velocity increases, raises *r* and thus diminishes the steam entrance, while, when the velocity decreases, *r* is lowered, and thus the supply of steam increased. Moreover, the weight of *r* is borne by a lever *T U* (Fig. 762) and a counterpoise *V*, in order that it may be readily moved by the governor.

We see also at *P Q*, Fig. 763, the feed pump, worked by the eccentric *F* and the eccentric rod *F N*.

Instead of the conical plug *K*, Fig. 762, the more easily

Fig. 763.

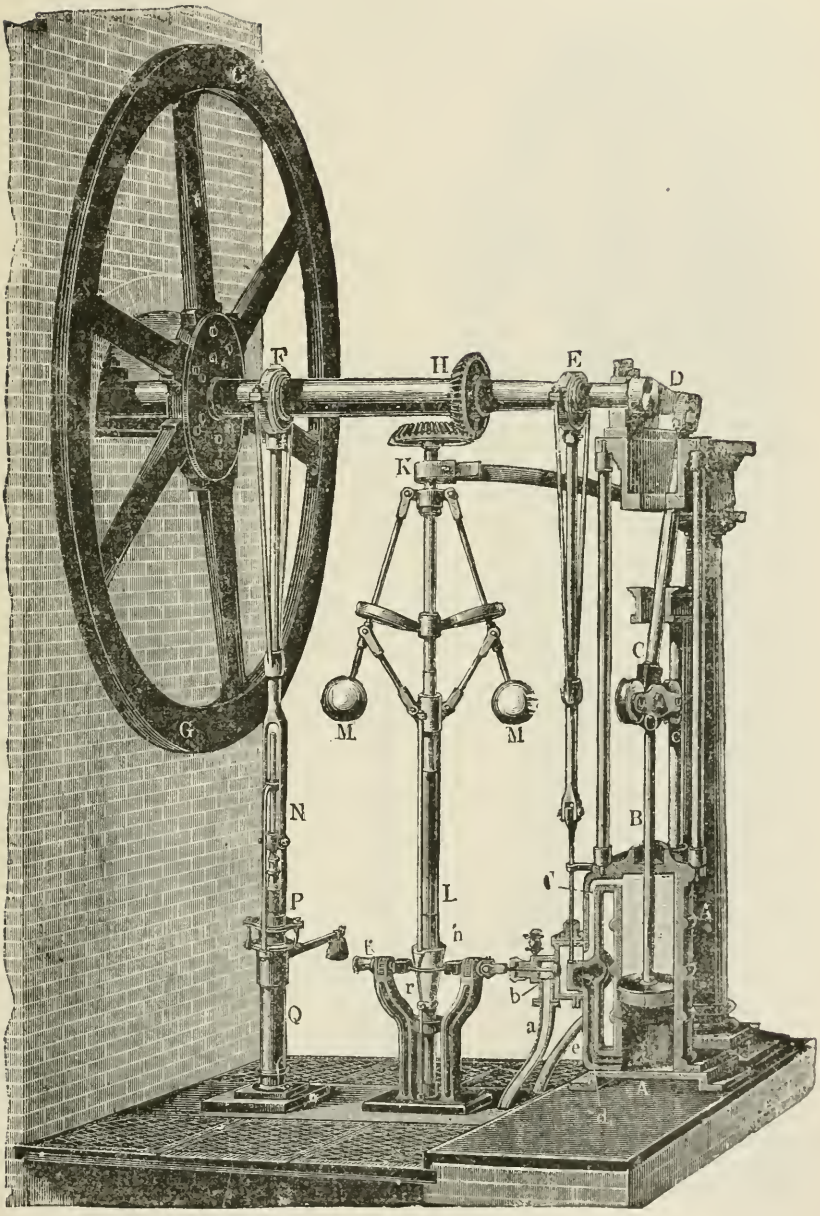
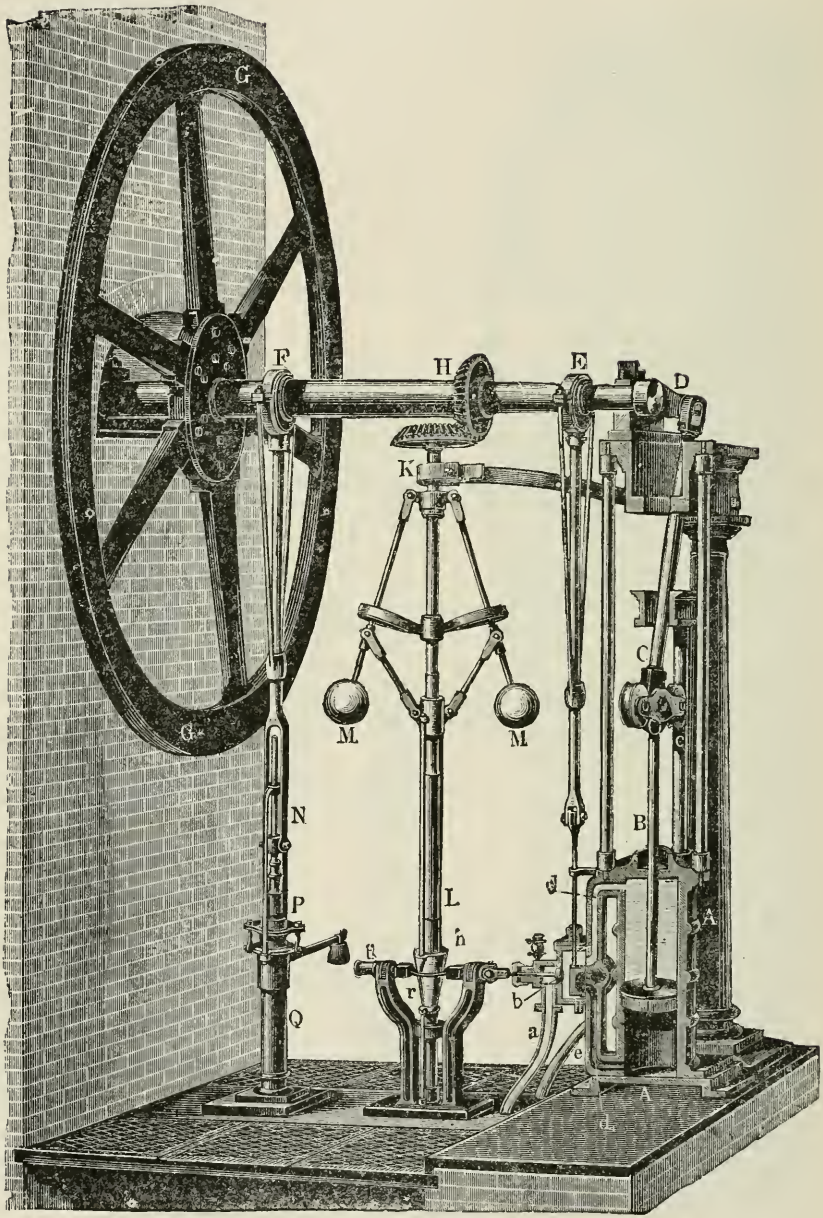


Fig. 764.



worked bell valve (§ 449) has recently been used, and by means of levers set in motion by the fly wheel. In Fig. 765, we have illustrated this mechanism. The admission valve A is closed by the rod $A F$ and the spring F, and

Fig. 765.

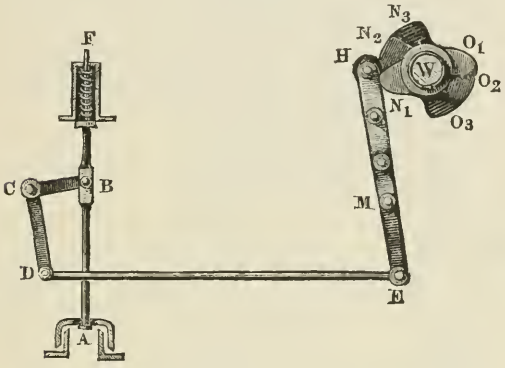
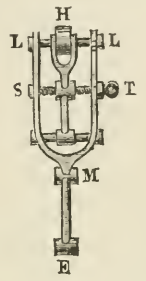


Fig. 766.

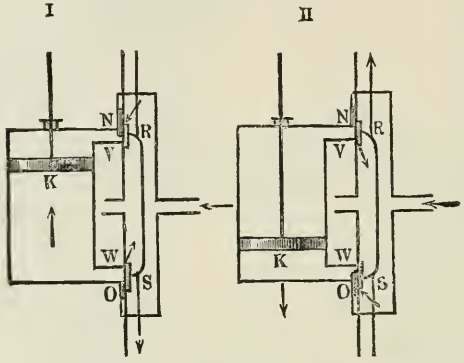


opened by means of the angle lever $B C D$, which is worked by the rod $D E$ attached to the lever $E H$ turning about M. This lever is set in motion by double cams, $N_1 - O_1$, $N_2 - O_2$, $N_3 - O_3$, upon the fly-wheel shaft. The friction wheel H at the end of the lever $E H$ can be moved, by means of the screw $S T$, horizontally along the axis $L L$, and thus, according as greater or less expansion is desired, the wheel moved by one or another of the cams.

§ **472.—Slide Valve with Movable Seat.**—In the ordinary valve motion with single slide, as shown in Fig. 736, both steam passages are opened simultaneously just before the end of stroke, and the entrance of steam upon one side of the piston takes place at the same time with its exit on the other. Since, however, for the better utilization of the steam, it is necessary that the exit should occur ahead of or before the entrance, the application of the simple slide causes an imperfect regulation. This is not the case with two slides, or with the slide with movable seat, as lately proposed by Napier and Rankine. An ideal representation

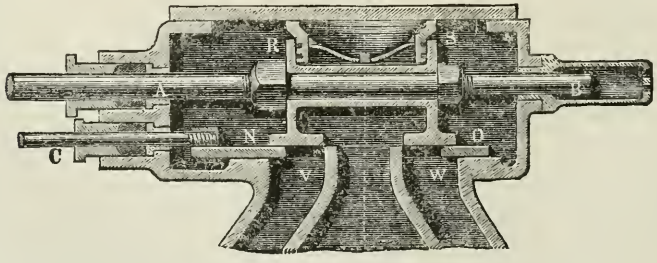
of such a slide is shown in Fig. 767, I. and II. In I., the piston K is nearly at the end of its upward and in II. of its downward stroke. The steam ports V and W are so covered by the slide, that upon its further descent, in the first case, steam will enter through V and depart through W, while upon its further ascent, in the second case, the steam will be discharged through V and will enter through W. In

FIG. 767.



order now to cause the exit to commence before the entrance upon the other side of the piston K, the breadth of the ports V and W can be varied by the movable seats N and O. In I., these seats are in the lowest position, so that the entrance through V is retarded, and in II. in the highest position, so that the entrance through W is retarded, while in both positions the exit is not affected.

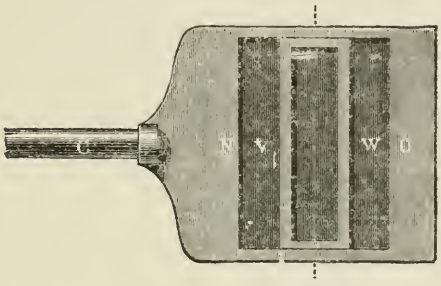
FIG. 768.



In Fig. 768 we have the arrangement of Adam's equilibrium slide with such a movable seat. $R\,S$ is the slide moved

by the rod $A\,B$, and $N\,O$ the movable seat moved by the rod $C\,N$. Fig. 769 shows the form of movable slide seat

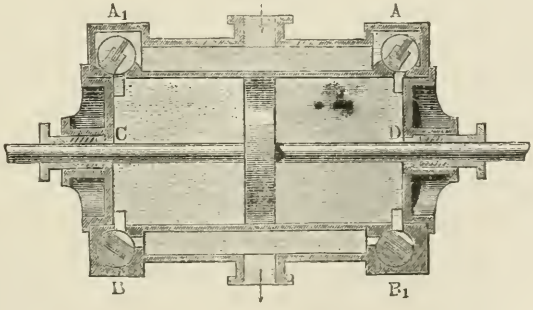
Fig. 769.



$N\,O$, which is set in motion by a separate eccentric. (See "Polytechn. Centralblatt," Jahrgang 1867, or "The Engineer," October 18th, 1867.)

§ **473.—Corliss Steam Engine.**—The cut-off of the Corliss steam engine is peculiar. In this machine, the steam does not leave the cylinder by the same way by which it entered. The valve motion consists of four rotating slides —two, as A and A_1, Fig. 770, for the admission, and two, as

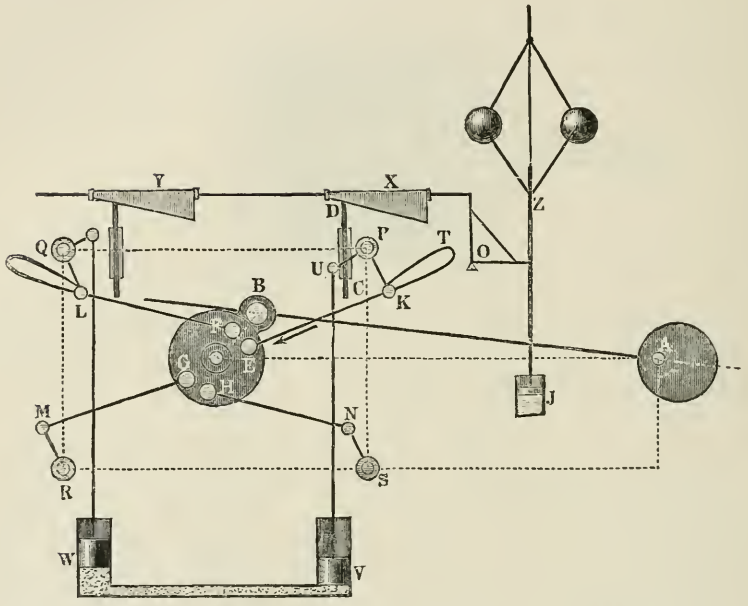
Fig. 770.



B and B_1, for the release of the steam. These valves are as close as possible to the ends of the cylinder, in order that the prejudicial space may be a minimum. The valve motion is effected partly by eccentrics and partly by weights. An

ordinary circular eccentric *A* sets in vibratory motion, by means of the rod *A B*, Fig. 771, a circular plate *E F G H*,

Fig. 771.



and this last, by means of the rods *E K*, *F L*, *G M*, and *H N*, and the corresponding arms *K P*, *L Q*, *M R*, and *N S*, sets in motion the four valves *P*, *Q*, *R*, *S*, part directly and partly indirectly. For the two emission valves *R* and *S*, the connection is direct; for the two admission valves *P* and *Q*, there is, however, a special mechanism, so that now one, and now the other, is turned back by a falling weight, and the steam thus cut off. For this purpose, the arm of the angle lever, by means of which the valve is set in motion, is furnished with a catch which engages with the rod *E K*, so that, as this latter moves in the direction of the arrow, the valve is opened and steam enters the cylinder. As the disc *E F G H* continues to revolve, *E K* comes in contact with the rod *D C*, the steel spring *T* gives and releases the catch so that *K* is set free. The angle lever is now rotated back by the weight *V* and rod *V U*, and thus the valve is closed

and the steam cut off. Towards the end of the stroke, the emission valve S is opened by means of the rod HN, and the exhaust commences.

Upon the return of the piston, the admission valve Q is first opened by means of FL. At the same time, EK also returns, and again engages K by means of the spring T and catch. Later, the catch at L is released, and the weight W acts to shut the valve Q and cut off the steam. The valve R then opens for the release, and so on.

The weights move in cylinders filled with air, in order to prevent sudden shocks.

In order to regulate the motion of the machine, a horizontal rod carrying two wedges, X and Y, is attached, by means of the angle lever, to the governor. According as the velocity of rotation of the governor increases or diminishes, the wedges are moved to the left or right, the rods DC moved down or up, and the amount of expansion increased or diminished.

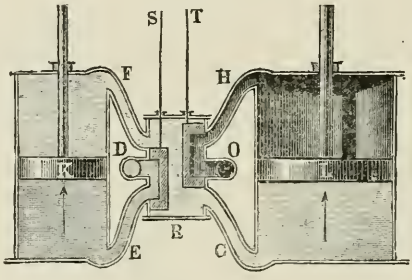
In order to diminish impact between the wedges and rods CD, the end of the governor rod has a piston which moves in a cylinder J filled with water, and hence the rapid rise and fall of the rod, as well as sudden motions of the wedges, are prevented. Fig. 771 is merely an ideal sketch.

§ 474.—**The Woolf Engine.**—We may also cause the steam to act expansively, by allowing it, after its action in one cylinder, to escape into another and larger one, and to act upon the piston there. Such an arrangement with two cylinders is called a *compound engine*, or, after its inventor, a Woolf engine.

In France, such engines were first introduced by Edward, and hence they are often called by his name. Steam of from 3 to 4 atmospheres is used, allowed to expand in the large cylinder about fourfold, and is then condensed in an ordinary condenser. The piston rods of both cylinders are in general attached to the same beam, that of the

small one within, and of the large one on the outside. Fig. 772 gives an ideal representation of the arrangement and mode of action of a Woolf steam engine. The steam is brought in at *D*, and, by means of the slide worked by the rod *S*, enters alternately through *E* and *F* the small cylinder, sets the piston *K* there in motion, and then, after complete action, passes by *F* and *E* into the steam chamber *R*. From here it passes, by means of the slide *T*, by *G* and *H* into the large cylinder, and then, after acting upon the piston *L*, passes out through the exhaust pipe *O* to the

FIG. 772.



condenser. Both pistons move simultaneously up and down, when the two slides have opposite positions.

The cut-off arrangements of such an engine may be seen in Fig. 773. *A A* is the small cylinder in which the steam first acts, if desired, without expansion, and *B B* the large cylinder (only partly visible), in which the steam acts expansively. The steam enters *A A* through a ring-shaped channel $C C_1$, surrounding the cylinder, which empties into the steam chamber *D D* through the spaces *d*, d_1. Into this chamber open three other passages, *E*, E_1, and *F*, which conduct the steam above and below the piston in *A A*, and into the steam chamber $G G_1$ of the second cylinder. The slide *S* always covers two of the three openings leading into *E*, E_1, and *F* in the steam chamber $D D_1$, so that the steam passes through the third, as, for instance, *e*, into one of the passages *E*, and thus into the cylinder *A A*, while the exhaust steam passes through the other E_1, and then

Fig. 773.

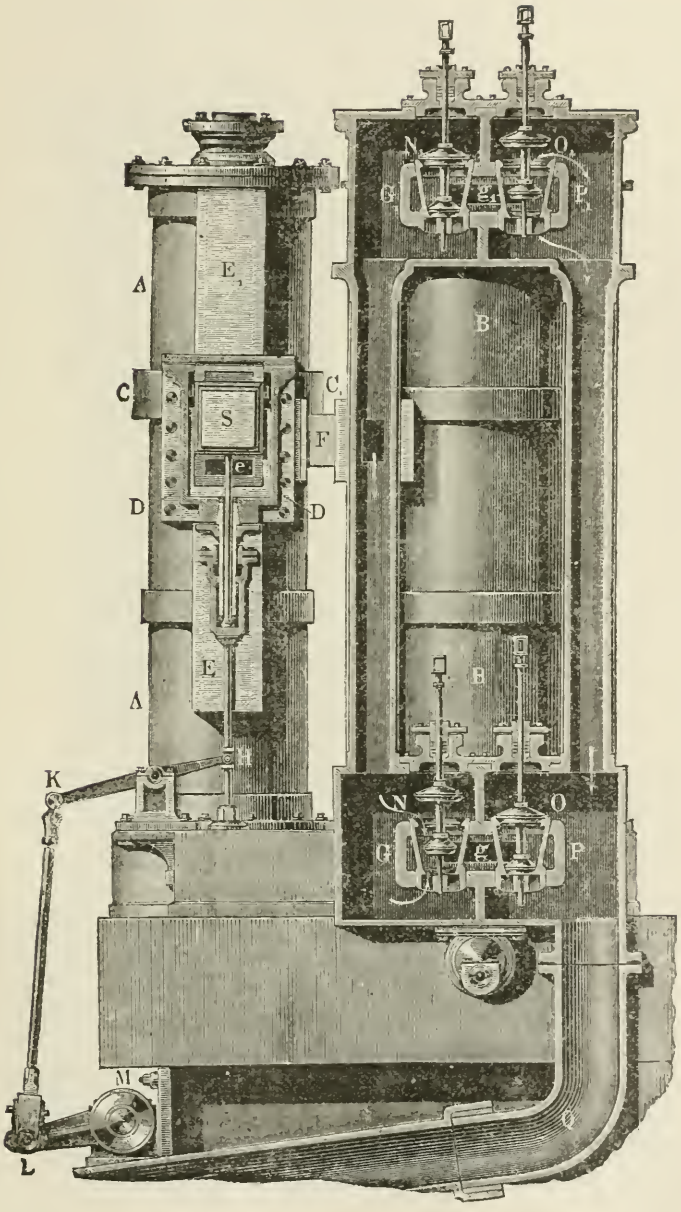
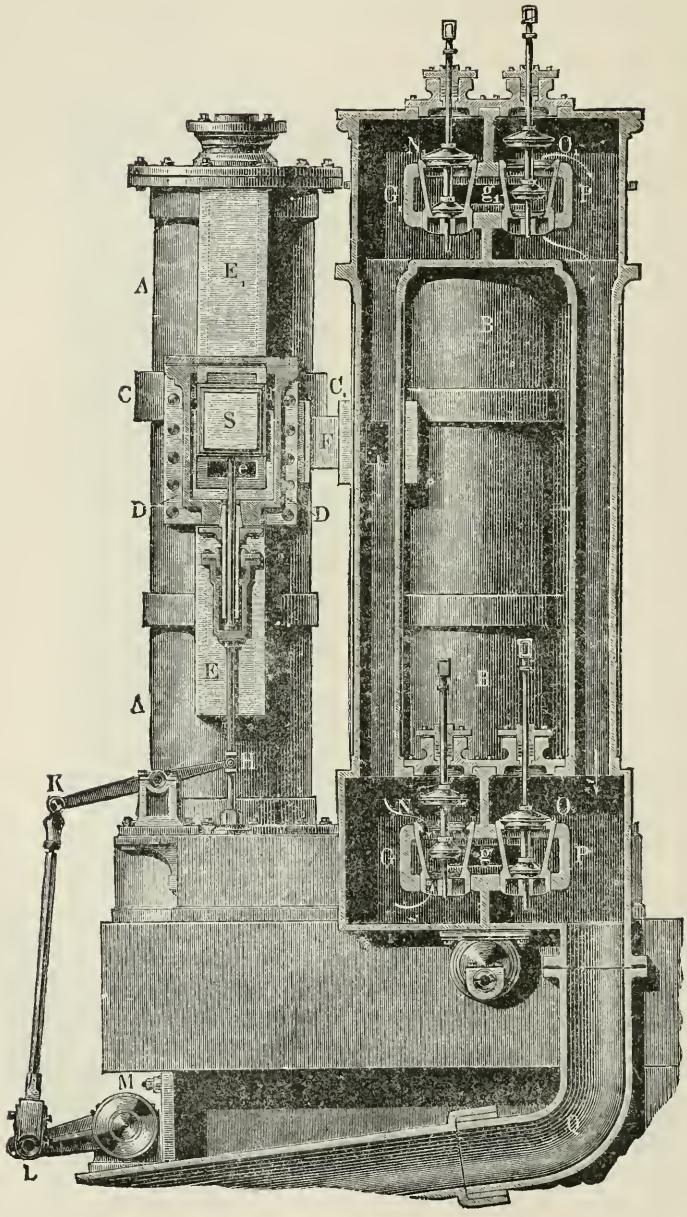


Fig. 774.



through F, into the chamber $G G_1$. The slide S is moved by an eccentric, which causes the shaft M to oscillate back and forth, and this shaft is connected with the slide rod $S H$ by means of the lever $H K$, the connecting rod $K L$, and the crank $M N$. In the steam chamber $G G_1$ are two double valves N and N_1, which open the ports g and g_1 for the admission of steam to the cylinder $B B$. Alongside of this chamber is the chamber $P P_1$, which communicates, by two other valves O and O_1, with the ports g and g_1, and also, by means of the pipe Q, with the condenser. When O and O_1 are raised, the steam which has expanded in $B B$ is thus allowed to escape to the condenser. The lifting of the valves N, N_1, O, O_1, is effected by means of levers and rods in connection with the shaft M. In the position of slide and valves, shown in the figure, the steam enters below the piston in $A A$ and drives it upwards; at the same time, the steam, already above this piston, passes through E_1 and F, enters through the valve N below the piston in $B B$, and forces it upwards also. Both pistons, therefore, rise and fall together.

The engine of Legavrian is upon the same principle as Woolf's, but has three cylinders. (See Fig. 724, § 453.)

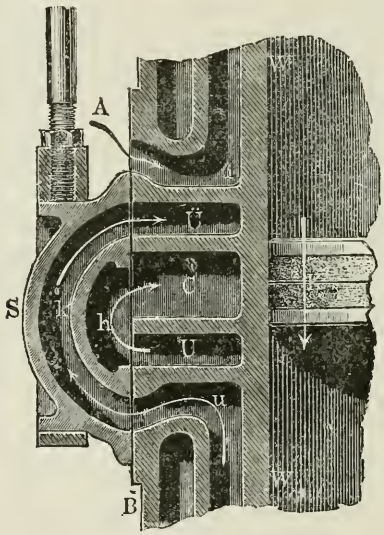
§ **475.**—Instead of slide-valves, the double slide of Hicks is used to advantage in the Woolf engine. This slide S, Fig. 775, contains two canals or steam passages h and k, and moves upon a seat $A B$ containing five ports, of which U and $\ddot{U}$ lead below and above the piston in the larger, and u and $\ddot{u}$ below and above the piston in the smaller cylinder, while C leads to the condenser. In the position shown in Fig. 775, steam enters by $\ddot{u}$ above the small piston, while the steam below this piston rushes through u, k, and $\ddot{U}$ above the large piston, and the steam below this piston escapes, through U and C, to the condenser.

When this double slide is at its upper position, the port u is uncovered and the port $\ddot{u}$ is in communication with k,

so that fresh steam passes through *u* below the small piston, and at the same time the steam above this piston passes by *ü* and *k* through *U*, below the large piston, while the exhaust steam passes through *Üh C* to the condenser.

To move the double slide, a device of recent and, according to Hornblower, of advantageous application, consists of a cam having the shape of a triangle instead of an eccentric. This cam, *A B C*, Fig. 777, is bounded by three equal arcs of circles of about 60°, and is fastened to the

FIG. 775.



revolving disc *S S* in such a manner that one side *A B* corresponds with the circumference, while the opposite corner *C* falls at the centre of the disc. The cam acts upon two straight edges *D E*, *F G*, at right angles to the slide rod, and whose distance apart, *D F* = *E G*, is the radius of the bounding arcs or of the disc.

In the position shown in Fig. 778, the slide has its highest position, and while now the disc revolves through the angle *B C O* = 60°, the side *A B* remains in contact with *D E*, and the slide remains stationary. At this latter posi-

tion, the forward side CB has just come into contact with the guide FG, and while now it continues to revolve

FIG. 776.

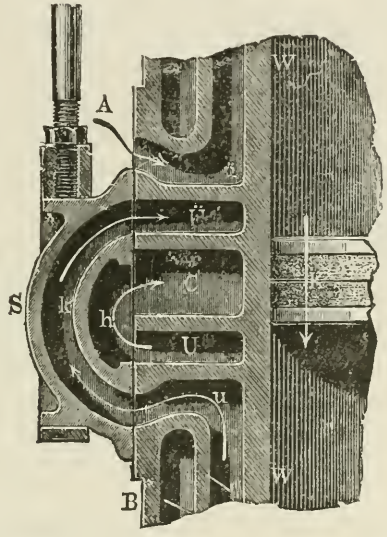


FIG. 777.

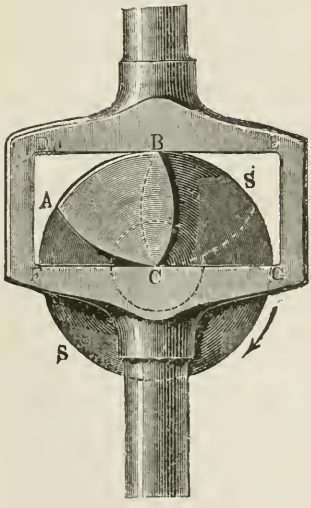
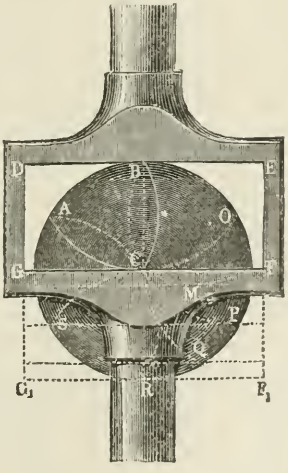


FIG. 778.



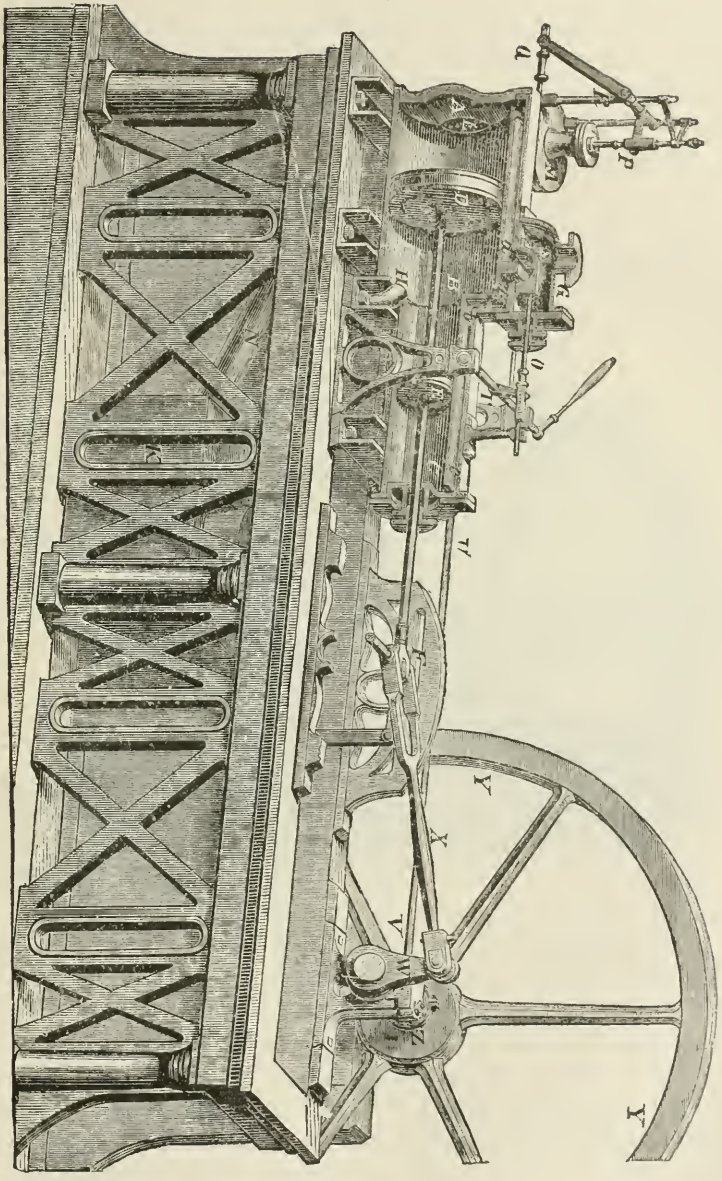
from CO to CP or through another angle of 60°, the point of contact M passes from C to P. Finally, after another

angle of 60° is passed, the forward side CB will pass from CP to CR, the point B will be at R, and the slide will have its lowest position. The return of the slide will take place precisely in the same way. The side AB, which now has the position PR, now slides upon $F_1 G_1$ without moving the slide, until the forward corner arrives at S. At this moment, the forward side just touches the upper guide, and from here on, the slide rises to its highest position again. The entire travel of the slide is thus twice the distance between the guides or the diameter of the disc. The slide is thus at rest during a third of the stroke, and during two thirds in motion. Its motion, therefore, as well as the opening and closing of the ports, is quicker than with the ordinary eccentric.

In Woolf's engine, if the steam is required to act expansively in the small cylinder also, there must be in addition a special expansion valve.

§ **476.—Sims' Engine.**—A peculiar arrangement is the compound engine with double horizontal cylinder, of Sims. This machine consists of two cylinders AB and BC, Fig. 779, of different diameters, laid end to end, and of two pistons D and E, upon the same piston rod DF, of which one (E) is forced in one direction by the steam entering from G through abc, and the other (D) in the other direction by the steam escaping from the small cylinder by cba and dc. The space DBE between the pistons is connected by a pipe H with the condenser, and there is hence a slight back pressure, which, since D is greater than E, assists somewhat the motion in the direction ED, and hinders that in the opposite direction. The exhaust steam flows, after having performed its work in the large cylinder, through L to a pipe M, and then by another pipe N to the condenser K. The admission and release of the steam is accomplished by means of a slide S in the steam chamber G, and by a valve in the pipe M (not visible), both of which are worked by the eccentric rod UV, by means of rods

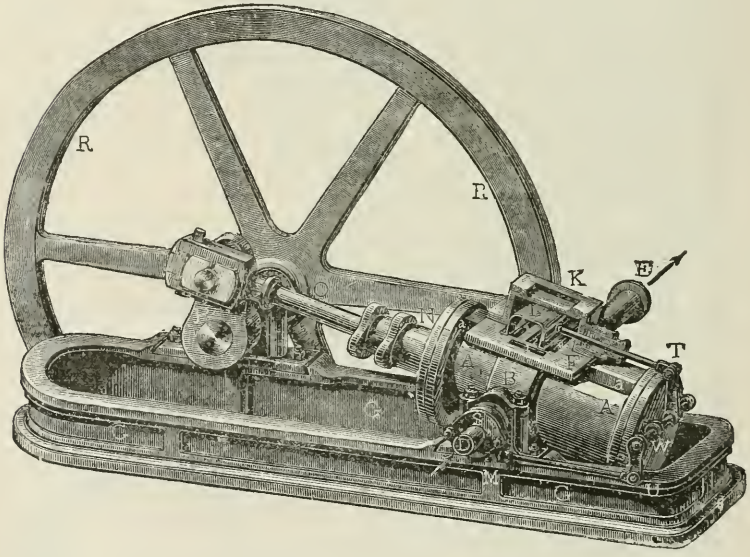
Fig. 779.



O, *P*, and *Q*, and the levers *R* and *T*. We also see in the figure the crank *W* and connecting rod *X*, as also the fly-wheel *Y Y*, by means of which the reciprocating motion of the piston rod is converted into a nearly uniform motion of rotation. (See "Practical Mechanics' Journal," 1849, July, p. 50, or the "Polyt. Centralblatt," 1851, Liefer. 1.)

§ **477.—Alban's Engine.**—A very simple *oscillating* engine, by Dr. Alban, in Plau, is shown in Fig. 780. (See

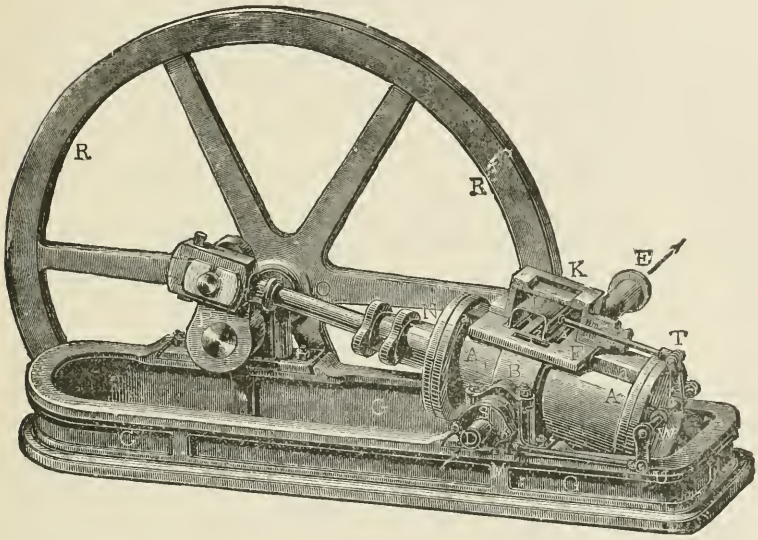
FIG. 780.



"Hochdruckdampfmaschine," von Alban, Rostock, etc.) The cylinder *A A* has two hollow cast trunnions, resting in ordinary pillow blocks, as *B*, bolted to a frame *G G G*. The pipes *D* and *E*, the former of which brings and the latter of which carries off the steam, communicate with the hollow trunnions and pass through stuffing boxes, as *S*. The foot plate *F* of the steam chamber *K* has four ports, of which the forward one (1) communicates through *B* and *S* with the steam pipe, and the middle one (2) in similar man-

ner with the exhaust pipe E. This port is always covered by the slide L, together with either (3) or (4), according to the position of the slide. The end of the slide L has been removed to allow a view of these ports. In the position of

FIG. 781.



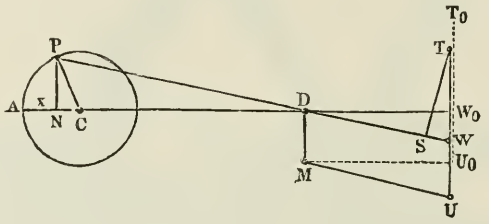
the slide represented in the figure, the steam eners through (1), and passes through (3) and a into the cylinder, while the exhaust steam passes through a_1 and (4) into the steam chamber, and from thence through (2) and E. In the opposite position of the slide, (4) is uncovered and (3) is within the slide; hence the steam enters the cylinder through (4) and a_1 and escapes through a, (3), and E. The piston rod O is attached directly to the crank P. The stuffing box N, with unusually long body, acts as a guide for the piston rod. To preserve uniformity of motion, the fly-wheel $R\,R$ is attached to the crank shaft.

In order to move the slide, we have an arrangement of levers, consisting of a shaft W attached to the end A of the cylinder. A link $M\,U$, turning about the fixed point M, turns the shaft W as the cylinder oscillates by means of the crank $U\,W$, and thus turns the crank $W\,T$ and moves the slide

back and forth. To illustrate this, we have the geometrical representation of Fig. 782. C is the axis of the fly-wheel, D the axis of oscillation of the cylinder, W the shaft for the slide, and M the fixed point of oscillation of the link MU. If the crank $CP = r_1$ turns through the angle $ACP = \beta$, the piston passes through the distance,

$$AN = x = r_1 (1 - cos.\ \beta) \text{ approximately,}$$

Fig. 782.



and the axis of the piston rod takes the inclination α given by the formula,

$$sin.\ \alpha = \frac{NP}{DP},$$

or approximately,

$$sin.\ \alpha = \frac{r_1\ sin.\ \beta}{d},$$

where d is the distance CD of the centres of crank and cylinder. If M is vertically below D, and WU equals DM, then $DMUW$ is in every position of the cylinder a parallelogram, and, at the end of the stroke, a rectangle, since then the axis of the cylinder is horizontal. If a denotes the length $WT = W_0 T_0$, we have for the travel of the slide corresponding to the angle $WD W_0 = STW = \alpha$,

$$WS = WT\ sin.\ STW,$$

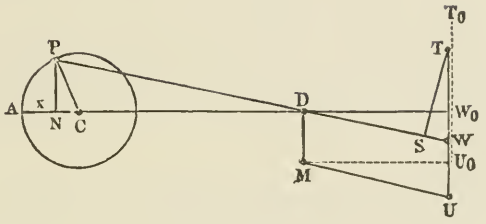
or

$$y = a\ sin.\ \alpha = \frac{a r_1\ sin.\ \beta}{d},$$

or, denoting $\frac{a\,r_1}{d}$ by r,

$$y = r\ sin.\ \beta.$$

Fig. 783.



These formulæ correspond entirely with those found for the valve motion with eccentric (see § 459), when the angle of advance is zero.

§ **478.—Work of Steam without Expansion.**—In the following we shall indicate how the performance of a steam engine can be calculated. Let us first take the simplest case, viz., a double-acting engine without expansion, and neglect at first all losses and resistances. We denote the steam pressure per unit of area (square inch) by p, and the area of the piston by F (in square inches); hence we have for the pressure of the steam upon the piston,

$$P = Fp.$$

If now s is the distance passed through by the piston in one stroke, we have for the work performed by each stroke,

$$Ps = Fps = Fs\,.\,p,$$

or, since Fs equals the volume V of steam used, per stroke,

$$Ps = Vp.$$

If the engine makes n revolutions per minute, and therefore the piston passes in the same time over the distance $2\,s \times n$, the mean piston velocity is

$$v = \frac{2\,s \times n}{60} = \frac{n\,s}{30},$$

and hence the theoretical delivery of the engine per second is

$$L = P v = \frac{n\,s}{30} F p = \frac{n}{30} V p = Q p,$$

where Q is the quantity of steam used per second.

This result holds good only when there is no back pressure upon the other side of the piston—*i. e.*, when there is perfect condensation. If, however, there is a back pressure q per square inch, or a total back pressure $F q$, the effective pressure is

$$P = F(p - q),$$

and hence the delivery per second is

$$L = \frac{n\,s}{30} F(p - q) = \frac{n}{30} V(p - q) = Q(p - q).$$

In condensing engines, q is the pressure in the condenser; in non-condensing engines, q is the atmospheric pressure = 14·7 lbs. per square inch = 1·033 kilogrammes per square centimeter. If V or Q is given in cubic feet and p and q are per square inch, we have

$$L = \frac{n}{30} V \,.\, 144\,(p - q) = Q \,.\, 144\,(p - q);$$

that is,

$$L = 4{\cdot}8\; n\, V(p - q) = 144\; Q(p - q) \text{ ft. lbs.}$$

If, however, V and Q are given in cubic meters, and p and q are per square centimeter, we have

$$L = 10000 \frac{n}{30} V(p - q) = 10000\, Q(p - q) \text{ kilogr'meters,}$$

since the pressure per square foot is $(12)^2 = 144$ times as great as that per square inch, and the pressure per square meter is $(100)^2 = 10000$ times as great as the pressure per square centimeter.

Example.—The interior cylinder diameter of a non-condensing steam engine is 18 inches, and the stroke 40 inches. The number of revolutions per minute is 24, and the steam tension $3\frac{1}{2}$ atmospheres. What is the pressure and work? The piston area is

$$F = \left(\frac{18}{2}\right)^2 \pi = 81\, \pi = 254{\cdot}47 \text{ square inches;}$$

hence the pressure is

$$P = F(p - q) = 254{\cdot}47 \times 14{\cdot}7(3{\cdot}5 - 1) = 9352 \text{ lbs.;}$$

now, $n = 24$ and $s = \frac{40}{12} = \frac{10}{3}$ ft., hence the theoretical delivery is

$$L = \frac{n s}{30} P = \frac{24 \times 10}{30 \times 3} \times 9352 = 24938 \text{ ft. lbs.}$$

$$= \frac{24938}{550} = 45{\cdot}34 \text{ horse-power.}$$

§ **479.—Action during Expansion.**—If the steam is cut off at the instant the piston has passed over the distance s, it acts during the rest of the stroke by its expansion. Here, several cases are conceivable. Either the temperature of the steam during expansion remains unchanged, or this temperature diminishes according as the steam expands, so that a portion is or is not condensed according to the initial temperature and the amount of its diminution. The first case can only happen when the cylinder is heated from without, as, for instance, when it is sur-

rounded by hot air or steam, and the motion of the piston is very slow, so that the steam can take up the heat necessary for its expansion. Under the supposition that unsaturated or superheated steam comports itself like atmospheric air, we may also assume that the expansive force of the cut-off steam follows the law of Mariotte. (See Vol. I., § 387 and § 388.)

The second case is conceivable under different conditions. If, as happens especially with very rapid steam generation, the cut-off steam is not dry, but contains water carried over with it, this water during the expansion will be turned into steam, and hence, under certain conditions, the entire amount of steam may be in the saturated condition. Under this assumption, we may find the tension of the steam, according to Pambour, from Navier's formula (§ 390),

$$\mu = \frac{\alpha}{\beta + p}$$

for the specific steam volume, while, according to Mariotte's law,

$$\mu = \frac{\alpha}{p}.$$

Under the supposition that heat is neither imparted to nor abstracted from the steam during its expansion, we can also assume that the expansive force, according to Poisson's law (§ 376), stands in an inverse ratio to some power of the volume, only, instead of the exponent κ for air, we must use an exponent ν determined by experiment.

Finally, the mechanical theory of heat gives also the means of determining the expansive force of steam in the cylinder.

REMARK.—Poncelet and Morin, as also Tredgold, etc., base their theory of the steam engine upon the first rule, while Pambour adopts the second. (See "Théorie des machines à vapeur, par Pambour, Paris, 1844, deux. édition," especially the Introduction.) Morin shows experimentally that Mariotte's law furnishes the basis for a theory of the steam engine in per-

fectly satisfactory accord with experience. (See "Leçons de mécanique pratique, 3me partie, par A. Morin, Paris, 1846.") Upon the application of the mechanical theory of heat to the steam engine, see Clausius, "Abhandlungen über die mechanische Wärmetheorie, Braunschweig, Vieweg und Sohn, 1864;" also, Zeuner, "Grundzüge der mechanischen Wärmetheorie," Leipzig, A. Felix, 1866; Combes, "Théorie mécanique de la chaleur et de ses applications," Paris, 1863; Hirn, "Théorie mécanique de la chaleur," Paris, 1865.

§ 480. — Expansion according to Mariotte's Law. — Upon the basis of Mariotte's law, the action of steam, like any gas, may be determined as in Vol. I., § 388. If one cubic foot of gas or steam passes from the greater tension p to the lesser tension p_1, the work performed is

$$A_1 = p \; log.\; nat. \left(\frac{p}{p_1}\right) = 2 \cdot 3026 \; p \; log. \left(\frac{p}{p_1}\right).$$

If the initial volume corresponding to the tension p is V, and that corresponding to p_1 is V_1, we have

$$\frac{p}{p_1} = \frac{V_1}{V},$$

and hence the mechanical effect performed by the gas in expanding from V to V_1 is

$$A_1 = V p \; log.\; nat. \left(\frac{V_1}{V}\right).$$

Applying this to the steam engine, if s is the distance passed over by the piston before expansion commences, and s_1 the entire length of stroke, we have

$$V = Fs \qquad \text{and} \qquad V_1 = Fs_1,$$

and hence the work performed is

$$A_1 = Fsp \; log.\; nat. \left(\frac{s_1}{s}\right).$$

If to this we add the work before expansion,

$$A_2 = F\,s\,p,$$

we have for the entire work

$$A = F\,s\,p + F\,s\,p\ log.\ nat.\left(\frac{s_1}{s}\right)$$

$$= F\,s\,p\left[1 + log.\ nat.\left(\frac{s_1}{s}\right)\right]$$

$$= F\,s_1\,p_1\left[1 + log.\ nat.\left(\frac{s_1}{s}\right)\right].$$

If we take into account the back pressure q upon the other side of the piston, which consumes the work $F s_1 q$, we have for the complete work of the steam per stroke

$$A = F\,s_1\,p_1\left[1 + log.\ nat.\left(\frac{s_1}{s}\right) - \frac{q}{p_1}\right]$$

$$= F\,s\,p\left[1 + log.\ nat.\left(\frac{s_1}{s}\right) - \frac{q}{p_1}\right].$$

The delivery of the machine per second is, then, as in § 478,

$$L = \frac{n}{30} F\,s\,p\left[1 + log.\ nat.\left(\frac{s_1}{s}\right) - \frac{q}{p_1}\right]$$

$$= 144\,\frac{n\,V\,p}{30}\left[1 + log.\ nat.\left(\frac{s_1}{s}\right) - \frac{q}{p_1}\right] \text{ ft. lbs.},$$

where V is the quantity of steam Fs used per stroke, or, finally,

$$L = 144\,Q\,p\left[1 + log.\ nat.\left(\frac{s_1}{s}\right) - \frac{q}{p_1}\right] \text{ ft. lbs.},$$

where Q is the quantity of steam used per second.

Example.—What is the work performed by the engine in the previous example when the steam is cut off at 0·4 of the entire stroke? Here $s_1 = 40$ inches $= \frac{10}{3}$ ft., $s = 0{\cdot}4 \times 40 = 16$ inches $= \frac{4}{3}$ ft. The pressure upon the piston is

$$Fp = 254{\cdot}47 \times 3{\cdot}5 \times 14{\cdot}7 = 13092,$$

and the work per stroke, then, is

$$L_1 = 13092 \times \tfrac{4}{3}\left(1 - \frac{1}{0{\cdot}4 \times 3{\cdot}5} + 2{\cdot}3026 \log. \tfrac{10}{4}\right)$$

$$= 17456\,(1 - 0{\cdot}71428 + 2{\cdot}3026 \times 0{\cdot}39794)$$

$$= 17456 \times 1{\cdot}20202 = 20982 \text{ ft. lbs.}$$

Hence the delivery per second is

$$L = \frac{n}{30} 20982 = \tfrac{24}{30} \times 20982 = 0{\cdot}8 \times 20982 = 16786 \text{ ft. lbs.} = 30{\cdot}5 \text{ horse-power.}$$

§ 481.—**Pambour's Theory.**—The delivery of a steam engine with expansion may be deduced from Navier's formula in the following manner: The specific steam volume, or the ratio of the steam volume to the volume of the equivalent water, is, for the tension p (§ 390),

$$\mu = \frac{\alpha}{\beta + p},$$

and hence for the tension p_1,

$$\mu_1 = \frac{\alpha}{\beta + p_1}.$$

We have, therefore,

$$\frac{\mu}{\mu_1} = \frac{\beta + p_1}{\beta + p},$$

or

$$p_1 = (\beta + p)\frac{\mu}{\mu_1} - \beta.$$

If s is the travel of the piston before expansion, and s_1 the entire distance up to any point of the expansion, the cor

responding tensions being p and p_1, we have for the pressure upon the piston at any moment of the expansion,

$$P = F p_1 = F\left(\frac{(\beta + p)\, s}{s_1} - \beta\right) = F\frac{(\beta + p)\, s}{s_1} - F\beta.$$

The first part of this expression is inversely proportional to the travel of the piston s_1, and the second part $F\beta$ is constant; hence, the work during expansion, corresponding to the first part, follows Mariotte's law as above, or

$$A_1 = F(\beta + p)\, s \; log.\; nat.\left(\frac{s_1}{s}\right),$$

and that corresponding to the second portion is, by multiplication by the distance $s_1 - s$,

$$A_2 = -F\beta\,(s_1 - s).$$

Hence the work of the steam during expansion is

$$A_1 + A_2 = F(\beta + p)\, s \; log.\; nat.\left(\frac{s_1}{s}\right) - F\beta\,(s_1 - s),$$

and the work during the entire stroke is

$$\begin{aligned} A &= F p\, s + A_1 + A_2 \\ &= F p\, s + F(\beta + p)\, s \; log.\; nat.\left(\frac{s_1}{s}\right) - F\beta\,(s_1 - s), \end{aligned}$$

or, taking into account the back pressure Fq which causes the loss of work $Fq\, s_1$,

$$A = Fs(\beta + p) + Fs(\beta + p) \; log.\; nat.\left(\frac{s_1}{s}\right) - F\beta s_1 - Fq s_1$$

$$= Fs(\beta + p)\left[1 + log.\; nat.\left(\frac{s_1}{s}\right) - \frac{\beta + q}{\beta + p}\cdot\frac{s_1}{s}\right],$$

or, since $\frac{s_1}{s} = \frac{\beta + p}{\beta + p_1}$,

$$A = Fs\,(\beta + p)\left[1 + log.\ nat.\left(\frac{s_1}{s}\right) - \frac{\beta + q}{\beta + p_1}\right]$$

$$= 144\,V\,(\beta + p)\left[1 + log.\ nat.\left(\frac{s_1}{s}\right) - \frac{\beta + q}{\beta + p_1}\right] \text{ ft. lbs.},$$

where V is the quantity of steam, in cubic feet, used per stroke.

The delivery per second is, for n revolutions per minute,

$$L = \frac{n}{30} A$$

$$= \frac{n}{30} \cdot 144\,V\,(\beta + p)\left[1 + log.\ nat.\left(\frac{s_1}{s}\right) - \frac{\beta + q}{\beta + p_1}\right]$$

$$= 144\,Q\,(\beta + p)\left[1 + log.\ nat.\left(\frac{s_1}{s}\right) - \frac{\beta + q}{\beta + p_1}\right] \text{ ft. lbs.},$$

where Q is the quantity of steam in cubic feet used per second.

If we make $\beta = 0$, this formula becomes identical with that already deduced from Mariotte's law.

EXAMPLE.—What is the delivery for the machine in the previous example according to the formula just found?

Here we have

$$144\,Q = \frac{n}{30} \cdot Fs = \tfrac{54}{30} \times 254{\cdot}47 \times \tfrac{4}{3} = 271{\cdot}44 \text{ cubic ft.};$$

also, according to § 390, $\beta = 0{\cdot}2922 \times 14{\cdot}7 = 4{\cdot}295$. Hence

$$\beta + p = 4{\cdot}295 + 3{\cdot}5 \times 14{\cdot}7 = 4{\cdot}295 + 51{\cdot}45 = 55{\cdot}745,$$

$$\beta + p_1 = \frac{s_1}{s}(\beta + p) = 0{\cdot}4 \times 55{\cdot}745 = 22{\cdot}298$$

and

$$\beta + q = 4{\cdot}295 + 14{\cdot}7 = 18{\cdot}995.$$

Hence the work required is

$$L = 271{\cdot}44 \times 55{\cdot}745 \left[1 + 0{\cdot}91630 - \frac{18{\cdot}995}{22{\cdot}298} \right]$$

$$= 271{\cdot}44 \times 55{\cdot}745 \left[1{\cdot}91630 - 0{\cdot}85188 \right] = 271{\cdot}44 \times 55{\cdot}745 \times 1{\cdot}06442$$

$$= 16106 \text{ ft. lbs.} = 29{\cdot}28 \text{ horse-power.}$$

The formula of the preceding article gave for the same case 30·5 horse-power.

§ 482.—Expansion in Two Cylinders. — The formula for the delivery of a compound engine can now be easily deduced. We assume that the steam acts without expansion in the small cylinder. Let the piston area in this cylinder be F and its stroke s, the area of the larger piston be F_1 and its stroke s_1. Also, let p be the tension of the steam in smaller cylinder, p_1 the tension of the expanded steam, and the back pressure per square inch upon the larger piston be q. We have, then, for each stroke, the work of the steam upon the small piston,

$$A_1 = F p s,$$

and the work lost by the back pressure upon the large piston

$$A_2 = F_1 q s_1.$$

Also the work during expansion, from Mariotte's law,

$$A_2 = V p \, log.\ nat. \frac{V_1}{V} = F s p \, log.\ nat. \left(\frac{F_1 s_1}{F s} \right).$$

Accordingly the entire work of both pistons during a single stroke is

$$A = A_1 - A_2 + A_3$$

$$= F s p \left[1 + log.\ nat. \left(\frac{F_1 s_1}{F s} \right) \right] - F_1 s_1 q$$

$$= F s p \left[1 + log.\ nat. \left(\frac{F_1 s_1}{F s} \right) - \frac{q}{p_1} \right]$$

$$= 144\, V p \left[1 + log.\ nat. \left(\frac{F_1 s_1}{F s} \right) - \frac{q}{p_1} \right]$$

$$= 144\, V p \left[1 + log.\ nat. \left(\frac{F_1 s_1}{F s} \right) - \frac{q}{p} \left(\frac{F_1 s_1}{F s} \right) \right] \text{ ft. lbs.}$$

The delivery per second is, therefore,

$$L = \frac{n}{30} \times 144\, V p \left[1 + log.\ nat. \left(\frac{F_1 s_1}{F s} \right) - \frac{q}{p_1} \right]$$

$$= 144\, Q p \left[1 + log.\ nat. \left(\frac{F_1 s_1}{F s} \right) - \frac{q}{p_1} \right] \text{ ft. lbs.}$$

If, according to Pambour, we start with Navier's rule, we obtain

$$L = 144\, Q (\beta + p) \left[1 + log.\ nat. \left(\frac{F_1 s_1}{F s} \right) - \frac{\beta + q}{\beta + p_1} \right] \text{ ft. lbs.}$$

Remark.—The work, during expansion, in a Woolf engine may be divided into work done and work lost. The first lifts the piston in the large and the second opposes the motion of the piston in the small cylinder. The above delivery is the difference. According to Mariotte's law, the delivery of the large piston during expansion is

$$\frac{F F_1 s s_1}{F_1 s_1 - F s}\, p\, log.\ nat. \left(\frac{F_1 s_1}{F s} \right),$$

and the loss of work of the small piston is

$$\frac{F^2 s^2}{F_1 s_1 - F s}\, p\, log.\ nat. \left(\frac{F_1 s_1}{F s} \right).$$

Hence the ratio of one to the other is $\frac{F_1 s_1}{F s}$, and their difference is, as above,

$$F p s\, log.\ nat. \left(\frac{F_1 s_1}{F s} \right).$$

Example.—What is the mechanical effect of a Woolf engine using steam at $3\frac{1}{2}$ atmospheres and discharging it into the condenser under $\frac{1}{3}$ atmosphere,

the dimensions being as follows: Diameter of small cylinder $d = 18$ inches, and stroke $s = 40$ inches; diameter of large cylinder $d_1 = 30$ inches, and stroke $s_1 = 50$ inches?

The expansion ratio is

$$\frac{V_1}{V} = \frac{F_1 s_1}{F s} = \frac{d_1^2 s_1}{d^2 s} = \frac{30^2 \cdot 50}{18^2 \cdot 40} = \frac{5^2 \cdot 5}{3^2 \cdot 4} = \frac{125}{36} = 3{\cdot}4722.$$

The first formula, based upon Mariotte's law, gives for the delivery per second, when the number of revolutions per minute is 24,

$$L = \tfrac{24}{60} \times (9)^2 \pi \times \tfrac{40}{12} \times 3{\cdot}5 \times 14{\cdot}7 \left(1 + \textit{log. nat.}\ 3{\cdot}4722 - \tfrac{1}{8} \cdot \frac{3{\cdot}4722}{3{\cdot}5}\right)$$

$$= 0{\cdot}8 \times 270 \times 51{\cdot}45\, \pi\, (1 + 2{\cdot}3026 \times 0{\cdot}5406 - 0{\cdot}1240)$$

$$= 11113\, \pi\, (0{\cdot}8760 + 1{\cdot}2448) = 11113 \times 2{\cdot}1208\, \pi$$

$$= 74042 \text{ ft. lbs.} = 134{\cdot}62 \text{ horse-power.}$$

According to Pambour's theory, we have

$$L = \tfrac{24}{60} \times (9)^2 \pi \times \tfrac{40}{12} \times 55{\cdot}745 \left(2{\cdot}2448 - \frac{4{\cdot}295 + \frac{1}{8} \times 14{\cdot}7}{55{\cdot}745} \times 3{\cdot}4722\right)$$

$$= 0{\cdot}8 \times 270 \times 55{\cdot}745\, \pi \left(2{\cdot}2448 - \frac{6{\cdot}1325 \times 3{\cdot}4722}{55{\cdot}745}\right)$$

$$= 12041\, \pi\, (2{\cdot}2448 - 0{\cdot}38197) = 12041 \times 1{\cdot}86283\, \pi$$

$$= 70466 \text{ ft. lbs.} = 128{\cdot}12 \text{ horse-power.}$$

§ **483.—Third Law of Expansion.**—If we assume that the tension of the steam during expansion is inversely as some power of the volume, we obtain for the work of the steam an expression similar to that for air (§ 378). If p is the tension before expansion, s the travel of the piston before expansion, and ν a number derived from experiment, we have for the tension corresponding to the piston travel x,

$$y = \left(\frac{s}{x}\right)^{\nu} p,$$

and hence the entire pressure upon the piston area F is

$$F y = F \left(\frac{s}{x}\right)^{\nu} p.$$

If, now, the piston moves through the elementary distance σ, it performs the elementary work,

$$F y \sigma = F p \left(\frac{s}{x}\right)^{\nu} \sigma = F p s^{\nu} x^{-\nu} \sigma,$$

and hence the work performed during the distance $x - s$ is

$A_1 = F p s_{\nu} \sigma$ times the sum of all the values of $x^{-\nu}$

$$= F p s^{\nu} \sigma \left[s^{-\nu} + (s+\sigma)^{-\nu} + (s+2\sigma)^{-\nu} + \ldots + x^{-\nu}\right]$$

$$= F p s^{\nu} \sigma \left\{ \begin{array}{c} (\sigma)^{-\nu} + (2\sigma)^{-\nu} + (3\sigma)^{-\nu} + \ldots + (m\sigma)^{-\nu} \\ + (m+1)\sigma^{-\nu} + \ldots + (n\sigma)^{-\nu} - (\sigma)^{-\nu} + (2\sigma)^{-\nu} \\ + (3\sigma)^{-\nu} + \ldots + (m\sigma)^{-\nu} \end{array} \right\}$$

$$= F p s^{\nu} \sigma \left\{ \begin{array}{l} (\sigma)^{-\nu}\left[1^{-\nu} + 2^{-\nu} + 3^{-\nu} + \ldots + m^{-\nu} + (m+1)^{-\nu} \right. \\ \left. + \ldots + n^{-\nu}\right] - \sigma^{-\nu}\left[1^{-\nu} + 2^{-\nu} + 3^{-\nu} + \ldots + m^{-\nu}\right], \end{array} \right\}$$

or, since

$$1^{-\nu} + 2^{-\nu} + 3^{-\nu} + \ldots + m^{-\nu} = \frac{m^{-\nu+1}}{-\nu+1},$$

$$A_1 = F p s^{\nu} \sigma^{-\nu+1} \left(\frac{n^{-\nu+1}}{-\nu+1} - \frac{m^{-\nu+1}}{-\nu+1}\right)$$

$$= \frac{F p s^{\nu} \sigma^{-\nu+1}}{\nu - 1} \left(m^{-\nu+1} - n^{-\nu+1}\right);$$

hence, since $s = m\sigma$ and $x = n\sigma$, or $m = \frac{s}{\sigma}$ and $n = \frac{x}{\sigma}$,

$$A_1 = \frac{F p s^{\nu} \sigma^{-\nu+1}}{\nu - 1} \left(\frac{s^{-\nu+1} - x^{-\nu+1}}{-\nu+1}\right)$$

$$= \frac{F p s^{\nu}}{\nu - 1} \left(\frac{1}{s^{\nu-1}} - \frac{1}{x^{\nu-1}}\right);$$

or placing for x the entire stroke s_1,

$$A_1 = \frac{F\,p\,s^{\nu}}{\nu - 1}\left(\frac{1}{s^{\nu-1}} - \frac{1}{s_1^{\nu-1}}\right).$$

If we add the work $F\,p\,s$ before expansion, and take into account the work consumed by the counter pressure $F\,q$, we have for the work per stroke,

$$A = F\,p\,s + \frac{F\,p\,s^{\nu}}{\nu - 1}\left(\frac{1}{s^{\nu-1}} - \frac{1}{s_1^{\nu-1}}\right) - F\,q\,s_1$$

$$= F\,p\,s\left[1 + \frac{1}{\nu - 1} - \frac{1}{\nu - 1}\left(\frac{s}{s_1}\right)^{\nu-1}\right] - F\,q\,s_1$$

$$= F\,p\,s\left[1 + \frac{1}{\nu - 1} - \frac{1}{\nu - 1}\left(\frac{s}{s_1}\right)^{\nu-1} - \frac{q\,s_1}{p\,s}\right].$$

According to Rankine ("Manual of Applied Mechanics"), for steam below 12 atmospheres, ν is approximately $= \frac{10}{9}$, hence $\nu - 1 = \frac{1}{9}$, and

$$A = F\,s\,p\left[10 - 9\left(\frac{s}{s_1}\right)^{\frac{1}{9}} - \frac{q\,s_1}{p\,s}\right].$$

If the machine makes n revolutions per minute, the steam used per second is

$$Q = \frac{2\,n}{60}\,F\,s = \frac{n\,F\,s}{30},$$

and the work per second is

$$L = Q\,p\left[10 - 9\left(\frac{s}{s_1}\right)^{\frac{1}{9}} - \frac{q\,s_1}{p\,s}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or if the pressure p is per square inch,

$$L = 144\,Q\,p\left[10 - 9\left(\frac{s}{s_1}\right)^{\frac{1}{9}} - \frac{q\,s_1}{p\,s}\right] \text{ ft. lbs.}$$

For $s_1 = s$, $\frac{s}{s_1} = 1$, and

$$L = 144\ Q\ p \left(1 - \frac{q}{p}\right) \text{ ft. lbs.},$$

as given in § 478.

For a Woolf or compound engine,

$$L = 144\ Q\ p \left[10 - 9 \left(\frac{F\ s}{F_1\ s_1}\right)^{\frac{1}{9}} - \frac{q\ F_1\ s_1}{p\ F\ s}\right] \text{ ft. lbs.}$$

According to Grashof, $\nu = 1 \cdot 14$, and Zeuner finds $\nu = 1 \cdot 135$. If we introduce then the value $\nu = 1 \cdot 135$, or $\nu - 1 = 0 \cdot 135$ and $\frac{1}{\nu - 1} = \frac{1}{0 \cdot 135} = 7 \cdot 4074$, we have

$$L = Q\ p \left[8 \cdot 4074 - 7 \cdot 4074 \left(\frac{s}{s_1}\right)^{0 \cdot 135} - \frac{q\ s_1}{p\ s}\right] \quad . \quad . \quad (2).$$

Example 1.—For the engine in the examples of §.480 and § 481,

$\frac{s_1}{s} = \frac{40}{16} = \frac{5}{2}$, $\frac{q}{p} = \frac{1}{3\frac{1}{2}} = \frac{2}{7}$ and $144\ Q\ p = \frac{n}{30} \times 17456 = 17456 \times \frac{24}{30} = 13965$ lbs

Hence the theoretical delivery, according to Rankine, is

$$L = 13965\ (10 - 8 \cdot 1288 - 0 \cdot 7143) = 13965 \times 1 \cdot 1569 = 16156 \text{ ft. lbs.}$$

$$= 29 \cdot 37 \text{ horse-power.}$$

The calculation, according to Mariotte's law, gave

$$L = 30 \cdot 5 \text{ horse-power,}$$

and that according to Pambour's formula,

$$L = 29 \cdot 28 \text{ horse-power.}$$

Example 2.—For the Woolf engine in § 482, we have

$$\frac{V_1}{V} = \frac{F_1\ s_1}{F\ s} = 3 \cdot 4722,\ \frac{q}{p} = \frac{1}{8 \times 3 \cdot 5} = \frac{1}{28},$$

and

$$144\ Q\ p = 11113\ \pi;$$

hence,

$$L = 11113\,\pi \left(10 - \frac{9}{\sqrt[9]{3 \cdot 4722}} - \frac{1}{10} \times 3 \cdot 4722\right)$$

$$= 11113\,\pi\,(10 - 7 \cdot 8375 - 0 \cdot 1240)$$

$$= 11113\,\pi \times 2 \cdot 0385 = 71169 \text{ ft. lbs.} = 129 \cdot 4 \text{ horse-power.}$$

We have already found above, by means of the first formula,

$$L = 134 \cdot 62 \text{ horse-power,}$$

and by means of the second,

$$L = 128 \cdot 12 \text{ horse-power.}$$

EXAMPLE 3.—For the single cylinder engine in the preceding examples, where $\frac{s_1}{s} = \frac{40}{10} = \frac{5}{2}$, $\frac{q}{p} = \frac{2}{7}$ and $144\,Qp = 13965$, we have, according to formula (2),

$$L = 13965\,[8 \cdot 4074 - 7 \cdot 4074\,(0 \cdot 4)^{0 \cdot 135} - 0 \cdot 7143]$$

$$= 13965\,(8 \cdot 4074 - 7 \cdot 2597) = 13965 \times 1 \cdot 1477$$

$$= 16027 \text{ ft. lbs.} = 29 \cdot 14 \text{ horse-power,}$$

while we have found above, by means of the other formulæ,

$$L = 29 \cdot 37,\ 30 \cdot 5, \text{ and } 29 \cdot 28 \text{ horse-power.}$$

§ 484.—Application of the Mechanical Theory of Heat.—If one unit in weight (one pound) of water has the volume σ, and is converted, under the constant pressure p, into steam of the volume ρ, the work performed by the steam is

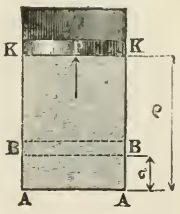
$$L = p\,(\rho - \sigma).$$

This is easily seen if we suppose that the piston $K\,K$, Fig. 784, whose area is unity, moves through the distance $B\,K = A\,K - A\,B = \rho - \sigma$ while overcoming the constant pressure p.

According to the mechanical theory of heat, the amount of heat equivalent to this work is

$$m = \frac{L}{A} = \frac{1}{A}\, p\,(\rho - \sigma),$$

Fig. 784.



where A is the mechanical equivalent of heat (§ 379). We can therefore write

$$m = \frac{1}{A}\, p\, u,$$

if we denote the difference $\rho - \sigma$ between the steam and water volumes by u.

This heat, which disappears during the performance of the work, is only a portion of the so-called *latent heat* or total heat of vaporization $W - \omega\, t$, which we denote by w. According to Regnault (§ 380), we have for this total latent heat,

$$w = W - \omega\, t = 606{\cdot}50 - 0{\cdot}695\, t - 0{\cdot}00002\, t^2 - 0{\cdot}0000003\, t^3.$$

We call the $m = \frac{1}{A}\, p\, u$ the *outer latent heat*, while that portion of the latent heat which remains, viz.,

$$r = w - m = w - \frac{1}{A}\, p\, u,$$

we call the *inner latent heat*.

According to Prof. Zeuner, we have,

$$r = 575{\cdot}40 - 0{\cdot}791\, t, \text{ and hence}$$

$$m = \frac{1}{A}\, p\, u = w - r$$

$$= 31{\cdot}10 + 0{\cdot}096\, t - 0{\cdot}00002\, t^2 - 0{\cdot}0000003\, t^3,$$

where t is the temperature of the steam generated from water at zero degrees centigrade.

From $m = w - r$, we obtain

$$u = \frac{A\,m}{p} = \frac{A}{p}(w - r),$$

and hence the volume of the unit of weight of steam

$$\rho = u + \sigma = \frac{A\,m}{p} + \sigma = \frac{424\,m}{p} + 0{\cdot}001,$$

since it may be assumed that 1 kilogramme of water has the volume $\sigma = 1$ decimeter $= 0{\cdot}001$ cubic meter, and that the mechanical equivalent is $A = 424$ meter kilogrammes.

We have now the so-called specific steam volume—*i.e.*, the ratio of the steam volume to that of the water for equal weights,

$$\mu = \frac{\rho}{\sigma} = 1 + \frac{424\,m}{\sigma\,p} = 1 + \frac{424000\,m}{p}$$

$$= 1 + \frac{424000}{p}(31{\cdot}10 + 0{\cdot}096\,t - 0{\cdot}00002\,t^2 - 0{\cdot}0000003\,t^3),$$

or if we express p in atmospheres, at 10335 kilogrammes per square meter,

$$\mu = 1 + \frac{1275{\cdot}9 + 3{\cdot}9385\,t - 0{\cdot}00082051\,t^2 - 0{\cdot}000012308\,t^3}{p},$$

as already given in § 391.

We have, according to Zeuner, with tolerable accuracy, when the steam pressure p is given in atmospheres,

$$p\,\rho^{1{\cdot}0646} = 1{\cdot}704,$$

whence

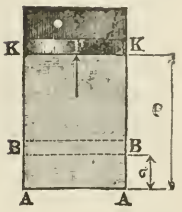
$$\rho = 1{\cdot}6498\,p^{-0{\cdot}9393},$$

and, therefore,

$$\mu = 1649{\cdot}8\,p^{-0{\cdot}9393}.$$

§ **485.**—If in a vessel AKK, Fig. 785, we have a unit's weight of liquid, of which a portion ξ is in the state of steam, and the remaining portion $1 - \xi$ is water, we have, according to the preceding, the volume of the steam

Fig. 785.



$$v_1 = \xi \rho,$$

and that of the water is

$$v_2 = (1 - \xi)\sigma;$$

hence, that of the mixture is

$$v = v_1 + v_2 = \xi \rho + (1 - \xi)\sigma = \xi(\rho - \sigma) + \sigma = \xi u + \sigma.$$

In order to increase v by the elementary amount $dv = u\,d\xi$, we must add the amount of heat,

$$dQ = w\,d\xi = \frac{w\,dv}{u} \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot (1).$$

According to the mechanical theory of heat, if Y is a function of the temperature and pressure,

$$dQ = \frac{1}{A} Y\,dv,$$

and

$$\frac{Y}{T} = \frac{dp}{dt},$$

where $T = a + t = 273° + t =$ the absolute temperature, and dp the elementary increase of pressure corresponding to dt. We have, therefore,

$$dQ = \frac{1}{A} T \frac{dp}{dt} dv \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot (2),$$

or, from (1) and (2),

$$\frac{w}{u} = \frac{T\,dp}{A\,dt} = \frac{a + t}{A}\left(\frac{dp}{dt}\right) \quad \cdot \cdot \cdot \cdot \cdot (3)$$

The heat in the mixture of steam, ξ, and water, $1 - \xi$, is $n + \xi r$, or the sum of the heat of the fluid $n = \int w\, dt$ and the inner latent heat ξr. This becomes $n_1 + \xi_1 r_1$, when the temperature t becomes t_1, the steam quantity ξ becomes ξ_1, the heat of the liquid n, n_1, and the inner latent heat r, r_1. For this change of condition, then, the original heat of the mixture is increased or diminished by

$$n_1 + \xi_1 r_1 - (n + \xi r)$$

or

$$n_1 - n + \xi_1 r_1 - \xi r,$$

according as t_1 is greater or less than t.

The corresponding *inner work* of the heat is

$$L = A\,(n_1 - n + \xi_1 r_1 - \xi r) \quad . \quad . \quad . \quad . (4);$$

hence, placing for $n_1 - n$, $d\,n$, and $\xi_1 r_1 - \xi r$, $d\,(\xi r)$,

$$d\,L = A\,[d\,n + d\,(\xi r)].$$

If to this we add the work $p\,d\,v$ due to the expansion, we have for the work corresponding to the change of condition,

$$d\,L = A\,[d\,n + d\,(\xi r)] + p\,d\,v,$$

and inversely, for the necessary amount of heat imparted to the liquid,

$$d\,Q = \frac{d\,L}{A} = d\,n + d\,(\xi r) + \frac{1}{A}\,p\,d\,v.$$

Since, now, $v = \xi u + \sigma$,

$$p\,d\,v = p\,d\,(\xi u) = d\,(\xi p\,u) - \xi u\,d\,p,$$

and, therefore,

$$d\,Q = d\,n + d\,(\xi r) + \frac{1}{A}\,[d\,(\xi p\,u) - \xi u\,d\,p].$$

From the preceding,

$$r = w - m = w - \frac{1}{A} p u,$$

therefore

$$\xi r = \xi w - \frac{1}{A} \xi p u,$$

and

$$d(\xi r) = d\left(\xi w - \frac{1}{A} \xi p u\right).$$

Also,

$$\frac{u T}{A} d p = w d t, \qquad \text{or} \qquad \frac{u d p}{A} = \frac{w d t}{T};$$

hence,

$$d Q = d n + d(\xi w) - \frac{\xi w d t}{T}.$$

Further, according to the known differential formula,

$$d\left(\frac{x}{y}\right) = \frac{y d x - x d y}{y^2},$$

$$d(\xi w) - \frac{\xi w d t}{T} = \frac{T d(\xi w) - \xi w d(T - a)}{T}$$

$$= T\left(\frac{T d(\xi w) - \xi w d T}{T^2}\right)$$

$$= T d\left(\frac{\xi w}{T}\right);$$

hence we have also,

$$d Q = d n + T d\left(\frac{\xi w}{T}\right) \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot (5).$$

Finally,

$$d n = w d t$$

and

$$x d n = w x d t;$$

hence

$$dQ = dn + d(\xi w) - \frac{\xi w\, dt}{T}$$

$$= w\, dt - \xi w\, dt + w\, d\xi + \xi\, dw + \xi w\, dt - \frac{\xi w\, dt}{T}$$

$$= (1 - \xi) w\, dt + w\, d\xi + \xi \left(w + \frac{dw}{dt} - \frac{w}{T} \right) dt.$$

We thus have, according to Clausius,

$$dQ = (1 - \xi) w\, dt + w\, d\xi + \xi h\, dt \quad . \quad . \quad . \quad . \quad . \quad . \quad (6),$$

where h denotes the expression

$$w + \frac{dw}{dt} - \frac{w}{T},$$

which depends upon the temperature t.

In formula (6), the member $(1 - \xi) w\, dt$ gives that portion of the heat which goes to raise the temperature of the liquid $(1 - \xi)$. Further, the member $w\, dt$ gives the expenditure of heat necessary to convert the quantity of liquid $d\xi$ into steam, and, finally, $\xi h\, dt$ gives that portion of the heat dQ which goes to increase the temperature of the steam, and may be regarded as the specific heat of the steam.

§ 486.—**The Adiabatic Law of Pressure.**—If, during the expansion or compression of a fluid, heat is neither added nor abstracted, the pressure p changes according to the so-called adiabatic law, and the curve of pressure is the adiabatic curve. For atmospheric air, this law corresponds with that of Poisson given in § 376. There is a special law, however, for steam which is expressed by equation (5) of the preceding article if dQ is made equal to zero. We thus obtain

$$dn + T d\left(\frac{\xi w}{T}\right) = 0,$$

or

$$d\left(\frac{\xi w}{T}\right) = -\frac{dn}{T};$$

hence,

$$\frac{\xi w}{T} = -\int\frac{dn}{T} = -\tau, \qquad \text{or} \qquad \frac{\xi w}{T} + \tau = 0,$$

if we denote the integral $\int\frac{dn}{T}$ between the limits $T = 0$ and $T = t$ by τ.

Since $n = \omega t = t + 0{\cdot}00002\ t^2 + 0{\cdot}0000003\ t^3$ (see § 380), we have

$$dn = (1 + 0{\cdot}00004\ t + 0{\cdot}0000009\ t^2)\ dt\cdot$$

hence,

$$\frac{dn}{T} = (1 + 0{\cdot}00004\ t + 0{\cdot}0000009\ t^2)\frac{dt}{T}$$

$$= [1 + 0{\cdot}00004\,(T - a) + 0{\cdot}0000009\,(T - a)^2]\ \frac{dT}{T}$$

$$= \frac{1 - 0{\cdot}00004\ a + 0{\cdot}0000009\ a^2}{T}\ dT + (0{\cdot}00004$$

$$- 0{\cdot}0000018\ a)\ dT + 0{\cdot}0000009\ T\,dT;$$

so that if we put $a = 273$,

$$\frac{dn}{T} = 1{\cdot}05615\ \frac{dT}{T} - 0{\cdot}0004514\ dT + 0{\cdot}0000009\ T\,dT.$$

Hence

$$\int\frac{dn}{T} = 1{\cdot}05615\ \textit{log. nat.}\ T - 0{\cdot}0004514\ T + 0{\cdot}00000045\ T^2,$$

and the temperature function sought is

$$(1)\quad \tau = \int_a^T \frac{dn}{T} = 1{\cdot}05615\ \textit{log. nat.}\left(\frac{T}{a}\right)$$

$$- 0{\cdot}0004514\,(T-a) + 0{\cdot}00000045\,(T^2 - a^2)$$

$$= 1{\cdot}05615\ \textit{log. nat.}\left(\frac{a+t}{a}\right) - 0{\cdot}0004514\,t$$

$$+ 0{\cdot}00000045\,(2\,a\,t + t^2).$$

If we take the approximate mean value of $\omega = 1{\cdot}0224$, we have

$$(2)\quad \tau = \omega \int_a^T \frac{a\,T}{T} = \omega\ \textit{log. nat.}\left(\frac{T}{a}\right)$$

$$= 1{\cdot}0224\ \textit{log. nat.}\left(\frac{a+t}{a}\right).$$

If for a certain initial temperature $T_1 = a + t_1 = 273 + t_1$, the values of w, n, and ξ are w_1, n_1, and ξ_1, we have

$$\frac{\xi_1\, w_1}{T_1} + \tau_1 = 0,$$

and hence

$$(3)\quad \frac{\xi\, w}{T} + \tau = \frac{\xi_1\, w_1}{T_1} + \tau_1.$$

If the values of ξ_1, w_1, T_1, and τ_1 are known for the initial condition, we can find by the last formula the specific steam quantity for any other temperature,

$$\xi = \frac{T}{w}\left(\frac{\xi_1\, w_1}{T_1} + \tau_1 - \tau\right).$$

If we also know the difference of volumes,

$$(4)\quad u = \rho - \sigma = \frac{A\,m}{p} = \frac{424}{p}\,(31\cdot1 + 0\cdot096\,t - 0\cdot00002\,t^2 - 0\cdot0000003\,t^3),$$

we can calculate the volume of the mixture of steam and water in both cases, viz.,

$$v_1 = \xi_1\,u_1 + \sigma \qquad \text{and} \qquad v = \xi\,u + \sigma,$$

as also the expansion or compression ratio,

$$\varepsilon = \frac{v}{v_1} = \frac{\xi\,u + \sigma}{\xi_1\,u_1 + \sigma_1}.$$

If ξ is found less than ξ_1, it follows that during the change of volume the specific steam quantity diminishes, and that therefore there is a partial condensation of the steam, as is generally the case in the cylinder of a steam engine.

EXAMPLE.—If the cylinder contains 1 kilogramme of saturated steam of 4 atmospheres, without admixture of water, and by expansion the tension is reduced to 1 atmosphere: what is the change of specific steam volume, density, etc.?

Here $p_1 = 4$ atmospheres and the temperature of the steam is $t_1 = 144°$ (see Table II., Art. 388); hence

$$T_1 = 273 + 144 = 417°,$$

and, according to formula (1),

$$\tau_1 = 1\cdot05615\ log.\ nat.\left(\frac{417}{273}\right) - 0\cdot0004514 \times 144 + 0\cdot00000045\,(144 \times 546 + 144^2)$$

$$= 0\cdot4474 - 0\cdot0650 + 0\cdot0447 = 0\cdot4271.$$

Also, for $t_1 = 144°$,

$$w_1 = 606\cdot5 - 0\cdot695 \times 144 - 0\cdot00002 \times 144^2 - 0\cdot0000003 \times 144^3$$

$$606\cdot5 - 100\cdot08 - 0\cdot41 - 0\cdot90 = 505\cdot11;$$

and hence

$$\frac{\xi_1 w_1}{T_1} + \tau_1 = \frac{505 \cdot 11}{417} + 0 \cdot 4271 = 1 \cdot 6384.$$

After the expansion, the steam pressure is $p = 1$ atmosphere; hence, under the supposition that the steam is still in the saturated state, the temperature is $t = 100°$ and $T = 373°$. Hence,

$$\tau = 1 \cdot 05615 \; log.\, nat, \left(\frac{373}{273}\right) - 0 \cdot 0004514 \times 100 + 0 \cdot 00000045 \, (100 \times 546 + 100^2)$$

$$= 0 \cdot 32964 - 0 \cdot 01607 = 0 \cdot 31356,$$

$$w = 606 \cdot 5 - 69 \cdot 50 - 0 \cdot 20 - 0 \cdot 30 = 536 \cdot 5,$$

and

$$\frac{\xi w}{T} + \tau = \frac{536 \cdot 5 \, \xi}{373} + 0 \cdot 31356 = \frac{\xi_1 w_1}{T_1} + \tau_1 = 1 \cdot 6384.$$

The solution of this equation gives for the specific steam quantity, after expansion,

$$\xi = \frac{(1 \cdot 6384 - 0 \cdot 31356) \times 373}{536 \cdot 5} = 0 \cdot 9211 \text{ kilogrammes.}$$

Since the original steam quantity was 1 kilogramme, 0·0789 kilogrammes have been condensed during the expansion, and hence the assumption that the steam remains saturated is justified.

Further, we have for the volume difference,

$$u_1 = \rho_1 - \sigma = \frac{424}{p_1} \, (31 \cdot 1 + 0 \cdot 096 \, t - 0 \cdot 00002 \, t^2 - 0 \cdot 0000003 \, t^3) \, ;$$

or, since the steam pressure per square meter, $p = 10335$ kilogrammes,

$$u_1 = \frac{424}{4 \times 10335} \, (31 \cdot 1 + 0 \cdot 096 \times 144 - 0 \cdot 00002 \times 144^2 - 0 \cdot 0000003 \times 144^3)$$

$$= \frac{106}{10335} \times 43 \cdot 613 = 0 \cdot 4474 \, ;$$

hence the initial steam volume is

$$v_1 = u_1 + \sigma = 0 \cdot 4484,$$

and the final volume is

$$u = \rho - \sigma = \frac{424}{10335}(31\cdot1 + 0\cdot096 \times 100 - 0\cdot00002 \times 100^2 - 0\cdot0000003 \times 100^3)$$

$$= \frac{424 \times 40\cdot2}{10335} = 1\cdot6492.$$

The volume of the steam and water is then

$$v = \xi u + \sigma = 1\cdot6492 \times 0\cdot9211 + 0\cdot0010 = 1\cdot5200,$$

and the expansion ratio is

$$\varepsilon = \frac{v}{v_1} = \frac{1\cdot5200}{0\cdot4484} = 3\cdot390.$$

If in the beginning we had 0·9 kilogrammes of steam and 0·1 kilogrammes of water, carried over with the steam, we should have

$$\frac{\xi_1 w_1}{T_1} + \tau_1 = \frac{0\cdot9 \times 505\cdot11}{417} + 0\cdot4271 = 1\cdot5173;$$

hence

$$\xi = \left(\frac{1\cdot5173 - 0\cdot3136}{536\cdot5}\right) \times 373 = 0\cdot8369 \text{ kilogrammes.}$$

Hence $0\cdot9000 - 0\cdot8369 = 0\cdot0631$ kilogrammes of steam would be condensed during expansion, and we should have

$$v_1 = \xi_1 u + 0\cdot001 = 0\cdot9 \times 0\cdot4474 + 0\cdot001 = 0\cdot4037,$$

and

$$v = \xi u + 0\cdot001 = 0\cdot8369 \times 1\cdot6492 + 0\cdot001 = 1\cdot3812,$$

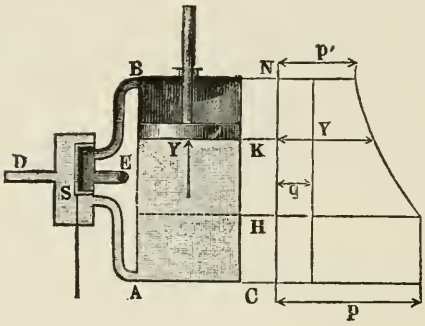
and the expansion ratio,

$$\varepsilon = \frac{v}{v_1} = \frac{1\cdot3812}{0\cdot4037} = 3\cdot419.$$

§ 487.—Theoretical Effect of a Steam Engine according to the Mechanical Theory of Heat.—In the steam engine $CABD$, Fig. 786, let s be the piston stroke CH, during full pressure p, s_1 the entire stroke CN, during which the pressure becomes p_1, and x the variable distance

CK, to which the varying tension y corresponds. Finally, let q be the back pressure which opposes the motion of the

FIG. 786.



piston during the entire stroke s_1. During the distance s, the pressure upon the piston, whose area is F, is $P = Fp$, and the work is

$$L_0 = Fps = Fsp = Vp = \frac{Mp}{\gamma} = \rho Mp,$$

where V is the volume of steam used, and $M = V\gamma = \frac{V}{\rho}$ is the weight of steam used.

When the steam is cut off by the slide S and the piston has passed through the distance $HN = s_1 - s$, the specific steam volume changes from ξ to ξ_1, the heat of the fluid from n to n_1, and the inner latent heat from r to r_1. According to the adiabatic law, therefore, which assumes that heat is neither added nor subtracted during the expansion, the work is

$$L_1 = AM(n - n_1 + \xi r - \xi_1 r_1)$$

(see formula (4), § 485).

The work consumed by the back pressure is

$$L_2 = Fqs_1 = Fs_1 q = \frac{Vs_1}{s} q = \frac{Ms_1 q}{\gamma s} = \rho Mq\frac{s_1}{s}.$$

Hence the total effect is

$$(1)\qquad L = L_0 + L_1 - L_2 = L_1 + L_0 - L_2$$
$$= A\,M(n - n_1 + \xi r - \xi_1 r_1) + V\left(p - \frac{s_1}{s} q\right)$$
$$= V\left[A\,\gamma\,(n - n_1 + \xi r - \xi_1 r_1) + p - \frac{s_1}{s} q\right]$$
$$= M\left[A\,(n - n_1 + \xi r - \xi_1 r_1) + \rho\left(p - \frac{s_1}{s} q\right)\right].$$

The specific steam volume of the expanded steam is to be found from the formula

$$(2)\qquad \xi_1 = \frac{T_1}{w_1}\left(\xi\frac{w}{T} + \tau - \tau_1\right).$$

If the engine makes n revolutions per minute, the steam quantity per second is

$$Q = \frac{2\,n\,F\,s}{60} = \frac{n\,V}{30},$$

and the work per second is therefore

$$(3)\qquad L = Q\left[A\,\gamma\,(n - n_1) + \xi r - \xi_1 r_1) + p - \frac{s_1}{s} q\right],$$

or, if p and q are the pressures per square inch, and the steam quantity Q is expressed in cubic feet,

$$(4)\ L = Q\left[A\,\gamma\,(n - n_1 + \xi r - \xi_1 r_1) + 144\left(p - \frac{s_1}{s} q\right)\right] \text{ ft. lbs.}$$

Example.—For the engine in the examples of § 480, etc., we had $p = 3\frac{1}{2}$ atmos., $q = 1$ atmos. $= 14{\cdot}7$ pounds per square inch, and the expansion ratio $\frac{s_1}{s} = \frac{5}{2}$; therefore

$$144\left(p - \frac{s_1}{s} q\right) = 144 \times 14{\cdot}7\,(3{\cdot}5 - 2{\cdot}5 \times 1) = 2116{\cdot}8 \text{ lbs.}$$

Also the area of piston was $F = 254{\cdot}47$ sq. inches and the portion of the stroke s was $0{\cdot}4 s_1 = 16$ inches ; therefore, the steam volume per stroke is

$$V = \frac{F s}{1728} = \frac{254{\cdot}47 \times 16}{1728} = \frac{254{\cdot}47}{108} = 2{\cdot}356 \text{ cubic ft.}$$

Since the engine makes $n = 24$ revolutions per minute, the steam quantity used per second is

$$Q = \frac{n V}{30} = 0{\cdot}8 \times 2{\cdot}356 = 1{\cdot}885 \text{ cubic ft.}$$

The temperature of saturated steam under a pressure of $3\frac{1}{2}$ atmospheres is $t = 139°$, and the absolute temperature is therefore $T = 273° + t = 412°$. Further, the temperature function

$$\tau = 1{\cdot}0224 \, \textit{log. nat.} \left(\frac{412}{273}\right) = 0{\cdot}4208,$$

and the density of the steam is, according to the Table, § 391,

$$\gamma = \frac{62{\cdot}5}{\mu} = \frac{62{\cdot}5}{508{\cdot}2} = 0{\cdot}1229 \text{ lbs.}$$

Assuming that the steam is still saturated at the end of the stroke, and that the pressure is then $p = 1{\cdot}3$ atmospheres, the temperature is $t_1 = 107{\cdot}5°$, so that $T_1 = 380{\cdot}5$ and $\tau_1 = 0{\cdot}3395$.

Since, now, the total latent heat or heat of vaporization for $t = 139°$ is

$$w = 606{\cdot}5 - 0{\cdot}695 \times 139 - 0{\cdot}00002 \times 139^2 - 0{\cdot}0000003 \times 139^3 = 508{\cdot}7,$$

and for

$$t_1 = 107{\cdot}5, \; w_1 = 531{\cdot}1, \quad \text{and} \quad \xi = 1$$

we have for the specific volume of the steam at the end of the stroke,

$$\xi_1 = \frac{380{\cdot}5}{531{\cdot}1}\left(\frac{508{\cdot}2}{412} + 0{\cdot}4208 - 0{\cdot}3395\right) = \frac{380{\cdot}5}{531{\cdot}1} \times 1{\cdot}316 = 0{\cdot}943.$$

Further, for $t = 139°$, the heat of the liquid is

$$n = 139 + 0{\cdot}00002 \times 139^2 + 0{\cdot}0000003 \times 139^3 = 140{\cdot}19$$

and for $t_1 = 107{\cdot}5°$ $n_1 = 107{\cdot}09$; hence

$$n - n_1 = 140{\cdot}19 - 107{\cdot}09 = 33{\cdot}10 \; ;$$

also for $t = 139°$, the inner latent heat is

$$r = 575{\cdot}4 - 0{\cdot}791 \, t = 575{\cdot}4 - 0{\cdot}791 \times 139 = 464{\cdot}5,$$

and for $t_1 = 107{\cdot}5°$, $r_1 = 490{\cdot}4$: hence since $\xi = 1{\cdot}00$ and $\xi_1 = 0{\cdot}943$,

$$\xi r - \xi_1 r_1 = 464{\cdot}5 - 0{\cdot}943 \times 490{\cdot}4 = 2{\cdot}10.$$

The mechanical equivalent of heat is $A = 1390$ ft. lbs. ; hence we have for the effect of the engine,

$$L = Q\left[A\gamma(n - n_1 + \xi r - \xi_1 r_1) + 144\left(p - \frac{s_1}{s}q\right)\right]$$
$$= 1\cdot885\,[1390 \times 0\cdot1229(33\cdot10 + 2\cdot10) + 2116\cdot8]$$
$$= 1\cdot885\,(6013\cdot2 + 2116\cdot8) = 1\cdot885 \times 8130 = 15325 \text{ ft. lbs.}$$

In the example of § 483, we found for the expansion ratio $\varepsilon = \frac{s_1}{s} = \frac{10}{4} = 2\cdot5$,

$$L = 16156 \text{ ft. lbs.},$$

while here

$$\varepsilon = \frac{\xi_1 u_1}{u} = 0\cdot943\,\frac{u_1}{u} = 0\cdot943\,\frac{1289}{508\cdot2} = 2\cdot39,$$

or $4\frac{1}{2}$ per cent less.

To facilitate the calculation of the steam engine according to the mechanical theory of heat, we append the following table, although in most cases it can only be applied by interpolation of intermediate values. As seen from formula (4), the values given in columns 7 and 10 are of especial service in such calculations.

TABLE

FOR CALCULATION OF THE EFFECT OF A STEAM ENGINE ACCORDING TO THE MECHANICAL THEORY OF HEAT.

Tension p in atmospheres.	Temperature t Centigrade.	Absolute Temperature $T = 273 + t$.	Temperature Function τ.	Specific Steam Volume μ.	Total heat H'.	Heat of Liquid u.	Vaporization Heat $w = H' - u$.	Outer Latent Heat $w_1 = \frac{1}{A}ps$.	Inner Latent Heat $r = w - w_1$.
0·1	46·2	319·2	0·1584	14556	620·6	46·3	574·3	35·5	538·8
0·5	81·7	354·7	0·2627	3172	631·4	82·0	549·4	38·6	510·8
1·0	100·0	373·0	0·3136	1650	637·0	100·5	536·5	40·2	496·3
1·5	111·7	384·7	0·3475	1127	640·6	112·4	528·2	41·2	487·0
2·0	120·6	393·6	0·3681	859·8	643·3	121·4	521·9	41·9	480·0
2·5	127·8	400·8	0·3890	697·1	645·5	128·8	516·7	42·4	474·3
3·0	133·9	406·9	0·4020	587·4	647·3	135·0	512·3	42·9	469·4
4·0	144·0	417·0	0·4271	448·4	650·4	145·3	505·1	43·6	461·3
5·0	152·2	425·2	0·4469	363·6	652.9	153·7	499·2	44·2	455·0
6·0	159·2	432·2	0·4639	306·4	655·1	160·9	494·2	44·7	449·5
7·0	165·3	438·3	0·4784	265·2	656·9	167·2	489·7	45·1	444·6
8·0	170·8	443·8	0·4912	233·9	658·6	172·9	485·7	45·4	440·3

§ **488.—Amount of Fuel.**—In the preceding, we have determined the efficiency of a steam engine from the quantity of steam used and its tension, but since this last depends upon the quantity of heat, and this again upon the consumption of fuel, we may express the efficiency in terms of this last.

If we put the specific steam volume, or the ratio of the steam volume to the water volume,

$$\mu = \frac{\alpha}{\beta + p},$$

we have, for the quantity of water in the steam Q,

$$Q_1 = \left(\frac{\beta + p}{\alpha}\right) Q,$$

and its weight, since one cubic foot of water weighs $62 \cdot 5$ lbs.,

$$Q_1 \gamma = 62 \cdot 5 \left(\frac{\beta + p}{\alpha}\right) Q \text{ lbs.}$$

According to § 401, the amount of heat necessary to convert $Q_1 \gamma$ pounds of water from $t°_1$ into steam at $t°$ is

$$W = (606 \cdot 5 + 0 \cdot 305\, t - t_1)\, Q_1 \gamma \text{ calories.}$$

If we take, however, as a mean value

$$W = (640 - t_1)\, Q_1 \gamma,$$

we have

$$W = 62 \cdot 5\, (640 - t_1) \frac{\beta + p}{\alpha} Q,$$

or, inversely,

$$Q = \frac{\alpha}{62 \cdot 5\, (640 - t_1)(\beta + p)}.$$

If we know the number ψ of heat-units given by the combustion of one pound of fuel, as given by the Table,

§ 400, we can find the number of pounds of fuel K necessary for the formation of the amount of steam Q,

$$K = \frac{W}{\psi} = 62 \cdot 5\,(640 - t_1)\,\frac{\beta + p}{\alpha\,\psi}\,Q,$$

or, inversely,

$$Q = \frac{\alpha\,\psi\,K}{62 \cdot 5\,(640 - t_1)\,(\beta + p)}.$$

If we assume that one pound of carbon gives 7500 heat-units, and that only 60 per cent takes effect (compare § 399), and put for t_1 the mean value 40°, we obtain

$$Q = \frac{0 \cdot 6 \times 7500\,\alpha\,K}{62 \cdot 5 \times 600\,(\beta + p)} = \frac{3}{25}\,\frac{\alpha}{\beta + p}\,K,$$

or

$$K = \frac{25}{3}\,\frac{\beta + p}{\alpha}\,Q.$$

For condensing engines, according to Pambour,

$$\mu = \frac{\alpha}{\beta + p} = \frac{27283}{1 \cdot 637 + p},$$

and for non-condensing,

$$\mu = \frac{\alpha}{\beta + p} = \frac{28961}{4 \cdot 120 + p};$$

therefore, in the first case,

$$(1) \qquad Q = \tfrac{3}{25}\,\frac{27283}{1 \cdot 637 + p}\,K = \frac{3274\,K}{1 \cdot 637 + p},$$

and in the second,

$$(2) \qquad Q = \tfrac{3}{25}\,\frac{28961}{4 \cdot 120 + p}\,K = \frac{3475\,K}{4 \cdot 120 + p}.$$

We may also calculate the specific steam volume according to the formula,

$$\mu = 1649 \cdot 8\,p^{-0 \cdot 9393},$$

or take it from the Table, § 391.

REMARK.—If we calculate from the formula given in § 393, Vol. I., the density of the steam,

$$(3) \qquad \gamma = \frac{0{\cdot}003539\,p}{1 + 0{\cdot}00367\,t},$$

we have for the weight of Q cubic feet,

$$Q\gamma = \frac{0{\cdot}003539\,p\,Q}{1 + 0{\cdot}00367\,t};$$

hence, the corresponding amount of heat is

$$W = \frac{0{\cdot}003539\,(640 - t_1)\,p\,Q}{1 + 0{\cdot}00367\,t},$$

and the amount of fuel

$$K = \frac{0{\cdot}003539\,(640 - t_1)\,p\,Q}{\psi\,(1 + 0{\cdot}00367\,t)}.$$

Hence for the amount of steam,

$$Q = \frac{(1 + 0{\cdot}00367\,t)\,\psi\,K}{0{\cdot}003539\,(640 - t_1)\,p}.$$

If we put $t_1 = 40$ and $\psi = 4500$, we have

$$Q = 2119\,(1 + 0{\cdot}00367\,t)\,\frac{K}{p},$$

or for $t = 100°$, $120°$, $140°$, $160°$,

$$Q = \frac{2897\,K}{p}, \quad \frac{3052\,K}{p}, \quad \frac{3708\,K}{p}, \quad \frac{3363\,K}{p} \text{ cubic ft.}$$

EXAMPLE.—How much steam of $3\frac{1}{2}$ atmospheres tension is given by the combustion of one pound of carbon? According to the table, § 391, we have here $\mu = 508{\cdot}2$. Hence

$$Q = \tfrac{4}{33} \times 508{\cdot}2 = 61{\cdot}6 \text{ cubic ft.}$$

From formula (1)

$$Q = \frac{3274\,K}{1{\cdot}637 + p} = \frac{3274}{1{\cdot}637 + 3{\cdot}5 \times 14{\cdot}7} = 61{\cdot}67 \text{ cubic ft.,}$$

and from formula (2)

$$Q = \frac{3475\,K}{4{\cdot}120 + p} = \frac{3475}{4{\cdot}120 + 3{\cdot}5 \times 14{\cdot}7} = 62{\cdot}53 \text{ cubic ft.}$$

Further, from the formula $\mu = 1649 \cdot 8\, p^{-0.9393}$, $\mu = 1649 \cdot 8 \times 3 \cdot 5^{-0.9393} = 535 \cdot 1$, and hence $Q = \frac{4}{33}\mu = 64 \cdot 21$ cubic ft. ; and, finally, from the above formula,

$$Q = 2119\,(1 + 0 \cdot 00367\, t)\,\frac{K}{p},$$

since for $p = 3 \cdot 5$, we have $t = 140°$,

$$Q = \frac{3207}{3 \cdot 5 \times 14 \cdot 7} = 62 \cdot 35 \text{ cubic ft.}$$

§ **489.—Formulæ for Delivery.**—If we unite the formulæ of the last article with those already found for the mechanical effect, we shall obtain a relation between the delivery and fuel consumed. If we take the general formula of Pambour,

$$L = 144\, Q\,(\beta + p)\left[1 + log.\ nat.\left(\frac{F_1 s_1}{F s}\right) - \frac{\beta + q}{\beta + p_1}\right] \text{ ft. lbs.},$$

and insert

$$Q = \frac{\psi}{640 - t_1} \cdot \frac{\alpha}{\beta + p} \cdot \frac{K}{62 \cdot 5},$$

we obtain

$$L = \frac{144\,\psi}{640 - t_1} \cdot \frac{\alpha}{\beta + p} \cdot \frac{K}{62 \cdot 5}\,(\beta + p)\left[1 + log.\ nat.\left(\frac{F_1 s_1}{F s}\right)\right.$$

$$\left. - \frac{\beta + q}{\beta + p_1}\right] = \tfrac{2}{1}\tfrac{8}{2}\tfrac{8}{5}\,\frac{\psi\,\alpha}{640 - t_1}\left[1 + log.\ nat.\left(\frac{F_1 s_1}{F s}\right) - \frac{\beta + q}{\beta + p_1}\right] K.$$

If we assume $t_1 = 40$ and $\psi = 4500$, we have

$$L = \tfrac{4}{2}\tfrac{3}{5}\tfrac{2}{}\left[1 + log.\ nat.\left(\frac{F_1 s_1}{F s}\right) - \frac{\beta + q}{\beta + p_1}\right] \alpha\, K \text{ ft. lbs.}$$

For low-pressure engines, $\alpha = 27283$, and hence

$$L = 471451\left[1 + log.\ nat.\left(\frac{F_1 s_1}{F s}\right) - \frac{\beta + q}{\beta + p_1}\right] K,$$

while for high-pressure engines, $\alpha = 28961$, and hence

$$L = 500446 \left[1 + log.\,nat. \left(\frac{F_1 s_1}{F s} \right) - \frac{\beta + q}{\beta + p_1} \right] K.$$

If we put $F_1 = F$, we have the formula for single-cylinder engines, and if we put also $s_1 = s$ and $p_1 = p$, we have the formulæ for condensing engines without expansion,

$$L = 471450 \left(1 - \frac{\beta + q}{\beta + p} \right) K \text{ ft. lbs.},$$

and for non-condensing engines without expansion,

$$L = 500446 \left(1 - \frac{\beta + q}{\beta + p} \right) K \text{ ft. lbs.}$$

In condensing engines, the condensation can only be carried as far as $\frac{1}{10}$ and the expansion as far as $\frac{1}{2}$ atmosphere, while in non-condensing engines the latter can only be carried as far as $\frac{3}{2}$ atmospheres. Assuming these relations, and expressing the tensions p, p_1 and q in atmospheres, we obtain

(1) For low-pressure engines with expansion,

$$\frac{F_1 s_1}{F s} = \frac{\beta + p}{\beta + p_1} = \frac{0{\cdot}1161 + p}{0{\cdot}1161 + 0{\cdot}5} = \frac{0{\cdot}1161\,p}{0{\cdot}6161} = 0{\cdot}188 + 0{\cdot}1623\,p,$$

and

$$\frac{\beta + q}{\beta + p_1} = \frac{0{\cdot}1161 + 0{\cdot}1}{0{\cdot}1161 + 0{\cdot}5} = \frac{0{\cdot}2161}{0{\cdot}6161} = 0{\cdot}351.$$

2. For high-pressure engines with condensation,

$$\frac{F_1 s_1}{F s} = \frac{\beta + p}{\beta + p_1} = \frac{0{\cdot}2922 + p}{0{\cdot}2922 + 0{\cdot}5} = \frac{0{\cdot}2922 + p}{0{\cdot}7922} = 0{\cdot}369 + 1{\cdot}262\,p,$$

and

$$\frac{\beta + q}{\beta + p_1} = \frac{0{\cdot}2922 + 0{\cdot}1}{0{\cdot}2922 + 0{\cdot}5} = \frac{0{\cdot}3922}{0{\cdot}7922} = 0{\cdot}495.$$

3. For high-pressure engines without condensation,

$$\frac{F_1 s_1}{F s} = \frac{\beta + p}{\beta_1 + p_1} = \frac{0\cdot2922 + p}{0\cdot2922 + 1\cdot5} = \frac{0\cdot2922 + p}{1\cdot7922} = 0\cdot163 + 0\cdot558p,$$

and

$$\frac{\beta + q}{\beta + p_1} = \frac{0\cdot2922 + 1}{0\cdot2922 + 1\cdot5} = \frac{1\cdot2922}{1\cdot7922} = 0\cdot721.$$

4. For low-pressure engines without expansion,

$$\frac{\beta + q}{\beta + p} = \frac{0\cdot1161 + 0\cdot1}{0\cdot1161 + p} = \frac{0\cdot2161}{0\cdot1161 + p}.$$

5. For high-pressure engines without expansion and with condensation,

$$\frac{\beta + q}{\beta + p} = \frac{0\cdot2922 + 0\cdot1}{0\cdot2922 + p} = \frac{0\cdot3922}{0\cdot2922 + p}.$$

6. For high-pressure engines without expansion and without condensation,

$$\frac{\beta + q}{\beta + p} = \frac{0\cdot2922 + 1}{0\cdot2922 + p} = \frac{1\cdot2922}{0\cdot2922 + p}.$$

Accordingly, we have for these several cases,

1. $L = 471451\,[0\cdot649 + 2\cdot3026\,log.\,nat.\,(0\cdot188 + 1\cdot623p)]K.$

2. $L = 500446\,[0\cdot505 + 2\cdot3026\,log.\,nat.\,(0\cdot369 + 1\cdot262p)]K.$

3. $L = 500446\,[0\cdot279 + 2\cdot3026\,log.\,nat.\,(0\cdot163 + 0\cdot558p)]K.$

4. $L = 471451\left[1 - \frac{0\cdot2161}{0\cdot1161 + p}\right]K.$

5. $L = 500446\left[1 - \frac{0\cdot3922}{0\cdot3922 + p}\right]K.$

6. $L = 500446\left[1 - \frac{1\cdot2922}{0\cdot2922 + p}\right]K$ ft. lbs.

EXAMPLE.—What is the delivery of a single-cylinder steam engine with expansion and condensation, using 40 lbs. of coal per hour, and steam at 4 atmospheres? According to 2, we have,

$$L = 500446\ [0\cdot505 + 2\cdot3026\ \textit{log. nat.}\ (0\cdot369 + 1\cdot262 \times 4)]\ K$$

$$= 500446\ (0\cdot505 + 2\cdot3026\ \textit{log. nat.}\ 5\cdot417)\ \frac{40}{3600}$$

$$= 500446\ (0\cdot505 + 1\cdot689)\ \tfrac{1}{90} = \frac{500446 \times 2\cdot194}{90}$$

$$= 12199 \text{ ft. lbs.} = 22\cdot18 \text{ horse-power.}$$

§ 490.—If we make $K = 1$, in formulæ 2, 3, 5, and 6, and $p = 1, 2, 3, 4$, etc., atmospheres, we obtain for these four cases the theoretical effect corresponding to one pound of carbon for different tensions.

The following table gives these effects in horse-powers, for one pound of carbon per hour:

Steam Tension in Atmospheres.			1	1½	2	3	4	5	6	7	8	∞
Machine with expansion,	with	Condensation.	905	1203	1426	1756	1996	2187	2341	2481	2536	∞
	without		0	254	477	807	1048	1239	1408	1530	1647	∞
Machine without expansion,	with		633	710	754	802	827	842	854	860	867	910
	without		0	254	397	553	634	690	723	749	768	910

We see, from this table, that engines with expansion and condensation have a much greater delivery than the others, and that the delivery is greater the greater the steam tension. While for a tension of 3 atmospheres, the delivery for each pound of carbon is 1756 horse-power, for 8 atmospheres we have 2536 horse-power. The table shows also that expansion engines without condensation have a less delivery than those with condensation, and

that in the latter the effect of expansion is greatest for high tensions. For tensions of from 3 to 4 atmospheres, for example, the delivery of engines with expansion and condensation is as great again as without condensation. We see further from the table, that engines without expansion and with condensation have a delivery which varies but slightly with increasing steam tension, and which at 3 atmospheres is nearly the same as for an expansion engine without condensation, and at 8 atmospheres about one half. There is no advantage, therefore, in this case in a high tension. Finally, the table shows that engines without expansion and condensation have a very small delivery for small and average tensions, and only approach the third class under high tensions.

Although, therefore, it is always more advantageous to use high-pressure steam, the tension must not be carried too far—for example, beyond 8 atmospheres—because in such case the prejudicial resistances, and especially the loss of heat, become very great, so that the gain is more than counterbalanced. To this must be added the danger of bursting the boiler.

If we put the mechanical equivalent of heat = 1390 ft. lbs. (see § 379), and the amount of heat generated by the combustion of one pound of carbon = 7500 calories, we have for the theoretical delivery of one pound of carbon,

$$1390 \times 7500 = 10425000 = 18954 \text{ horse-power},$$

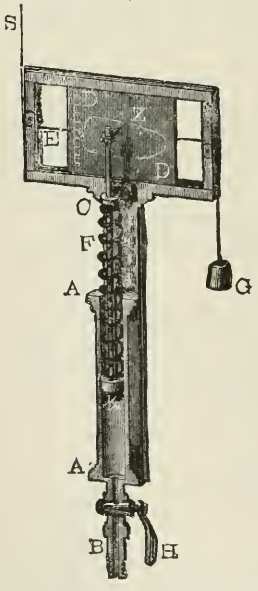
or more than seven times the greatest value (2536) in the preceding table. If the amount of heat due to the combustion of one pound of coal is only 4500 calories, the corresponding mechanical effect is

$$1390 \times 4500 = 6255000 \text{ ft. lbs.} = 11372 \text{ horse-power},$$

or more than four times as great as the greatest value in the table.

§ **491.** — **Steam Indicator.** — The tension of the steam in the cylinder is given by an instrument called the indicator. Fig. 787 gives a very simple contrivance of this kind by Watt. Its arrangement is as follows: AA is a cylinder about $1\frac{1}{2}$ inches in diameter and 1 ft. long, ending below in a narrower tube B and closed above by a piston. The screw end B is screwed into a hole in the cylinder cover, so that by opening the cock H the steam enters the cylinder AA and presses up the piston K. The piston rod KC passes through a guide C and is surrounded by a spiral spring, which is compressed by the pressure upon K until equilibrium occurs. The pointer Z, on the end of the piston rod, shows by its position the intensity of the pressure. Since this force, especially in machines working expansively, varies during the stroke, it is required to determine its mean value, or, what is the same thing, the mean position of Z. For this purpose, we have a pencil at Z, which presses against a plate DD, which latter is made to move horizontally by means of the string ES attached to the piston rod, and carrying the counter-weight G at its other end. The pencil during the stroke thus describes a curve DD, the area of which measures the work performed by the piston. If, therefore, we divide the work thus determined by the length of stroke, we obtain the mean pressure of the steam upon the piston.

FIG. 787.



If the tension of the steam below K is p, and the atmospheric pressure above K is a, and the pressure of the spring per square inch of piston area is y_1, we have during the rising of K,

$$p = y_1 + a.$$

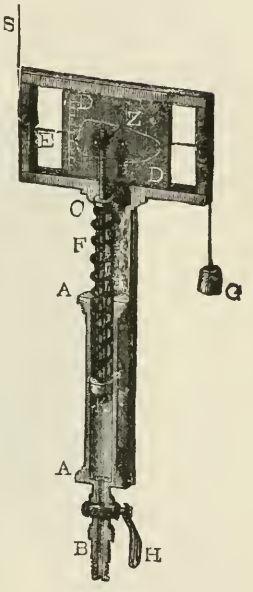
If, however, q is the tension during descent of K, and y_2 the force necessary to extend the spring,

$$a = q + y_2.$$

Uniting these equations, we have for the moving force of K per square inch,

$$p - q = y_1 + a - (a - y_2) = y_1 + y_2.$$

Fig. 788.

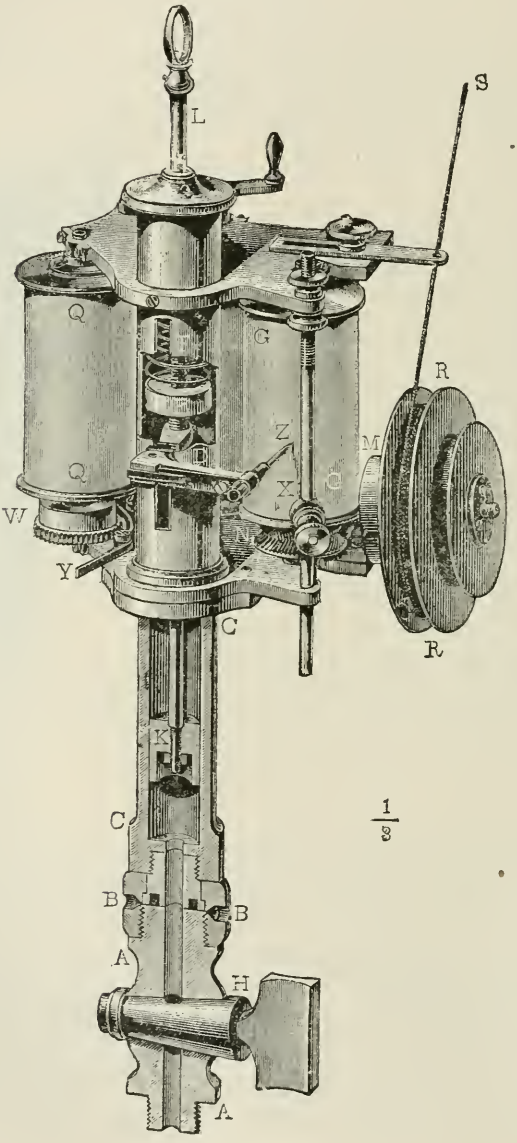


If the forces of compression and extension of the spring are proportional to the elongation and compression, y_1 and y_2 can be measured by the distance of the pencil from the horizontal line which would be described by it if the spring were neither extended nor compressed, or the piston subjected to atmospheric pressure above and below. If, now, the plate and piston both move, the product of the mean distance of the pencil from this line into the distance moved through by the plate, or the area described by the pencil, will give the measure of the work of the steam for a half revolution or one stroke.

§ **492.**—In Fig. 789, we have represented an indicator by Clair, of Paris. CC is the cylinder in which the piston K moves, AA is the foot piece furnished with a cock H, which screws into the steam-cylinder cover and into the end C of the indicator cylinder. Around the piston rod KL is a spiral spring F which presses against the plate E. Below this plate, we have an arm D and link attached to the rod, which carry the pencil Z. The point of this pencil presses upon a strip of paper which is wound upon the hollow

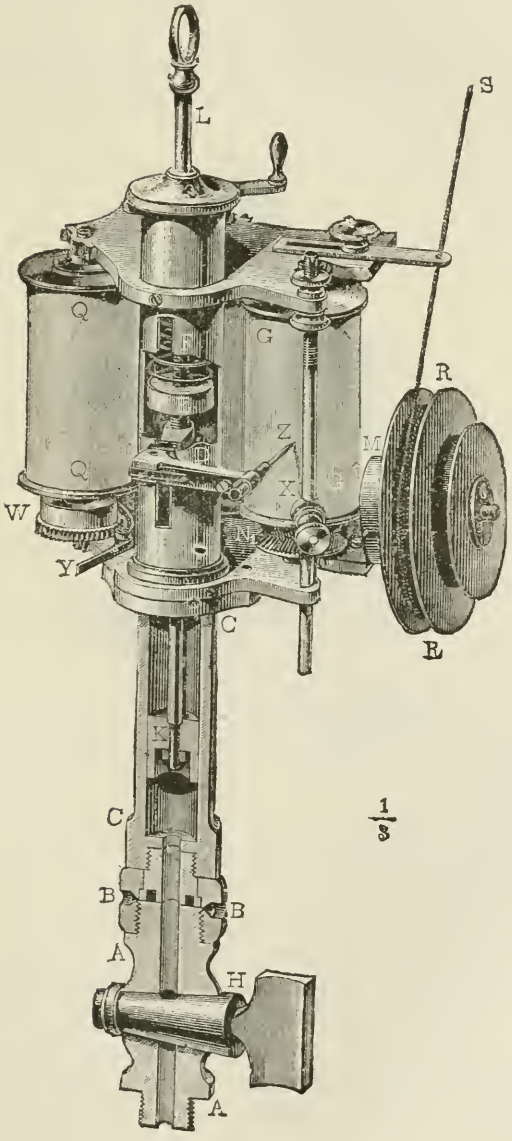
cylinder *G G*, so that, when the paper is moved while the pencil goes up and down, a curve is described upon it

FIG. 789.



whose vertical ordinates are proportional to the pressure of the steam. The motion of the cylinder and paper is

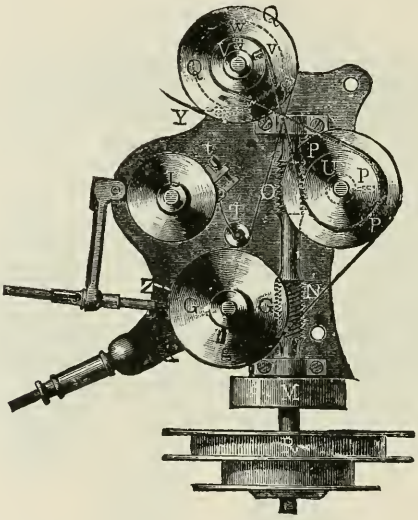
Fig. 790.



caused by the piston rod of the engine, by means of the string SR and the drum RR. Since this drum is thus turned only in one direction by the action of the engine, we have a spiral spring in the box M, which turns the drum back upon the return stroke.

The axle NO of the drum RR, as shown in Fig. 791, has endless screws at N and O, which engage with cog-wheels

FIG. 791.

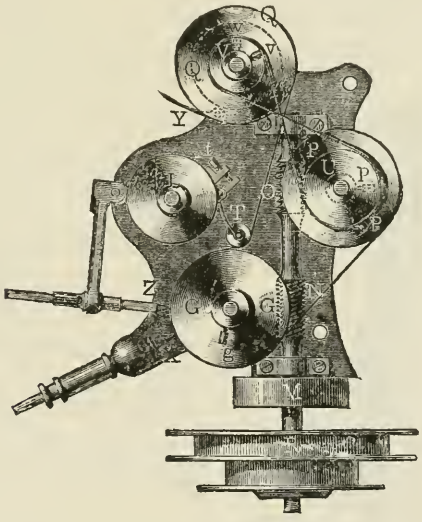


N_1 and O_1 upon the axles of the cylinders GG and PP_1, and thus turns them in opposite directions. Since this axle, during one revolution of the engine, turns through a certain angle and back again, the paper strip unrolls from PP upon GG and back from GG to PP, and the pencil Z describes a closed curve. The area of this curve gives, as already indicated, the variation of the steam pressure, as well as its mean intensity and the work performed.

The indicator of Clair above represented is distinguished from the indicator of MacNaught, in that it gives not only closed curves, but also continuous ones, as in the case of a dynamometer (§ 125). For this purpose, the axle of the cylinder GG is furnished with two toothed wheels as N_1,

and the portion N of the horizontal axis $N O$ engages with two toothed wheels. When, by loosening the screw p, the toothed wheel O is disconnected from the axle of $P P$, and by tightening the screw g the toothed wheel N_1 is connected with the axis of the cylinder $G G$, then, if the drum $R R$ and the axle $N O$ turn back and forth through a certain angle, still the cylinder $G G$ has a forward motion, and hence the pencil Z moves onward continuously. In order that the paper strip may be uniformly wound off from $P P$

FIG. 792.

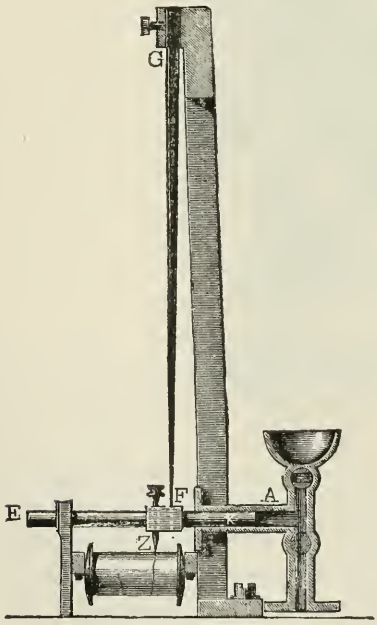


and upon $Q Q$, it is necessary that the plate V should be connected with the axis of the cylinder $Q Q$ by the screw v, because then, by means of the band about U and V, the motion of the drum $P P$ will be in opposite direction to that of $Q Q$. In order to keep the paper in tension, we have the roller T, which is pressed against the strip by means of the screw t. We have also a second pencil X, which describes the base line upon the paper in the case of continuous curves. In order to keep the paper strip always stretched between G and Q, when describing a closed curve, we have

a spiral spring about the axis of $Q\,Q$, which can be wound up to any required tension by means of the toothed wheel W and the detent Y. In order to cause this spring to act, we have only to tighten the screw w, which thus unites the end of the spring with the axle.

In order, finally, to find the relation between the steam pressure and the distance passed over by the pencil, special experiments must be made with weights upon the spring F. In the instrument possessed by the author, the diameter of the piston K is 22 millimeters, and under 1 kilogramme one spiral spring gives 2 millimeters, and the other 5 millimeters travel of pencil. In order that the piston friction may be as near as possible constant and independent of the pressure, the piston K is not packed, but turned carefully and covered with oil. If, therefore, the piston friction, in the preliminary experiments, where the spring is compressed by weights, is the same as when the steam is admitted, the results of the indicator are independent of the friction and may be disregarded.

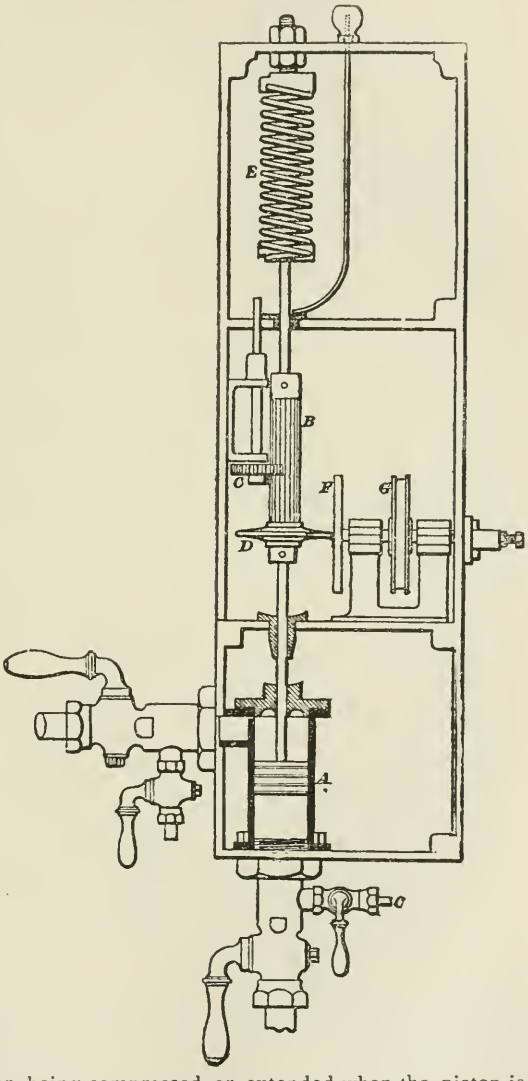
FIG. 793.



REMARK.—Recently, straight springs have been applied, according to Poncelet, instead of spiral springs. The arrangement of such an indicator is seen in Fig. 793. Here the cylinder A is horizontal and the spring FG and pencil Z are fastened to the rod KE. The curve is described upon a paper strip wound upon the drum below. (Compare § 125 and § 127; also, Morin, "Leçon de mécanique pratique, 1re partie," 1855.) Another indicator with two springs has been constructed by Welkner. (See his article, "Die Locomotive," Göttingen, 1859.)

[Ashton and Storey's power meter, Fig. 793*a*, is a continuous indicator, which measures the effective pressure at every instant, and multiplying this pressure by the piston speed, by means of an integrating apparatus, records the power on a dial. It is thus described by the manufacturers: "The Steam Power Meter and Continuous Indicator consists of a small cylinder, *A*, similar to that of an ordinary indicator, except that each end is here connected by means of a pipe with the corresponding end of the steam engine cylinder; these connections are made as short and direct as possible. The piston-rod of the indicator carries a long pinion, *B*, which revolves loosely upon it, but is held endwise between two collars on the rod. This pinion gears into a toothed wheel, *C*, which drives the index on the registering dial of the instrument. On the lower end of the long, loose pinion is fixed a wheel, *D*, called the integrating wheel, having a smooth rim with a rounded edge. To the upper end of the piston-rod is attached a spiral spring, *E*, which acts like the spring of an ordinary indicator, being compressed or extended when the piston is moved above or below its middle position in the cylinder.

FIG. 793*a*.



"On a short horizontal shaft is mounted a circular disc, *F*, the face of

which is constantly pressed against the rim of the integrating wheel *D*, by means of a light spring bearing against the end of the shaft; the pressure of the spring is sufficient to cause any motion of rotation in the disc wheel *F* to be transmitted to the edge of the integrating wheel without slipping. On the shaft of the disc-wheel is keyed a small pulley, *G*, round which is wound a cord, the two ends of which are thence carried around loose guide pulleys, fixed at the back of the machine, and separately attached to the cross-head, or other convenient part of the engine, the reciprocating motion of the engine being thus converted into a rotary motion of the disc, taking place in alternate opposite directions.

"When there is no pressure on the piston of the engine, and consequently none on the piston of the indicator, the integrating wheel is so adjusted that the point of contact of its rim with the disc is at the centre of the disc, that being the zero point of the instrument, and in this position any rotation may take place of the disc which is geared with the engine, without transmitting motion to the integrating wheel. But when the pressure of the steam is admitted so as to act on the piston of the indicator, the integrating wheel *D* traverses either upwards or downwards from the centre towards the circumference of the disc *F*, the distance traversed being proportionate to the pressure of the steam on the piston, which is measured by the compression or extension of the spiral spring *E*. Supposing the engine, and with it the disc, be moving, the motion will be communicated by the disc to the integrating wheel, and through this to the index of the instrument. The extent of the motion so given to the index during any stroke of the engine is proportionate to the pressure of the steam on the indicator piston during that stroke, because the rate at which the integrating wheel is driven by the disc is directly proportionate to the distance that the integrating wheel is raised or lowered from the centre of the disc, and this distance is the same as the amount of compression or extension of the indicator spring.

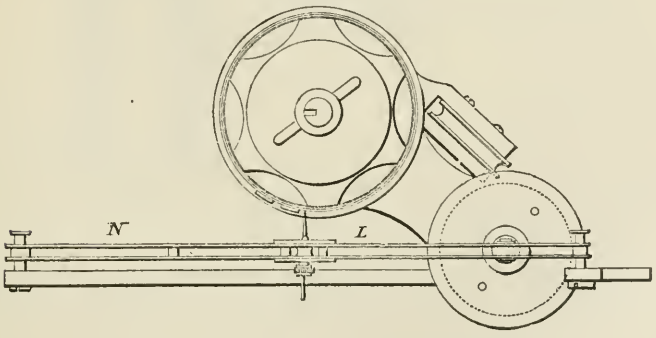
"When the stroke is finished, and the return stroke commences, the disc will rotate in the opposite direction; and if the steam acting upon the piston were pressing in the same direction as before, the integrating wheel and index would necessarily go backward. If, however, the steam acts on the opposite side of the piston when the motion of the engine's piston is reversed, the integrating wheel will be moved to the opposite side of the centre of the disc, so that the integrating wheel and index will continue to be moved in the same direction as before, and the quantity of motion through the return stroke of the engine will again be proportionate to the pressure of steam on the piston during the return stroke, and will be added to the movement of the index during the preceding stroke. By this means, therefore, the registering index is moved during each stroke of the engine through a space proportionate to the sum of the moments of pressure exerted during that stroke, and consequently the total amount of power developed during the stroke is thus indicated.

"The following is the mode in which the graduation of the index and the measurement of power by the meter is effected. Assuming the indicator spring to be of such strength as to yield one inch with a pressure of 25 lbs. per

circular inch in the cylinder, the integrating wheel will be in contact with the driving disc at a distance of one inch from its centre when there is that pressure in the engine cylinder; and if the radius of the integrating wheel is one inch, one revolution of the disc will drive it one revolution. Then taking the circumference of the pulley where the motion of the engine is applied to be one foot, this movement of one revolution of the integrating wheel will represent a motion of one foot of the engine piston under a pressure of 25 lbs. per circular inch, or a moving power of 25 foot lbs. for each circular inch in the area of the piston; and if four revolutions of the integrating wheel and long pinion are required to give one revolution to the index driving wheel, that will represent four times 25 foot lbs. or 100 foot lbs. per circular inch. In the actual instrument the proportions of the moving parts and the gearing for reducing the motion down to the registering index are so arranged that each unit on the dial represents 1000 foot lbs. per circular inch of the engine piston.

"The total work done by an engine in foot lbs. during any period is therefore measured at once by simply multiplying the number indicated on the dial during the period of work by the square of the diameter of the engine cylinder in inches. The average horse-power of the engine during the time is obtained by dividing this total amount of foot lbs. by the number of minutes during which the metre has been in action, and also by 33,000. For convenience of reference, where the indicator is intended to be employed constantly upon a particular engine, a separate index may be provided on the dial for showing at any time the load on the engine in horse-power, by simply observing the movement recorded by this special index during one minute."

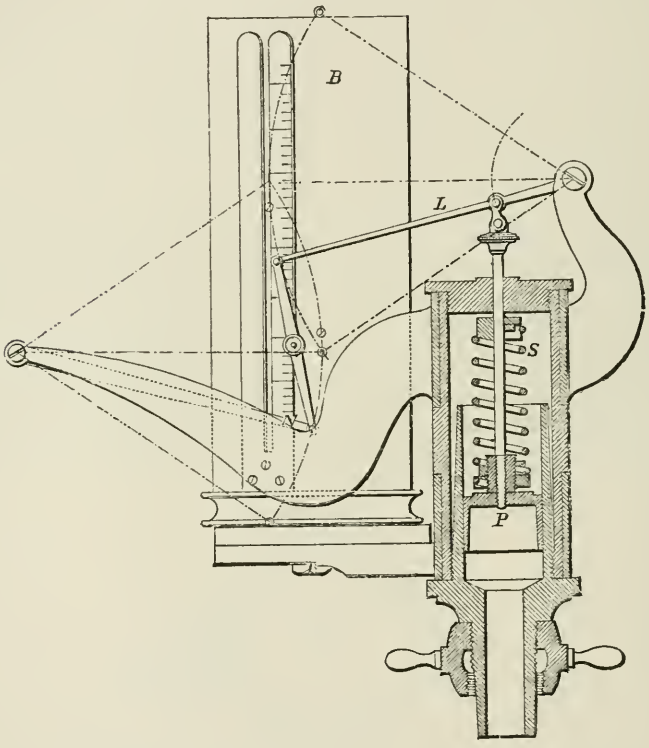
FIG. 793*b*.



The indicator most commonly employed for temporary use is the improved form invented by Charles B. Richards, in 1862, Figs. 793*b* and 793*c*. The movement of the indicator piston, *P*, is multiplied by connecting the piston-rod to the short arm of a lever, *L*, the pencil being attached to a lever jointed to the other end, and to a second lever, *N*, forming a parallel motion, to guide it in a straight line. As ordinarily built, the indicator piston has a

cross-section of half a square inch, and the pencil moves four times as far as the piston, the latter having a maximum movement upward, to compress the spring *S*, of $\frac{25}{32}$ of an inch, corresponding to a movement of the pencil of $3\frac{1}{8}$ inches. The paper on which the diagram is to be traced is placed upon the

FIG. 793*c*.

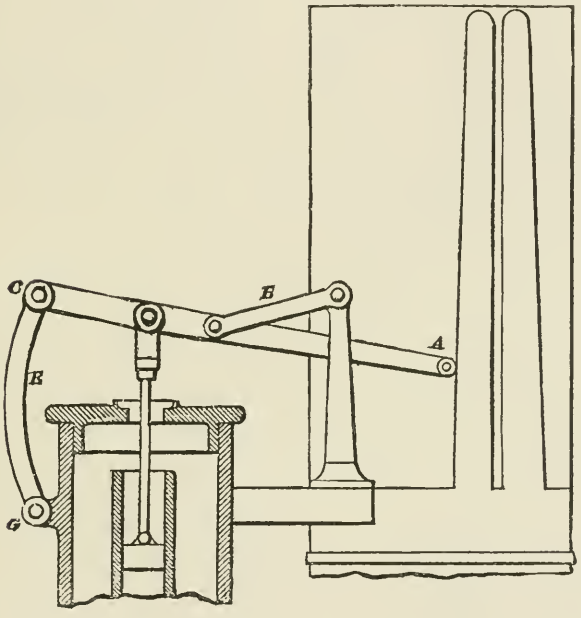


barrel *B*, and held there by spring clips. Prepared paper, upon which a black mark can be made with a metallic point, is often used for taking diagrams, a wire with a fine point being substituted for the pencil.

The indicator patented by J. W. Thompson, 1875, the upper portion of which is shown in Fig. 793*d*, is a modification of the Richards indicator just described, having a lighter system of levers to carry the pencil and form the parallel motion. The pencil is inserted at the end of a lever, *A C*, the other end of which is jointed to the rocking lever *C G*, which, with the guide link *B*, causes the point *A* to move in a straight line as the indicator piston rises or falls.

The arrangements of levers employed to guide the pencils in the indicators patented by Richards and Thompson are not exact parallel motions, but are practically correct within the range of the movement that takes place.

FIG. 793*d*.



A few notes are added in relation to the method of attaching and using the indicator.

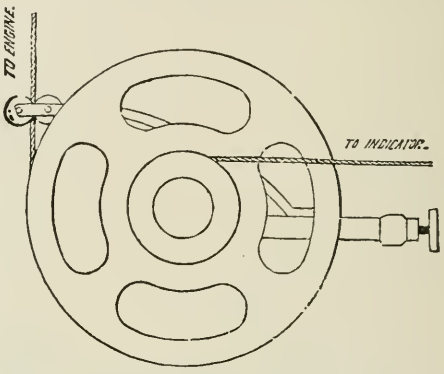
To attach the indicator to the cylinder of an engine, drill a hole in the cylinder, either in the head or close to the end, and tap it out for a half-inch iron nipple. The indicator cock must be connected to this, using an elbow if necessary, so as to bring it into a convenient position to attach the indicator. In drilling this hole, care should be taken not to locate it close to the ports. Sometimes the connections from the two ends of a cylinder are brought together, and one indicator is made to answer for both ends, a cock being fitted in each pipe, so that either can be opened to the indicator, as desired. In such a case, the holes in the cylinder should be larger, for three-quarter-inch pipe at least, so as to prevent any loss of pressure. It is obvious, however, that as the indicator is designed to show the pressure in a cylinder, it is best to make the connection as direct as possible.

Since the diagram taken by the indicator is intended to show the pressure at every point of the stroke, it is necessary that the motion of the paper bar-

rel should coincide with that of the piston. The paper barrel ordinarily has a movement of between five and six inches, so that some means of reducing the engine motion must generally be adopted.

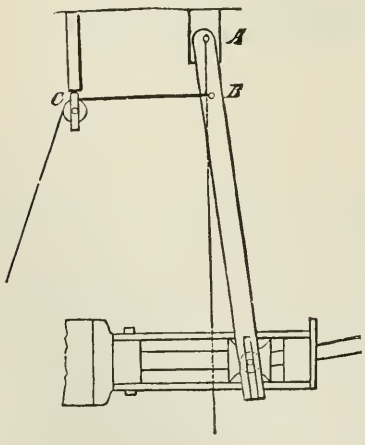
The reducing wheel, a simple form of which is shown in Fig. 793*e*, is often employed for this purpose. This is attached to some part of the engine frame, and the cord marked "to engine" is made fast to the cross-head, being carried over a pulley, if necessary, so that it leads parallel to the guides. The other cord, "to indicator," is fastened to the cord that is wound round the paper barrel of the indicator. The two wheels bear the same proportion to each other as the stroke of the engine does to the desired range of motion for the paper barrel. There is a coiled spring in the reducing-wheel, which makes it turn back on the return movement of the cross-head. By having different-sized wheels to carry the cord leading to the indicator, this arrangement can be adapted to engines with different strokes.

Fig. 793*e*.



A swinging board, receiving motion from the cross-head, is one of the most common reducing attachments, since it can readily be fitted with such facilities as are afforded in most engine rooms. There are two general varieties of this arrangement to be noticed.

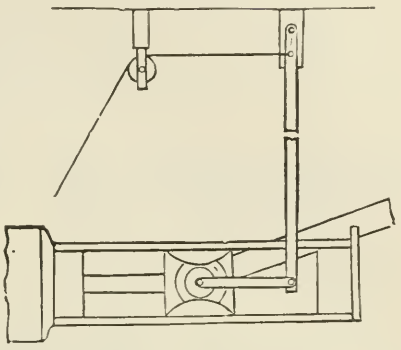
Fig. 793*f*.



In the swinging board with slot, Fig. 793*f*, a point, *A*, is marked at some convenient distance from the cross-head, on a line perpendicular to the guides at the centre of the stroke, and a board is attached so that it can swing freely from this point, and is connected by a slot in the other end to a pin in the cross-head. Then, as the cross-head moves, the board will swing to and fro. At some point, *B*, of the board, which has the proper range of movement for the paper cylinder, a cord or wire is attached, and carried over a pulley, *C*, adjusted at such a height that the part of the cord, *B C*, is parallel to the guides when the engine is at half stroke. The cord is then brought down and attached to the cord of the paper barrel.

Sometimes it is not practicable to attach the board directly to the cross-head by a pin, and it is more convenient to use a link connection for the swinging board, as shown in Fig. 793*g*.

Fig. 793*g*.



It is easy to see that a number of arrangements could be devised on the general principle of the swinging board. Sometimes it is attached to the guides by a standard, and sometimes one end is connected to the cross-head and the other to the indicator, the point on which the board swings being between, at distances from the two ends proportional respectively to the stroke of the engine and the movement of the paper barrel. Whatever the special arrangement, attention should be paid to the following points:

1st. To have the board perpendicular to the guides when the engine is at half stroke.

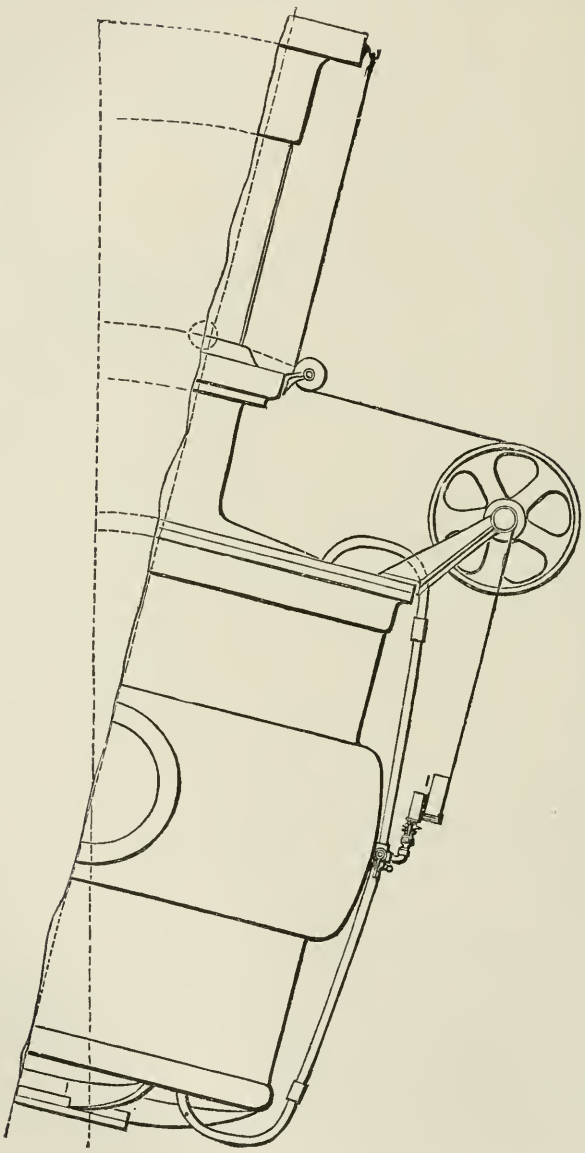
2d. To lead the cord off in a direction parallel to the guides.

Fig. 793*h* shows a method of connecting the indicator to the cylinder of an oscillating engine, a reducing-wheel being used to give motion to the paper barrel.

In taking an indicator diagram, if cord is used for the motion, it should have a slide on it, as shown in Fig. 793*i*, so that its length can readily be adjusted, and usually there is a hook in the loop so formed, so that the cord from the engine can be connected to the cord of the paper barrel and detached, at pleasure. Fine wire is better than cord, as it is quite flexible, and does not stretch so readily. If several cards, taken at intervals, are of the same length, the connection is all right. Messrs. Elliott Brothers apply a detent to the paper barrel (see *Engineering*, xx., 474) for preventing its movement except when a diagram is to be taken, so that the cord need not be unhooked during the progress of an experiment, and the editor of *Engineering* thus describes a simple device for the same purpose: "This plan consists simply in leading off from the cord which connects the indicator drum with the engine another cord which can be attached to any convenient fixed point. Instead of unhooking the connecting cord, the paper drum is pulled round to the extent of its travel by means of the supplementary cord, and the latter is then made fast. Under these circumstances, of course, no motion is communicated to the paper drum, the motion of the engine simply alternately slackening and tightening the cord between the engine and the indicator. When a diagram is to be taken, the supplementary cord is simply slacked off when the instrument acts in the ordinary way. All unhooking of the connecting cord is thus dispensed with."—*Engineering*, xx., 493.

Having got the motion of the paper barrel properly adjusted, turn the cock of the indicator so that the steam will flow through, having first put a

Fig. 793*h*.



piece of paper on the paper barrel. Then turn the cock so as to let steam into the indicator, and press the pencil lightly against the paper. Draw it back as soon as the card is traced, shut the indicator cock, and apply the pencil again, to trace the atmospheric line. Be particular not to trace the atmospheric line until after the diagram has been taken. Then stop the motion of the paper barrel, remove the diagram, and mark on it the pressure of the steam, the revolutions per minute, the height of the barometer, and the temperature of the engine-room—if these data can be obtained—and the vacuum and temperatures of hot well and injection water, if the diagram has been taken from a condensing engine. All the dimensions of the engine should also be noted, for future reference, together with such particulars in regard to the dimensions and performance of the boiler as can be obtained.

FIG. 793*i*.

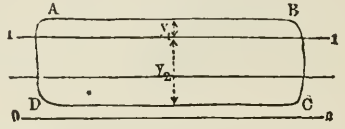
A counter should always be employed to determine the number of revolutions per minute at the time the diagram is taken. It can be connected to the indicator motion, or to some moving part of the engine. Take its reading at the beginning of a minute, and at the end, having indicated the engine meanwhile. This gives the revolutions at the time the card was taken, with considerable accuracy.

The cylinder and all moving parts of the indicator should be oiled before the instrument is applied to an engine. The best grade of oil, such as is specially prepared for sewing machines or clocks, is the most suitable. The cylinder of the indicator should be examined after one or two diagrams have been taken, and if any grit has entered, it should be removed by a piece of soft cotton waste on the end of a white-pine stick. It is important to attend to this on indicating an engine whose condition is not known, and if it is found that grit or dirt is forced into the indicator, it should be cleaned at frequent intervals during the experiment. At the conclusion of an experiment the indicator should be removed, and as soon as it has cooled sufficiently, wiped out and oiled. The spring should be thoroughly dried and covered with oil. Never put any hard substance into the cylinder, but use a white-pine stick and soft cotton waste. After use, take out the plug of the indicator cock, apply oil, and after replacing adjust it so that it moves freely and does not leak. The indicator will then be in good order for subsequent use, and will remain servicable for years, while a neglect of these simple methods of preservation will destroy its free action in a very short time.]

§ **493.—Indicator Diagram.**—According as an engine is high or low pressure, with or without expansion, according as the slide valve is in advance of the piston, etc., the indicator diagram varies. In a low-pressure engine with-

out expansion, the curve has the general shape of a rectangle, as *A B C D*, Fig. 794. When the piston of the engine

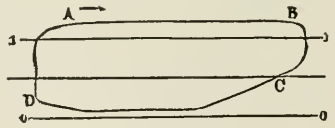
Fig. 794.



commences to descend, the pencil is at *A*. During the descent it describes a line *A B* parallel or nearly so with the line *O O*. At the lowest position of the piston, the steam being released, the pencil describes the line *B C*, and on the return stroke the line *D C* but little above the zero line *O O*, the height above representing the back pressure. When at the highest point of the piston, the steam is again admitted, the pencil rises to *A* again, and so on. The ordinates y_1, above the line 1 1, corresponding to a pressure of one atmosphere, are much less than the ordinates y_2 below this line, because the first represent the excess of the steam pressure above one atmosphere, and the latter the excess of the atmospheric pressure above the pressure in the condenser. An indicator, communicating with the lower portion of the cylinder, would of course give a reversed curve.

If the steam is first admitted when the piston is just about descending or rising, the curve is not so complete, but we have at *A* and *C*, Fig. 795, the corners rounded off.

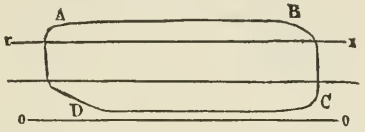
Fig. 795.



This rounding-off is especially great when the opening of the port is gradual, so that the steam flows through a narrow orifice at first and loses in pressure. By the slow opening of the exhaust port, we have the corner at *C* rounded, and this is especially great because the departing steam reacts upon the piston at first nearly with full force. In order to

diminish this great rounding-off at *C*, it is absolutely necessary then that the valve should be in advance of the piston and the steam released before the end of the stroke. Too great an advance, however, in the admission and exhaust,

FIG. 796.

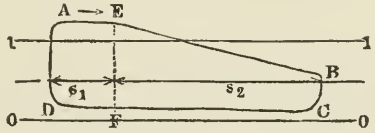


may cause, as in Fig. 796, a large rounding-off at the corners *B* and *D*.

§ **494.**—In engines working expansively, the indicator diagram takes a form resembling a rectangle and a trapezoid combined. The rectangular portion corresponds to the action of the steam before expansion, and the trapezoidal portion during expansion.

A low-pressure engine with expansion gives a diagram similar to Fig. 797. The portion of the stroke s_1, before

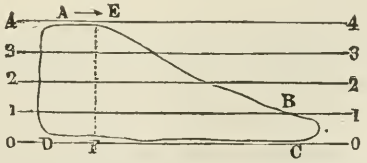
FIG. 797.



expansion commences, corresponds to the line *A E*, which is parallel or nearly so to 0 0 or 1 1. The rest of the stroke s_2 corresponds to *E B*, which gradually approaches the line 0 0, or the line of no pressure. The area *E B C F* gives the delivery during expansion.

The diagram *A C*, in Fig. 798, is for a high-pressure

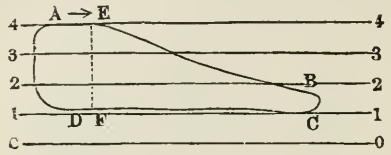
FIG. 798.



engine with condensation and expansion, and in Fig. 799

without condensation. While in the first, the portion *C D*

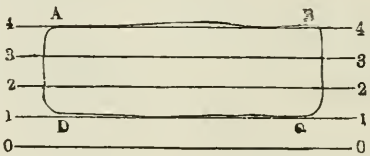
FIG. 799.



is but little above the zero line, the second is but little above the line 1 1, and the measure of the work in the second case is less by the area of the rectangle between the lines 0 0 and 1 1.

In Fig. 800, we have, finally, the diagram for a high-

FIG. 800.



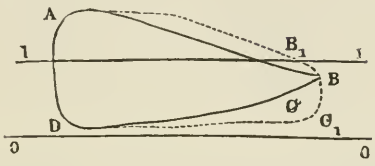
pressure engine (4 atmospheres) without expansion and condensation. Here also the work is less than in the case of a condensing engine by the rectangle between the lines 0 0 and 1 1.

§ **495.**—The indicator is not only an excellent instrument for determining the force and delivery of an engine, but is also the best means of investigating the action of the slide valve or regulator, as the shape of the diagram gives full information of all defects. The defects of the valve gear may be as follows:

1. The steam passages may not have sufficient width. If the cross-section of the steam passages is too small, the steam enters and departs with too great velocity, and thus loses a large amount of its tension. The diagram takes then the shape shown in *A B C D*, Fig. 801. For proper

opening of the ports, it should have the shape $A B_1 C_1 D$ shown by the dotted lines.

FIG. 801.



2. The valve rod is not long enough, so that the slide passes through a greater distance on one side of the ports than on the other. We have then one port opened longer than the other, so that the length of the diagram on one side is greater than on the other.

There is also in a certain degree a difference in the time of opening of the steam passages, because the piston does not make one half of its stroke in the same time as the other. If r is the radius of the crank and l the length of the connecting rod, the piston travel for the first and fourth quadrants, as the crank revolves (§ 458), is

$$s_1 = r - \frac{r^2}{2\,l},$$

and in the second and third quadrants,

$$s_2 = r + \frac{r^2}{2\,l}.$$

The difference is, therefore,

$$s_2 - s_1 = \frac{r^2}{l},$$

and hence its ratio to the entire stroke $2r$ is

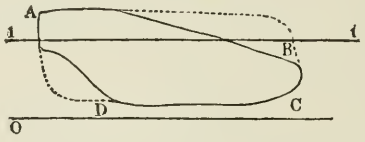
$$\frac{s_2 - s_1}{2\,r} = \frac{r}{2\,l}.$$

Since the expansion of the steam takes place especially in

the second half of the stroke, the action of the steam upon one side of the piston is not precisely the same as upon the other, and therefore it is necessary, for the exact determination of the delivery, to take an indicator diagram from the other end of the cylinder also. For this purpose, we may have the indicator connected by a pipe with either end of the cylinder. It is best to use simultaneously two indicators.

3. The surface of the slide has not the proper breadth. Thus, for example, there may be too much lap, which is

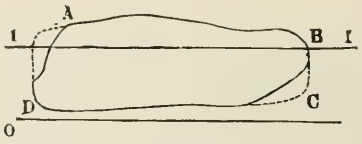
FIG. 802.



shown (Fig. 802) by one side of the diagram falling off and the other rising too quickly.

4. The eccentric has not the proper position with reference to the crank, and hence the valve has not the proper amount of advance. If the advance or the lead is too great, the indicator curve is similar to Fig. 802. If, on the other hand, it is too little, we have the reverse, or a rounding-off

FIG. 803.

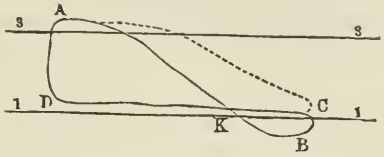


of the angles *A* and *C*, Fig. 803.

5. The eccentric has not the proper eccentricity or the travel of the slide is not large enough. If the travel is too small, the ports are not opened sufficiently, and we have a curve like Fig. 802 ; if, however, it is too great, the expansion is diminished, and the consumption of steam is too large, as in the case of too little lap.

The diagram takes a peculiar form, Fig. 804, in the case

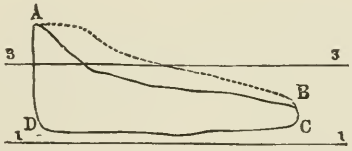
Fig. 804.



of an engine without condensation, when the expansion of the steam is carried too far. In such case, the counter pressure towards the end of the stroke is greater than the steam pressure.

If, further, the valve in the steam pipe is not sufficiently opened, there is a poor use of the steam, as indicated by

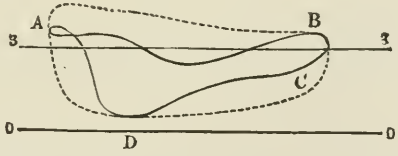
Fig. 805.



the curve in Fig. 805.

If the piston is not steam-tight, the diagram is also a peculiar one, because the steam pressure is thus diminished and the counter pressure increased. The curve may even

Fig. 806.



take such a shape as shown in Fig. 806. We have a similar result when the slide is not steam-tight.

The indicator may also be applied to the slide so that we obtain a slide diagram, which gives the steam tension at various positions of the slide, and allows the action of the

valve at the beginning and end of the stroke to be discussed. In order to obtain full information as to the action of the slide valve, we may take a third diagram which gives the relation between the travel of piston and valve directly, and which, according to § 459, has the shape of an ellipse.

The indicator is also useful when applied to the air and hot-water pump.

REMARK.—Full discussions of indicator curves, obtained from different machines, are given by Morin in the third part of his "Leçons de mécanique pratique." See also the "Catéchisme du mécanicien à vapeur," by E. Paris; article, "Indicateur de P. Garnier," as also Bornemann's article upon the indicator (Combes) in "Der Ingenieur." Especially to be recommended is "Der Indicator, Anleitung zum Gebrauch desselben bei der Prüfung von Dampfmaschinen," by J. Völckers, Berlin, 1863.

The indicator diagram from one end of the cylinder only shows what takes place on the side of the piston to which the indicator is applied; but at the same time there is some back pressure on the other side, opposing the motion of the piston. To get the actual diagram, therefore, it is necessary to take diagrams from both ends of the cylinder simultaneously, and then combine the parts of each that were traced at the same time. Such a diagram represents the true distribution of the pressure throughout the stroke, so that, in making calculations from an uncorrected diagram, the actual effective pressure is not obtained; but there is no error in practice if diagrams are taken from both ends of the cylinder, since the inaccuracies of the diagrams from the two sides balance each other. If, however, the object of the diagram is to show the distribution of the pressure throughout the stroke, it must be corrected, as already explained, by combining the return-stroke line of one diagram with the forward stroke line of the diagram taken from the other end of the cylinder at the same instant.

There is another correction which must also be made in cases where the weight of the reciprocating parts of the engine is sufficiently large to render it necessary. It is obvious that these reciprocating parts (the piston, piston-rod, cross-head, and connecting-rod) are put in motion and brought to rest at the commencement and termination of each stroke respectively, so that a certain amount of force or pressure must be deducted from the effective pressure on the piston at the beginning of the stroke, and added to it at the termination. Mr. Charles T. Porter was the first to call attention to the important modification produced on the distribution of the pressure by heavy reciprocating parts, and has discussed the subject quite thoroughly in his "Treatise on the Richards Steam Engine Indicator," New York, 1874. Dr. F. A. P. Barnard has developed the mathematical theory quite fully in a paper published in the "Transactions of the American Institute," 1871–1872, and further

information may be found in "A Practical Treatise on the Steam Engine," by Arthur Rigg, London, 1878, and "Ueber Damfmaschinen mit hoher Kolbengeschwindigkeit," by Joh. Fried. Radinger, Wien, 1872. In the present note the practical application of the theory will be briefly described.

In an engine with a connecting-rod of infinite length let

W = weight of reciprocating parts in pounds.
L = radius of the crank in feet.
R = revolutions of engine per minute.
A = area of the piston in square inches.

Then the pressure per square inch of piston area required to develop the initial accelerating force is

$$p = \frac{0{\cdot}000341 \times W \times L \times R^2}{a}.$$

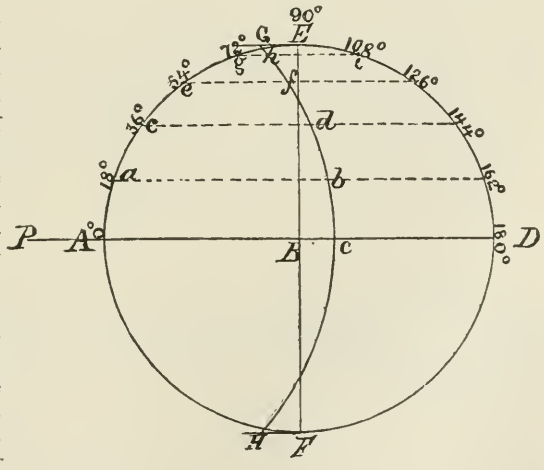
If, for example, the reciprocating parts of an engine 30 inches in diameter and 4 feet stroke, making 60 revolutions a minute, weigh 2000 pounds, the initial accelerating force is

$$p = \frac{0{\cdot}000341 \times 2000 \times 2 \times 3600}{707} = 6{\cdot}95$$

pounds per square inch of piston area; and this pressure diminishes to half stroke in the ratio of the cosine of the crank angle, and increases in the same ratio from mid-stroke to termination.

In the case of an engine with a connecting-rod of finite length, the initial accelerating force is increased at the commencement of the forward stroke, and diminished at the commencement of the return. By the aid of a simple graphical construction, the force required for acceleration or given out during retardation at any point of the stroke can easily be found.

FIG. 806*a*.



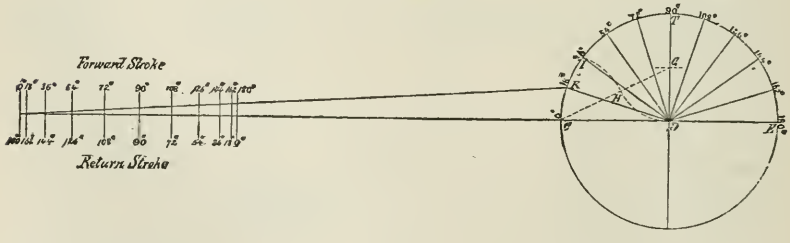
Assuming the ratio of the length of the connecting-rod to the length of the crank to be *a*, on a straight line *A D*, Fig. 806*a*, lay off a distance *A B*, equal to *p*, the initial accelerating force in pounds per square inch of piston area for a connecting-rod of infinite length. With this line,

A B, as a radius, describe a circle and draw a diameter, *E F*, perpendicular to *A B*. Make *B c* equal to $\frac{AB}{a}$, and on tangents to the circle at *E* and *F* lay off distances, *E G*, *F H*, each equal to *B c*. Through the points *G c H* draw a circular arc, and dividing the circle, whose radius is *A B*, into any number of equal parts, draw lines through the points of division parallel to *A D*. Then the distances from the circumference to the curve *G c H*, measured on the lines so drawn, are the accelerating or retarding forces for the different positions of the crank. To illustrate, for the forward stroke the initial accelerating force in pounds per square inch when the crank is at 0°, is *A c*; at 18° it is *a b*, *c d* at 36°, etc.; while all distances measured on the right of the curve represent retarding forces, the final retarding force, when the crank is at 180°, being *D C*. On the return stroke the forces of acceleration and retardation are the same as for the forward stroke, except that they are arranged in an inverse order, *D C* being the initial accelerating force, and *A C* the final retarding force, in pounds per square inch of piston area.

It is obvious that when the connecting-rod is of infinite length, $\frac{AB}{a} = 0$, and the initial and final forces are equal, and symmetrically arranged from mid-stroke to the two extremities, and are also the same for similar crank positions on forward and return strokes.

The rotative effect of the pressure on the piston, or the tangential component of the pressure that is transmitted from the piston to the crank-pin through the connecting-rod, really shows the effect of the distribution of the pressure on the piston more accurately than the corrected diagram from the cylinder of the engine, and it is often required to derive the diagram of rotative effect from the indicator diagram. Calling *P* the pressure in pounds per square inch on the piston at any point of the stroke, and α the angle made by the connecting-rod with the guides at this point, the component of this pressure transmitted through the connecting-rodto the crank-pin is $P \times Sec.\alpha$; and resolving this force into two components respectively parallel to and at right angles with the position of the crank, the component of rotative effect is obtained. The rotative effect can also be determined by the following graphical method, with sufficient accuracy for most practical purposes.

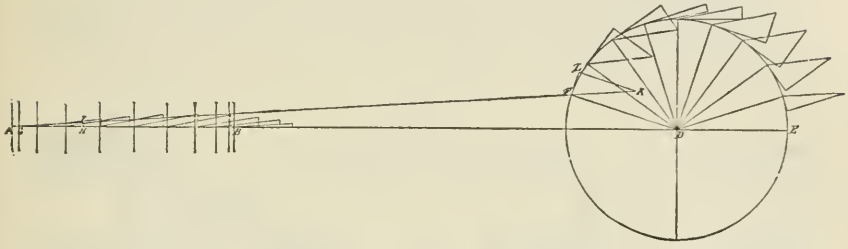
FIG. 806*b*.



Draw a circle to represent the path of the crank, with a radius *D C*, Fig. 806*b*, equal to the length of the crank, and divide this circle into any number

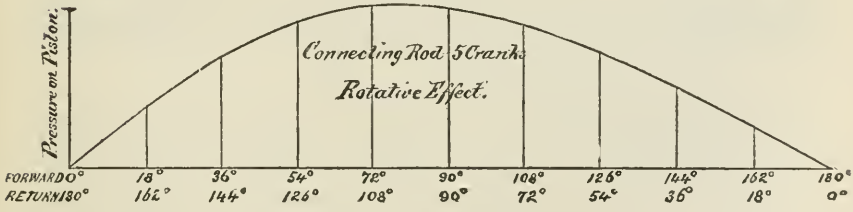
of equal parts—in the present instance 20. (A convenient method of effecting this division is shown in the figure. Bisect the radius $D\,T$ in G, and draw the straight line $C\,G$. From G as a centre describe the arc $D\,H$, and from C as a centre the arc $H\,I$, thus determining the length of the arc $C\,I$, which is one tenth of the circumference). Continue the line $E\,C$, and on the prolongation lay off a distance equal to $E\,C$, to represent the stroke, and in such a position that the distance from C to the further extremity, or from E to the

FIG. 806*c*.



nearer extremity, is equal to the length of the connecting-rod. Then from each division of the circumference as a centre, with a radius equal to the length of the connecting-rod, describe an arc cutting the line that represents the stroke, thus determining the piston positions corresponding to the crank positions assumed by the division of the circle, as will be evident by an inspection of the figure. Supposing the pressures on the piston at the positions so determined to be known, from G, the position of the piston corresponding

FIG. 806*d*.

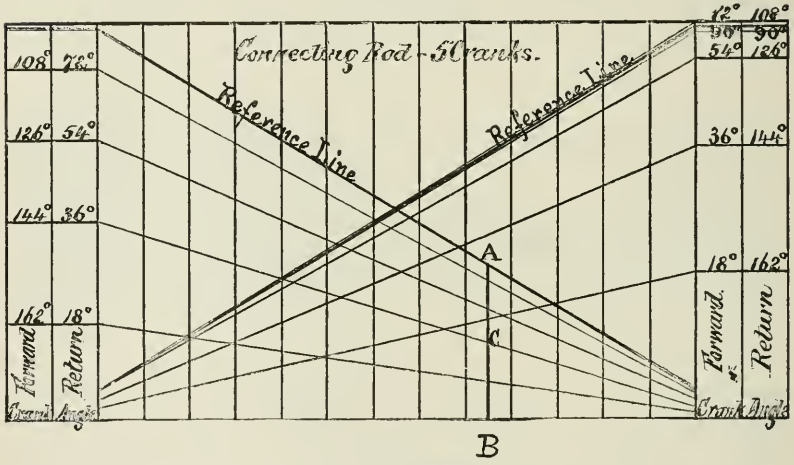

Pressure on Piston.
Connecting Rod & Crank.
Rotative Effect.
FORWARD 0° 18° 36° 54° 72° 90° 108° 126° 144° 162° 180°
RETURN 180° 162° 144° 126° 108° 90° 72° 54° 36° 18° 0°

to the crank position $D\,F$, Fig. 806*c*, lay off on the line $A\,B$ a distance $G\,H$ to represent the pressure on the piston at the point G, and connect the points G and F by a straight line. At H erect the perpendicular $H\,I$, which gives $G\,I$ as the component of the pressure transmitted through the connecting-rod to the crank-pin at this point. Continue the line $G\,F$ beyond the point F, and make $F\,K$ equal to $G\,I$. From F draw a straight line to $F\,L$, perpendicular to the position of the crank (or straight line connecting the centre of the shaft and centre of the crank-pin) $D\,F$, and from K draw another straight line parallel to $D\,F$. The point of intersection L of these two lines fixes the length $F\,L$ of the line which represents the rotative effect on the crank of the

pressure GH on the piston. Similar constructions are made at the other points of division of the stroke and corresponding crank positions, as indicated in the figure. By drawing a base line, Fig. 806*d*, to represent the developed path of the crank during a stroke of the piston, dividing it into the same number of equal parts as those into which the semi-circumference, Fig. 806*c*, was divided, and laying off on perpendiculars erected at the points of division the rotative pressures previously constructed, a curve can be passed through the extremities of the ordinates, which will represent the rotative effect of a given pressure on the piston at every point of the revolution.

A diagram for determining the rotative effect of any piston pressure can readily be laid down from the data furnished by the construction explained in Fig. 806*c*. It is merely necessary to lay off at the two extremities of a base

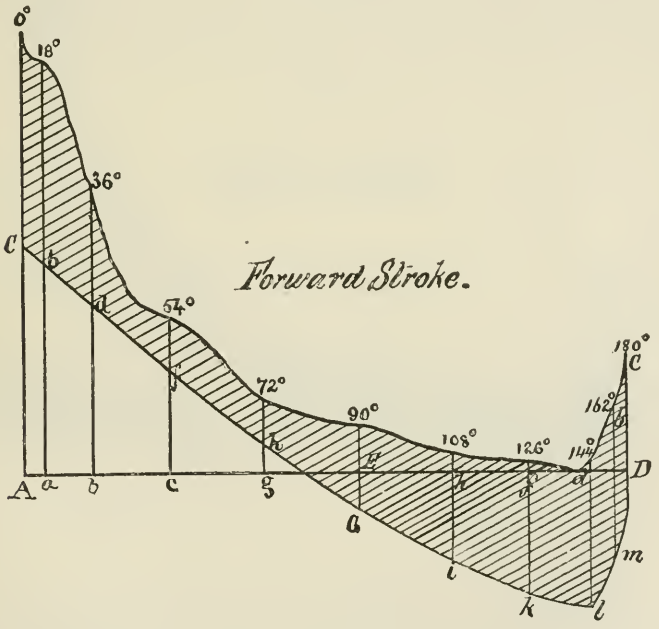
FIG. 806*e*.



line, Fig. 806*e*, any multiples of a given piston pressure and the corresponding rotative effects for various crank angles, and connect each pair of points by a straight line. Then it is evident that for a pressure represented by a straight line of any length within the limits of the diagram, the corresponding rotative effect for any crank angle can be measured from the base line at once. For example, in Fig. 806*e*, which is a diagram representing rotative effect for the case in which the connecting-rod is five times the length of the crank, and for crank angles corresponding to every twentieth of a revolution, suppose the pressure on the piston when it is at a point of the forward stroke corresponding to a crank angle of 144°, or a point on the return stroke corresponding to a crank angle of 36°, is represented by a straight line whose length is AB, and it is required to find the corresponding rotative effect on the crank at this point. It is evidently CB, since C is on the vertical line AB, which represents the given pressure, and also on the line which represents the rotative effect for the given crank angle. In using such a diagram in practice, a pair of dividers opened to the given distance AB is moved

along the base line until it encloses a vertical distance $B\,A$ between the base line and the reference line. Then, keeping one point of the dividers at B, close them until the other point cuts the line representing rotative effect for the given crank angle, when the distance between the points of the dividers will evidently be the length of a line representing the corresponding rotative effect. Perpendiculars to the base line at short intervals render it easy to measure distances vertically with the dividers. With a diagram of this nature, carefully constructed, the rotative pressures corresponding to given piston pressures can be determined with considerable accuracy, and more quickly than by the aid of tabular coefficients.

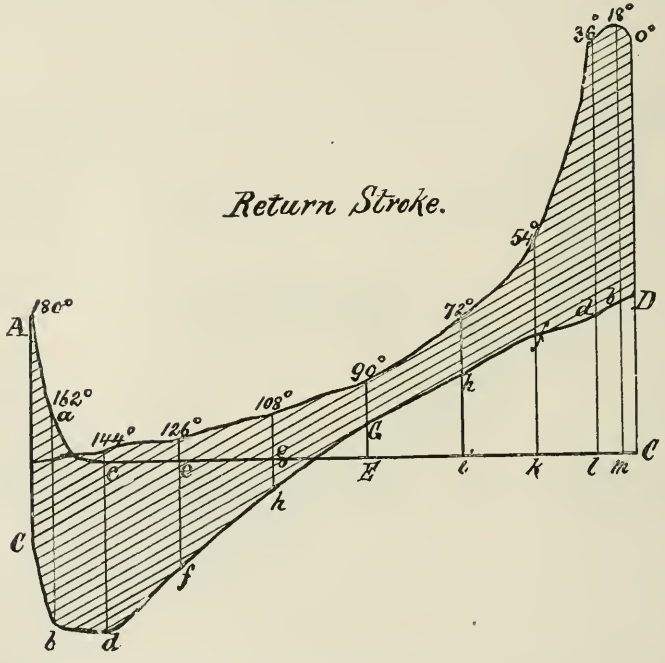
FIG. 806*f*.



In Figs. 806*f*, 806*g*, $0°\ d\ C\ D\ A$ and $0°\ a\ A\ C$ are intended to represent the true indicator diagrams taken from the two ends of a cylinder, the lines traced at the same time at opposite ends of the cylinder being combined in each diagram, and it is required to deduce from these diagrams the corresponding diagram representing the distribution of useful pressure on the crank-pin. The length of the diagrams $A\ D$, Fig. 806*f*, is first to be laid off to represent the stroke of the engine, and the points of the stroke corresponding to successive crank positions varying 18° from each other are to be constructed as explained in Fig. 806*b*. This gives the points $a\ b\ c\ g$, etc., Fig. 806*f*, and $m\ l\ k\ i$, etc., Fig. 806*g*, and at each of these points a perpendicular to the base line of the diagram is to be drawn. Then by constructing the lines repre-

senting the pressures required to accelerate and retard the reciprocating parts of the engine, as explained in connection with Fig. 806*a*, these pressures can be subtracted from or added to the indicated pressures at corresponding points of the stroke, according as they represent forces of acceleration or retardation. For example, in Figs. 806*f* and 806*a*, if the accelerating force at a piston position corresponding to a crank position of $O°$ is $A\,c$, Fig. 806*a*, this pressure is to be subtracted from the indicated pressure at that point, $A\,O°$, Fig. 806*f*, thus determining the pressure $CO°$ that is transmitted through the connecting-rod to the crank-pin.

FIG. 806*g*.

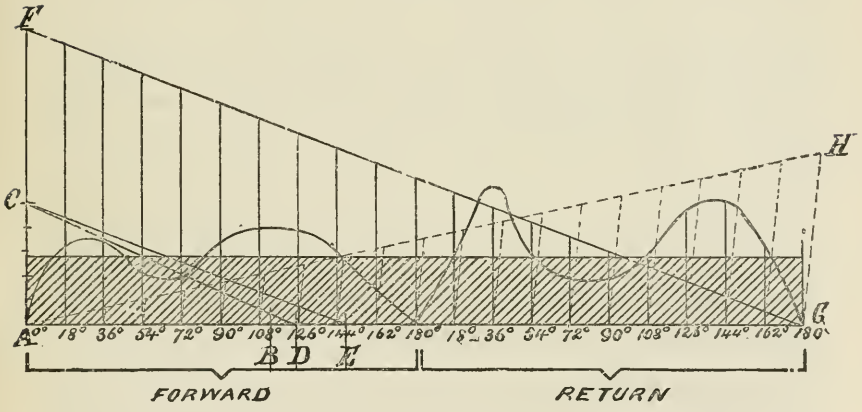


Similarly $a\,b$, Fig. 806*a*, is to be subtracted from a 18°, Fig. 806*f*; $c\,d$, Fig. 806*a*, is to be subtracted from b 36°, Fig. 806*f*; and so on, until retardation commences, as indicated in Fig. 806*a*, and then the pressures given out during retardation are to be added to the indicated pressures. Thus $E\,G$, Fig. 806*a*, is to be added to E 90°, Fig. 806*f*; $c\,h$, Fig. 806*a*, is to be added to h 108°, Fig. 806*f*; and so on in regular order, thus determining the diagram, in shaded portion of Fig. 806*f*, of pressure transmitted to the crank-pin on the forward stroke. In a similar manner the diagram of pressure transmitted to the crank-pin on the return stroke, being the shaded portion in Fig. 806*g*, is determined.

Having now the pressure on the piston that is transmitted to the crank-

pin at various points, the rotative effect of the pressure at each point can be laid off at once by the aid of a diagram similar to Fig. 806*e*. In Fig. 806*h*, draw a straight line $A\,G$ to represent the developed path of the centre of the crank-pin in a revolution. (A simple method of constructing the length of the circumference, when the diameter is given, is shown in the figure. On any straight line $A\,G$ lay off the diameter of the circle $A\,B$; at A draw a line $A\,F$ perpendicular to $A\,G$; make $A\,C$ equal to the radius of the circle, and divide it into five equal parts. From B lay off on $A\,G$ a distance $B\,D = \frac{1}{5}\,A\,C$, and a distance $B\,E = \frac{3}{5}\,A\,C$. Draw straight lines, $C\,D$, $C\,E$, make $A\,F$ equal to $C\,D$, and from F draw a straight line $F\,G$ parallel to $C\,E$, thus determining $A\,G$ the circumference of a circle whose diameter is $A\,B$. This construction is theoretically correct to within a half-millionth part of the circumference, and gives a very close approximation in practice if carefully drawn.) Divide this line into twenty equal parts, to represent the crank positions for

FIG. 806*h*.



every 18° of the revolution. Erect perpendiculars to $A\,G$ at the several points of division, and lay off on these perpendiculars distances representing the pressures producing rotative effect, as determined from diagrams similar to Figs. 806*e*, 806*f*, and 806*g*. The points so fixed may then be connected by a curve, Fig. 806*h*, representing the rotative effect at every point of the revolution; and if the area between this curve and the base line $A\,G$ be computed, a line can be drawn parallel to $A\,G$, enclosing an equal area, in the shaded portion, showing the mean rotative effect throughout the revolution, and the fluctuations of energy for which provision must be made by a fly-wheel or otherwise. It is easy to see that a variation in steam pressure, point of cut-off, piston speed, or weight of reciprocating parts, may produce marked variations in the diagram of rotative effect, and it is often useful, in comparing different engines, to make the diagrams of rotative effect one of the elements of comparison.]

§ **496.—Loss of Work in the Steam Engine.**—The theoretical delivery of the steam engine, as found by means of the above formulæ, is considerably diminished by many resistances, such as piston friction, cooling, loss of pressure in the passages, etc., so that the effective delivery comes out only from 40 to 70 per cent of the theoretical, as is shown by experiments with friction brake and dynamometer. As to the passages, by means of which the steam is conducted from the boiler to the steam chest and from there to the cylinder, they cause a diminution of the pressure, and the tension p of the steam in the cylinder which we have to introduce in the formula (see § 478) is not the same as the steam tension p_0 in the boiler, but less according to the losses in the passages. These losses are due to friction of the steam, to eddies and whirls caused by changes of direction and cross-section, and cooling of the enclosing walls. The diminution of the pressure in the conducting pipes, for fully opened steam valve, is only 1 to 5 per cent. By the position of this valve, which is usually a throttle, the difference $p_0 - p$ between the boiler and cylinder pressure may be increased at pleasure. In passing the steam valve, the steam remains in its saturated condition; the density, therefore, diminishes with the tension, and its capability of work remains almost unchanged. The relations of motion are very different from water. The work $\frac{(v_1 - v)^2}{2g} Q\gamma$ lost by a quantity of fluid $Q\gamma$, when its velocity is diminished by whirls and eddies from v_1 to v, furnishes an amount of heat which only in the case of water is lost, but for steam is utilized during expansion.

Another loss of work is due to the necessary excess of the steam pressure above that in the condenser or above the exterior air. Also sometimes considerable loss is occasioned by the boiler, water being carried out in mechanical mixture with the steam.

The piston friction is to be estimated precisely as in water-column engines (§ 320), as are also the losses caused by the slide valve.

§ **497.—Prejudicial Space.**—We have another loss of effect due to the prejudicial space in the steam engine. We understand by this that space which exists between the piston, when at the end of a stroke, and the slide valve or exhaust valve, and which must be again filled with fresh steam before it can act upon the piston. This space consists of two unequal portions: one portion is formed by the steam passage and the other by a part of the cylinder. If F_2 is the cross-section and l_2 the length of the steam passage, its contents are $F_2 l_2$, and if we put the height of the clearance between the piston and cylinder end $= \sigma_1$, we have for the contents, $F\sigma_1$. The entire prejudicial space is therefore

$$V_1 = F_2 l_2 + F\sigma_1 = F\left(\sigma_1 + \frac{F_2}{F} l_2\right).$$

For the sake of simplicity, we may express the space in the steam passage by a portion of the cylinder, and thus take for the height of the prejudicial space,

$$\sigma = \sigma_1 + \frac{F_2}{F} l_2,$$

and for this space itself

$$V_1 = F\sigma.$$

As a general rule, σ is not greater than $\frac{s}{20}$, or 5 per cent of the entire piston travel, hence the prejudicial space $= \frac{1}{20}$ of the entire space described by the piston. If the prejudicial space were zero, the amount of steam used per stroke would be $V = Fs$, but since it has always a certain size $F\sigma$, is filled at first with steam of the tension q, and afterwards, at the end of the stroke, with steam of the tension p, we have for each stroke the loss of steam $F\sigma\left(1 - \frac{q}{p}\right)$, or approximately $F\sigma$, since especially for condensing engines $\frac{q}{p}$ is a small fraction. Accordingly, for machines without expansion, the steam used per stroke is

$$V = F(s + \sigma),$$

hence

$$F s = \frac{s}{s+\sigma} V,$$

and the delivery, according to § 478, is

$$L = \frac{n}{30} F s (p - q) = \frac{n}{30} \frac{s}{s+\sigma} V (p - q),$$

or

$$L = \frac{s}{s+\sigma} (p - q) Q,$$

or

$$L = 144 \frac{s}{s+\sigma} (p - q) Q \text{ ft. lbs.}$$

EXAMPLE.—A steam engine without expansion has for the prejudicial space $\sigma = 0{\cdot}05\ s$, the delivery

$$L = \frac{s}{s + 0{\cdot}05\, s} \cdot 144 (p - q) Q = 0{\cdot}952 \times 144 (p - q) Q,$$

or about 5 per cent less than without prejudicial space. If therefore the theoretical delivery (see § 478, Example) were $L = 50$ horse-power, we should have, by reason of the prejudicial space,

$$L = 50 \times 0{\cdot}95 = 47{\cdot}5 \text{ horse-power.}$$

§ **498.**—In machines working with expansion, the prejudical space has considerable influence, since here, in each stroke, the steam volume $F(s + \sigma)$ becomes $F(s_1 + \sigma)$. The work during expansion is, therefore, according to Mariotte's law,

$$A_1 = F(s + \sigma) p \log. nat. \left(\frac{s_1 + \sigma}{s + \sigma}\right) = V p \log. nat. \left(\frac{s_1 + \sigma}{s + \sigma}\right).$$

The work lost by the back pressure, moreover, is not $F s q$, but $F s_1 q = \frac{V s_1}{s + \sigma} q$; hence the total effect per stroke is

$$A = \frac{s}{s+\sigma} Vp - \frac{s_1}{s+\sigma} Vq + Vp \; log.\; nat. \left(\frac{s_1+\sigma}{s+\sigma}\right)$$

$$= Vp \left[\frac{s}{s+\sigma} - \frac{s_1}{s+\sigma} \cdot \frac{q}{p} + log.\; nat. \left(\frac{s_1+\sigma}{s+\sigma}\right)\right],$$

or, per second,

$$L = 144\, Qp \left[\frac{s}{s+\sigma} + log.\; nat. \left(\frac{s_1+\sigma}{s+\sigma}\right) - \frac{s_1}{s+\sigma} \frac{q}{p}\right].$$

If we base our calculation upon Pambour's theory, we have, according to § 481, for the work, per revolution, during expansion, by putting $s + \sigma$ in place of s, and $s_1 + \sigma$ in place of s_1,

$$A_1 = F(\beta + p)(s + \sigma)\; log.\; nat. \left(\frac{s_1+\sigma}{s+\sigma}\right) - F\beta(s_1 - s)$$

$$= V\left[(\beta + p)\; log.\; nat. \left(\frac{s_1+\sigma}{s+\sigma}\right) - \frac{\beta(s_1 - s)}{s+\sigma}\right].$$

Hence, the total delivery per stroke is

$$A = \left[\frac{p\,s}{s+\sigma} - \frac{q\,s_1}{s+\sigma} + (\beta + p)\; log.\; nat. \left(\frac{s_1+\sigma}{s+\sigma}\right) - \beta \frac{(s_1 - s)}{s+\sigma}\right] V$$

$$= \left[\frac{s}{s+\sigma}(\beta + p) + (\beta + p)\; log.\; nat. \left(\frac{s_1+\sigma}{s+\sigma}\right) - \frac{s_1}{s+\sigma}(\beta + q)\right] V$$

$$= (\beta + p)\, V\left[\frac{s}{s+\sigma} + log.\; nat. \left(\frac{s_1+\sigma}{s+\sigma}\right) - \frac{s_1}{s+\sigma} \frac{\beta + q}{\beta + p}\right].$$

Finally, the delivery per second is

$$L = 144\, Q(\beta + p)\left[\frac{s}{s+\sigma} + log.\; nat. \left(\frac{s_1+\sigma}{s+\sigma}\right) - \frac{s_1}{s+\sigma} \cdot \frac{\beta + q}{\beta + p}\right]$$

$$= \frac{288}{125} \frac{\psi a}{640 - t_1} \cdot K\left[\frac{s}{s+\sigma} + log.\; nat. \left(\frac{s_1+\sigma}{s+\sigma}\right) - \frac{s_1}{s+\sigma} \cdot \frac{\beta + q}{\beta + p}\right]$$

ft. lbs. For the double cylinder or Woolf's engine, we have two prejudicial spaces, σ and σ_1, the first in the small and the second in the large cylinder, and hence, according to Mariotte's law,

$$\frac{p}{p_1} = \frac{F_1(s_1+\sigma_1)+F\sigma}{F(s+\sigma)+F_1\sigma_1};$$

therefore, the delivery per stroke is

$$A = Vp\left[\frac{s}{s+\sigma} - \frac{s_1}{s+\sigma}\cdot\frac{F_1}{F}\cdot\frac{q}{p} + log.\,nat.\left(\frac{F_1(s_1+\sigma_1)+F\sigma}{F(s+\sigma)+F_1\sigma_1}\right)\right],$$

and per second,

$$L = 144\,Qp\left[\frac{s}{s+\sigma} - \frac{s_1}{s+\sigma}\cdot\frac{F_1}{F}\cdot\frac{q}{p} + log.\,nat.\left(\frac{F_1(s_1+\sigma_1)+F\sigma}{F(s+\sigma)+F_1\sigma_1}\right)\right].$$

According to Pambour's theory, on the other hand, we have

$$L = 144\,Q\,(\beta+p)\left[\frac{s}{s+\sigma} - \frac{s_1}{s+\sigma}\cdot\frac{F_1}{F}\cdot\frac{\beta+q}{\beta+p} + log.nat.\left(\frac{F_1(s_1+\sigma_1)+F\sigma}{F(s+\sigma)+F_1\sigma_1}\right)\right] \text{ ft. lbs.}$$

Example.—How much effect is lost by a single-cylinder engine by reason of the prejudicial space, when this is $\frac{1}{20}$ of the stroke, the steam working at 4 atmospheres and without condensation, the cut-off taking place at $\frac{3}{8}$ of the stroke?

Without prejudicial space, we should have

$$L = 144\left(1 + log.\,nat.\,\tfrac{8}{3} - \tfrac{8}{3}\,\frac{0\cdot2922+1}{0\cdot2922+4}\right)(\beta+p)\,Q$$

$$= 144\,(1 + 0\cdot9808 - 0\cdot8028)\,(\beta+p)\,Q = 169\cdot6\,(\beta+p)\,Q \text{ ft. lbs.}$$

With prejudicial space, on the other hand, since $\frac{\sigma}{s_1} = \frac{1}{20}$ and $s = \frac{3}{8}\,s_1$, hence

$$\frac{\sigma}{s} = \frac{8}{3\times20} = \frac{2}{15},$$

$$L = 144 \left(\frac{15}{15+12} + log.\ nat.\ \frac{40+2}{15+2} - \frac{40}{17} \cdot \frac{1 \cdot 2922}{4 \cdot 2922} \right) (\beta + p)\, Q$$

$$= 144\,(0 \cdot 8823 + 0 \cdot 9045 - 0 \cdot 7083)\,(\beta = p)\, Q = 155 \cdot 3\ (\beta + p)\, Q.$$

Hence, by reason of the prejudical space, the loss of mechanical effect is

$$\frac{169 \cdot 6 - 155 \cdot 3}{169 \cdot 6}\, 100 = 8 \cdot 4 \text{ per cent.}$$

§ **499.—Piston Friction.**—The friction of the piston causes a considerable loss of work. This is to be estimated as in the case of water-pressure engines (§ 320). If the depth of packing is e, the piston diameter d, and the tension p, the force with which the piston presses against the sides of the cylinder is $\pi d e p$, and hence the corresponding friction is

$$R = \phi \pi d e p.$$

Since now the steam pressure is $\frac{\pi d^2}{4} p$, we have the ratio

$$\frac{R}{P} = \frac{4 \phi e}{d},$$

and hence we must multiply the steam pressure by $1 - \frac{4 \phi e}{d}$, in order to obtain the effective pressure upon the piston. We take, according to Vol. I., § 174, and in accordance also with Tredgold's assumption, for metal packing, $\phi = 0 \cdot 08$, and for hemp packing, $\phi = 0 \cdot 15$.

Since during expansion the tension decreases, the piston friction would decrease if the packing were pressed against the cylinder walls by the steam. But as this is accomplished in general by springs, we can take it as constant during the whole stroke. We must, moreover, take into account the back pressure, since this acts to oppose the penetration of the steam between the cylinder walls and the packing. We have accordingly, for every stroke, the effect absorbed by the piston friction,

$$\frac{4 \phi e}{d} F (p - q) s_1,$$

so that for a machine without expansion, where $s_1 = s$,

$$L = 144\, Q\,(p - q)\left(1 - \frac{4\,\phi\, e}{d}\right) \text{ ft. lbs.},$$

and with expansion,

$$L = 144\, Q\, p\left[1 + log.\ nat.\left(\frac{s_1}{s}\right) - \frac{q}{p_1} - \frac{4\,\phi\, e}{d}\cdot\frac{s_1}{s}\cdot\frac{p-q}{p}\right]$$

$$= 144\, Q\, p\left[1 + log.\ nat.\frac{s_1}{s} - \frac{q}{p_1} - \frac{4\,\phi\, e}{d}\cdot\frac{p-q}{p_1}\right],$$

or, acording to Pambour,

$$L = 144\,(\beta+p)\,Q\left[1 + log.\ nat.\left(\frac{s_1}{s}\right) - \frac{\beta+q}{\beta+p_1} - \frac{4\,\phi\, e}{d}\cdot\frac{s_1}{s}\cdot\frac{p-q}{\beta+p}\right]$$

$$= 144\,(\beta+p)\,Q\left[1 + log.\ nat.\left(\frac{s_1}{s}\right) - \frac{\beta+q+\frac{4\,\phi\, e}{d}(p-q)}{\beta+p_1}\right].$$

We have also friction of the piston rod in the stuffing box, which may be estimated precisely as the piston friction. If d_1 is the diameter of the rod and e_1 the depth of the stuffing box, we have for the friction,

$$R_1 = \phi\,\pi\, d_1\, e_1\,(p - q),$$

where q is again the back pressure. For similar packing, therefore,

$$\frac{R_1}{R} = \frac{d_1\, e_1}{d\, e},$$

and we have therefore to increase the piston friction by the amount $\frac{d_1\, e_1}{d\, e}$, in order to obtain the total friction.

The area of pressure is diminished by the cross-section of the piston rod, so that the pressure during descent of the piston is less than during its ascent. Since, however,

less steam is required in the first case, it is not necessary to consider this specially, but only to use in the calculation, in place of F, the mean value,

$$F = \frac{\pi}{4}\left(d^2 - \frac{d_1^2}{2}\right).$$

REMARK.—The losses of effect due to the valve gear are too numerous for special rules to be given for them. We have to do here only with a superficial calculation or estimate.

EXAMPLE.—What is the loss of effect due to piston friction in the machine given in the examples § 478 and § 480?

If we take, according to § 320, $\frac{\ell}{d} = \frac{1}{8}$ and $\phi = 0{\cdot}08$, we have, since

$$p - q = (3{\cdot}5 - 1) \times 14{\cdot}7 = 36{\cdot}75,$$

and, since $d = 1{\cdot}5$ ft., the piston friction,

$$R = 0{\cdot}08\ \pi \times \tfrac{1}{8} \times 1{\cdot}5^2 \times 36{\cdot}75 \times 144 = 3{\cdot}24 \times 36{\cdot}75 \times \pi = 374 \text{ lbs.};$$

hence the work consumed by this friction is

$$R\,s = \tfrac{10}{3} \times 374 = 1246 \text{ ft. lbs},$$

and therefore, for 24 revolutions per minute, the loss of work per second by reason of friction is

$$R\,v = 1246 \times \tfrac{24}{60} = 1246 \times \tfrac{1}{5} = 997 \text{ ft. lbs.} = 1{\cdot}81 \text{ horse-power.}$$

Since, in the example of § 478, the delivery is 45·34 horse-power, the friction consumes $\frac{1{\cdot}81}{45{\cdot}34} \cdot 100 = 3.9$ per cent.

§ 500.—Maximum Delivery.—For the sake of simplicity, we may take the piston friction R, together with the other resistances, as a pressure Fr; which, together with the back pressure Fq in the condenser, etc., opposes the motion of the piston, and thus, in the preceding formulæ, insert, instead of q, $q + r$. Here, of course, r is that part of the piston friction, etc., which comes upon each square inch of the piston area, and is equal to

$$\frac{R}{F} + \ldots = \frac{4\,\phi\,\ell}{d}(p - q) + \ldots.$$

The most general formula of Pambour for the delivery of a single-cylinder engine with expansion, takes then the form,

$$L = 144\, Q\,(\beta + p)\left[\frac{s}{s+\sigma} + log.\,nat.\left(\frac{s_1+\sigma}{s+\sigma}\right) + \frac{s_1}{s+\sigma}\cdot\frac{\beta+q+r}{\beta+p}\right].$$

The question now is, how far must the expansion be carried in order for a given quantity of steam to obtain the greatest effect, and what therefore should be the ratio $\frac{s_1}{s}$? Expansion is evidently advantageous so long as it gives an effect which exceeds that of the back pressure, piston friction, etc.—*i. e.*, so long as the steam pressure is greater than the back pressure $q + r$. If, however, it were less than the back pressure, the acting force would be negative, and the expansion would be carried further only by reason of the inertia of the machine at the expense of the total effect. In order that no such loss may take place and that the greatest result may be obtained, it is necessary to expand to such an extent that the steam tension at the end of the stroke p_1 shall be just equal to the back pressure $q + r$. Now, according to Navier's rule,

$$\frac{s+\sigma}{s_1+\sigma} = \frac{\beta+p_1}{\beta+p};$$

if therefore, we put $q + r$ in place of p_1, we have

$$\frac{s+\sigma}{s_1+\sigma} = \frac{\beta+q+r}{\beta+p},$$

or, neglecting σ,

$$\frac{s}{s_1} = \frac{\beta+q+r}{\beta+p},$$

or,

$$\frac{s}{s_1} = \frac{1}{\beta+p} \div \frac{1}{\beta+q+r}.$$

Hence, denoting the specific steam volumes $\frac{\alpha}{\beta+p}$ and

$\frac{\alpha}{\beta + q + r}$, corresponding to the tensions p and $q + r$, by μ and μ_1,

$$\frac{s}{s_1} = \frac{\mu}{\mu_1};$$

i. e., the best effect is produced when the travel of the piston before expansion is to the entire stroke as the specific steam volume of the entering steam is to that corresponding to the back pressure $q + r$.

If, according to Mariotte's law, we take $\beta = 0$, we have the rule,

$$\frac{s}{s_1} = \frac{q + r}{p},$$

which, for considerable tensions, gives too small values.

EXAMPLE.—How far should the expansion be carried in the examples to § 480 and § 481, in order to obtain the best effect?

Here $p = 3{\cdot}5 \times 14{\cdot}7 = 51{\cdot}45$, $q = 14{\cdot}7$, and $r = \frac{R}{F} = \frac{374}{254{\cdot}47} = 1{\cdot}469$. If, however, we take, by reason of other losses, twice this or $r = 2{\cdot}938$, we have $q + r = 17.638$.

But, now, the tension $p = 3{\cdot}5$ atmospheres corresponds to the specific steam volume 508, and the tension $q + r = 17{\cdot}638$ lbs. $= 1{\cdot}2$ atmospheres to the specific steam volume 1390. Therefore, the best expansion ratio is

$$\frac{s}{s_1} = \frac{508}{1390}, \text{ or about } \frac{4}{11}.$$

According to Mariotte's law, it is

$$\frac{s}{s_1} = \frac{1{\cdot}2}{3{\cdot}5}, \text{ or about } \frac{40}{117}.$$

§ **501.—Efficiency of the Steam Engine.**—The effective delivery of the steam engine may be approximately estimated by the aid of experimental coefficients, which vary somewhat for different sizes and systems, and the formulæ for the theoretical delivery. This method of calculation is due especially to Poncelet and Morin, and

the latter gives in his "Aide Mémoire de Mécanique pratique," and in his "Leçons de Mécanique pratique," the experimental numbers.

For machines without expansion, the delivery is

$$L_1 = \eta \cdot 144\ Q\ (p_0 - q_0) \text{ ft. lbs.},$$

where Q is the steam used per second, p_0 the steam tension in the boiler, and q_0 that in the condenser or in the air. The coefficient η, or the efficiency, increases with the size of the machine, but for a certain size seems to attain a maximum. The following table gives its values:

1. *For Low-pressure Engines.*

Size in Horse-powers.	Efficiency η.	
	Working condition.	
	Good	Ordinary
4 to 8	0·50	0·42
10 " 20	0·56	0·47
30 " 50	0·60	0·54
60 " 100	0·60	0·54

2. *For High-pressure Engines.*

Size in Horse-powers.	Efficiency η.	
	Working condition.	
	Good	Ordinary
below 10	0·50	0·40
10 to 20	0·55	0·44
20 " 30	0·60	0·48
30 " 40	0·65	0·52
40 " 50	0·70	0·56

EXAMPLE.—What is the delivery of a low-pressure steam engine without expansion, which has a stroke of 6 ft., diameter of cylinder 2½ ft., and makes 18 revolutions per minute, the steam having a temperature of 104° and the condenser 35°? The steam used per stroke is

$$V = \pi\,(\tfrac{5}{4})^2 \times 6 = 29{\cdot}45 \text{ cubic ft.},$$

and the tensions corresponding to the temperatures 104° and 35° are 1·148 and 0·057 atmospheres, hence the theoretical delivery of this machine per stroke is

$$P s = 144\ V(p_0 - q_0) = 144 \times 29{\cdot}45 \times 14{\cdot}7\ (1{\cdot}148 - 0{\cdot}057)$$
$$= 4240{\cdot}8 \times 14{\cdot}7 \times 1{\cdot}091 = 68012,$$

or, since the machine in one second goes through $\frac{2 \times 18}{60} = 0{\cdot}6$ of its stroke, the theoretical delivery per second is

$$L_1 = 0{\cdot}6 \times 68012 = 40807 = 74{\cdot}2 \text{ horse-power.}$$

If, now, we take the efficiency $\eta = 0{\cdot}60$, we have for the effective delivery,

$$L_1 = 0{\cdot}6 \times 40807 = 24484 \text{ ft. lbs.} = 44{\cdot}5 \text{ horse-power.}$$

The steam quantity $Q = 0{\cdot}6 \times 29{\cdot}45 = 17{\cdot}67$ cubic ft., which this machine uses per second, weighs, according to the table, § 391, for 1·152 atmospheres tension,

$$Q\gamma = \frac{62{\cdot}5 \times 17{\cdot}67}{1451} = \frac{1104{\cdot}37}{1451} = 0{\cdot}7611 \text{ lbs.}$$

and requires, if the feed water enters the boiler with a temperature of 30°, the amount of heat, approximately,

$$W = (640 - 30)\ 0{\cdot}7611 = 610 \times 0{\cdot}7611 = 464 \text{ calories.}$$

If, now, one pound of the fuel used gives only $\frac{9}{10} \times 7500 = 5625$ calories, and only 0·6 of this goes to the generation of steam, the consumption of fuel per hour is

$$\frac{60 \times 60 \times 464}{0{\cdot}6 \times 5625} = \frac{2784000}{5625} = 494 \text{ lbs.}$$

Since, now, the delivery is 44·5 horse-power, the consumption of fuel per hour for each horse-power is

$$K = \frac{494}{44{\cdot}5} = 11 \text{ lbs.}$$

§ **502.**—For engines with expansion, the effective delivery is in like manner,

$$L_1 = \eta \cdot 144\ Q\ p_0 \left(1 + \textit{log. nat.}\ \frac{p_0}{p_1} - \frac{q_0}{p_1}\right) \text{ ft. lbs.,}$$

where for $\frac{p_0}{p_1}$ we may put $\frac{F_1 s_1}{F s}$. Moreover, p_0 is the tension of steam in the boiler, and q_0 in the condenser. The efficiency η increases here also with the size. Its value may be found from the following table:

Size in Horse-powers.	Efficiency η.	
	Working condition.	
	Good	Ordinary
4 to 8	0·33	0·30
10 " 20	0·42	0·35
20 " 30	0·47	0·38
30 " 40	0·49	0·39
40 " 50	0·57	0·46
50 " 60	0·62	0·50
60 " 70	0·66	0·53
70 " 100	0·76	0·61

These coefficients can be applied to double as well as single cylinder expansion engines.

It is of course evident that these coefficients only hold good for mean velocity, mean cross-section of steam passages, etc.

REMARK.—The delivery of the locomotive and single-acting machines for raising water, such as the Cornish engine, will be treated of in the third volume. The theory of the link and valve motion will also be fully treated there.

EXAMPLE.—What is the delivery for a Woolf expansion engine, which, as in § 482, has the dimensions $d = 18$ inches, $s = 40$ inches, $d_1 = 30$ inches, and $s_1 = 50$ inches, makes 24 revolutions per minute, and uses steam at $3\frac{1}{2}$ atmospheres in the boiler and $\frac{1}{3}$ atmosphere in the condenser? According to the calculation in the above article, the theoretical delivery is $L = 134$ horse-power. If we take $\eta = 0{\cdot}7$, we have the effective delivery,

$$L_1 = 0{\cdot}7 \times 134 = 94 \text{ horse-power},$$

and we may therefore take in round numbers 90 horse-power. The steam quantity per second is

$$Q = \frac{24}{30}\left(\frac{3}{4}\right)^2 \pi \cdot \frac{40}{12} = \frac{3\pi}{2} = 4{\cdot}7124 \text{ cubic ft.}$$

It weighs $\frac{62\cdot5 \times 4\cdot7124}{535} = 0\cdot5505$ lbs., and requires $610 \times 0\cdot5505 = 336$ calories for its generation. If, now, one pound of fuel gives 5625 calories and only 0·6 take effect, the machine uses hourly $\frac{60 \times 60 \times 336}{0\cdot6 \times 5625} = 358$ lbs., and hence the amount used per hour per horse-power is

$$K = \frac{358}{90} = 3\cdot9 \text{ lbs.}$$

§ **503.—Pambour's Theory.**—Pambour, in his theory of the steam engine, takes the pressure upon the piston equal to the load of the machine reduced to the piston area, and takes this last as consisting of three parts—namely, of the useful load P_1, of a constant resistance R and of a changeable resistance δP_1, depending upon P_1 (compare § 140). We have, therefore, the mean piston pressure

$$P = P_1 + R + \delta P_1 = P_1 (1 + \delta) + R,$$

and inversely, the useful load,

$$P_1 = \frac{P - R}{1 + \delta}.$$

He also reduces these forces to the unit of area of the piston area $F = \frac{\pi d^2}{4}$, so that

$$P = Fp, \quad P_1 = Fp_1, \quad \text{and} \quad R = Fr_1.$$

Accordingly,

$$p = (1 + \delta) p_1 + r_1,$$

and the useful load per square inch of piston area is

$$p_1 = \frac{p - r_1}{1 + \delta}.$$

The constant loss of pressure r_1, corresponding to the constant resistance R, consists again of two parts, the back

pressure q and the piston friction, etc., r. Pambour takes for this last,

$$r = \frac{300}{d} \text{ pounds}$$

per square foot, or $r = \frac{25}{d}$ pounds per square inch, where d is taken in inches. The coefficient δ is taken $= 0 \cdot 14$, so that we have

$$p = 1 \cdot 14\, p_1 + q + r,$$

or

$$p_1 = \frac{p - (q + r)}{1 \cdot 14} = 0 \cdot 878\, [p - (q + r)].$$

The useful load in an engine without expansion is, therefore,

$$P_1 = F p_1 = \frac{F[p - (q + r)]}{1 + \delta} = 0 \cdot 878\, F\, [p - (q + r)] \text{ pounds,}$$

and the useful effect is

$$\begin{aligned} L_1 = P_1 v &= \frac{Fv}{1 + \delta}[p - (q + r)] \\ &= \frac{144}{1 + \delta}\, Q\, [p - (q + r)] \\ &= 0 \cdot 878 \times 144\, Q\, [p - (q + r)] \\ &= 126 \cdot 4\, Q\, [p - (q + r)] \text{ foot pounds.} \end{aligned}$$

For engines working expansively, p is variable, and, therefore, according to § 500,

$$L_1 = \frac{144\, Q}{1 + \delta}\left[\left(\frac{s}{s + \sigma} + log.\, nat.\, \frac{s_1 + \sigma}{s + \sigma}\right)(\beta + p) - \frac{s_1}{s + \sigma}(\beta + q + r)\right]$$

$$= 126 \cdot 4\, Q\left(\left[\frac{s}{s + \sigma} + log.\, nat.\, \left(\frac{s_1 + \sigma}{s + \sigma}\right)\right](\beta + p) - \frac{s_1}{s + \sigma}\left(\beta + q + r\right)\right)$$

foot pounds.

Völkers takes the back pressure per square inch for en-

gines with condensation $q = 2{\cdot}4$ pounds, and without condensation $q = 15$ pounds. Also he takes

$$r = r_1 + r_2 + r_3 + r_4,$$

where r_1 is the friction of the fly wheel, r_2 of the piston, r_3 the resistance of the air pump, and r_4 of the cold-water pump. According to his experiments, we have

1. For engines without condensation,

$$r = 0{\cdot}00033\frac{G}{d^2} + \frac{1{\cdot}212}{d}.$$

2. For ordinary engines with condensation,

$$r = 0{\cdot}00033\frac{G}{d^2} + \frac{1{\cdot}212}{d} + 0{\cdot}48 + 0{\cdot}009h.$$

3. For Woolf's engine,

$$r = 0{\cdot}00024\frac{G}{d^2} + \frac{1{\cdot}32}{d} + 0{\cdot}41 + 0{\cdot}008\,h.$$

4. For the Corliss engine,

$$r = 0{\cdot}00033\frac{G}{d^2} + \frac{1{\cdot}212}{d} + 0{\cdot}41 + 0{\cdot}008\,h,$$

where G is the weight of the fly wheel in pounds, d the diameter of the piston in inches, h the height to which the cold-water pump raises the water, and r the constant resistance reduced to pounds per square inch of piston area.

EXAMPLE.—For the single-cylinder expansion engine given in the examples, § 481 and § 482, we have, according to Morin, since the theoretical delivery has been found to be $L = 29$ horse-power, and therefore $\eta = 0{\cdot}50$, the effective delivery $L_1 = 0{\cdot}50 \times 29 = 14{\cdot}5$ horse-power. According to Pambour's theory, if $\sigma = \frac{1}{20}s_1$, $r = \frac{25}{18} = 1{\cdot}39$, and the tension p in the cylinder is 10 per cent less than in the boiler, therefore $p = 0{\cdot}9$, $p_0 = 0{\cdot}9 \times 3{\cdot}5 \times 14{\cdot}7 = 46{\cdot}3$ pounds, and if the tension of the steam at exit is 10 per cent greater

than in the condenser, or $q = 1 \cdot 1\, q_0 = 1 \cdot 1 \times 14 \cdot 7 = 16 \cdot 17$ pounds, we have the effective delivery,

$$L_1 = 0 \cdot 878 \times 271 \cdot 44 \left(\left[\frac{0 \cdot 4}{0 \cdot 4 + 0 \cdot 05} + log.\ nat. \left(\frac{1 \cdot 05}{0 \cdot 45} \right) \right] 50 \cdot 5 - \frac{21 \cdot 85}{0 \cdot 45} \right)$$

$$= 238 \cdot 3 \left[(0 \cdot 8888 + 0 \cdot 8473)\ 50 \cdot 5 - 48 \cdot 55 \right]$$

$$= 238 \cdot 3\, (1 \cdot 7361 \times 50 \cdot 5 - 48 \cdot 55) = 238 \cdot 3\, (87 \cdot 67 - 48 \cdot 55)$$

$$= 238 \cdot 3 \times 39 \cdot 12 = 9322 \cdot 3 \text{ foot pounds} = 17 \text{ horse-power},$$

or 17 per cent greater than according to Morin. By taking a greater difference of tensions, these results would agree more closely.

§ 504.—Formulæ for Delivery according to Pambour's Theory.—If we introduce, instead of the steam quantity Q, the corresponding amount of feed water M, and therefore put

$$Q = \frac{\alpha M}{\beta + p},$$

we have

$$L_1 = \frac{144\, \alpha}{1 + \delta} M \left[\frac{s}{s + \sigma} + log.\ nat. \left(\frac{s_1 + \sigma}{s + \sigma} \right) - \frac{s_1}{s + \sigma} \cdot \frac{\beta + q + r}{\beta + p} \right],$$

or

$$L_1 = \frac{144}{1 + \delta} \left(\left[\frac{s}{s + \sigma} + log.\ nat. \left(\frac{s_1 + \sigma}{s + \sigma} \right) \right] \alpha M - \frac{s_1}{s + \sigma} \left(\beta + q + r \right) Q \right),$$

and therefore the tension p of the steam in the cylinder is not necessary to be known.

We have still $Q = \frac{n}{30} F\ (s + \sigma)$ and $v = \frac{n}{30} s_1$, hence we may also introduce

$$Q = \frac{v}{s_1} F\, (s + \sigma) = \frac{s + \sigma}{s_1} F\, v,$$

and thus obtain

(1) $$L_1 = \frac{144}{1 + \delta} \left(\left[\frac{s}{s + \sigma} + log.\ nat. \left(\frac{s_1 + \sigma}{s + \sigma} \right) \right] \alpha M - (\beta + q + r)\, F\, v \right)$$

foot pounds.

By means of this formula, the delivery may be calculated from the *evaporative capacity* of the boiler, or from the amount of water M, converted into steam per second.

If we put still

$$M = \frac{\psi K}{62 \cdot 5\,(640 - t_1)},$$

where ψ is the heat furnished per pound of fuel, we have the delivery in terms of the consumption of fuel K,

$$(2)\quad L_1 = \frac{144}{1+\delta}\left(\left[\frac{s}{s+\sigma} + log.\,nat.\left(\frac{s_1+\sigma}{s+\sigma}\right)\right]\frac{\alpha\,\psi\,K}{62 \cdot 5\,(640 - t_1)} - (\beta + q + r)\,Fv\right) \text{ foot pounds.}$$

Völker calls the ratio $\frac{L_1}{M\gamma}$ of the useful delivery to the quantity of steam $M\gamma = \frac{Q\,\gamma}{\mu}$, the *value* (Güteverhältniss) of the machine. This ratio is, from the preceding,

$$\frac{L_1}{M\gamma} = \frac{144}{1+\delta} \cdot \frac{\alpha}{\gamma}\left[\frac{s}{s+\sigma} + log.\,nat.\left(\frac{s_1+\sigma}{s+\sigma}\right) - \frac{s_1}{s+\sigma} \cdot \frac{\beta+q+r}{6+p}\right],$$

and it increases with the tension p and the expansion ratio,

$$\varepsilon = \frac{s_1}{s}.$$

Pambour gives no rule for the determination of the tension p_0 in the boiler. In order to determine it from M and Q, or by means of the formula

$$p = \frac{\alpha M}{Q} - \beta,$$

it is only necessary to find the losses of tension by experiment, and add them to the tension p in the cylinder.

Völker takes, on the ground of his experiments, the loss of tension for steam valve entirely opened, $p_0 - p = 0 \cdot 031\,\frac{Fv}{F_1}$

pounds, where F is the cross-section of the piston, F_1 that of the steam passages, and v the velocity of the piston in feet.

Having thus determined the tension p_0 in the boiler, we have the corresponding steam volume under this tension,

$$Q_0 = \left(\frac{\beta + p}{\beta + p_0}\right) Q,$$

while the steam quantity measured under the mean pressure in the cylinder is

$$Q = \frac{s + \sigma}{s_1} F v.$$

In order to find by experiment the factor r of the constant resistance, we diminish the tension p of the steam in the boiler until it just suffices to put the unloaded machine in motion. Then the useful delivery of the machine is zero, or

$$\frac{s_1}{s + \sigma}(\beta + q + r) = \left[\frac{s}{s + \sigma} + log.\,nat.\left(\frac{s_1 + \sigma}{s + \sigma}\right)\right](\beta + p_m),$$

where p_m is the corresponding steam tension, and hence the constant resistance is

$$r = \left[\frac{s}{s_1} + \frac{s + \sigma}{s_1} log.\,nat.\left(\frac{s_1 + \sigma}{s + \sigma}\right)\right](\beta + p_m) - (\beta + q).$$

In order to determine the factor $1 + \delta$ of the variable resistance, we increase, for fully opened steam valve, the load until the machine is brought to rest, and observe the corresponding steam tension p_n. We have then

$$p_n = \frac{L_1}{F v} = \frac{30}{\pi s_1} \cdot \frac{L_1}{F} = \frac{s + \sigma}{s_1} \cdot \frac{L_1}{144\, Q}$$

$$= \frac{1}{1 + \delta}\left(\left[\frac{1}{1 + \delta} + log.\,nat.\left(\frac{s_1 + \sigma}{s + \sigma}\right)\right](\beta + p_n)\frac{s + \sigma}{s_1} - (\beta + q + r),\right.$$

and hence

$$1+\delta=\frac{1}{p_n}\left(\left[s+(s+\sigma)\,log.\,nat.\left(\frac{s_1+\sigma}{s+\sigma}\right)\right]\frac{\beta+p_n}{s_1}-(\beta+q+r).\right.$$

Example 1.—A high-pressure engine evaporates 20 cubic feet of water per hour, has a cylinder $1\frac{3}{4}$ ft. in diameter, makes 24 revolutions per minute, has a stroke of 5 ft., cuts off the steam at $\frac{1}{4}$ of the stroke, and the tension in the condenser is $\frac{1}{10}$ of an atmosphere. What is the delivery?

According to formula (1), since $\sigma=\frac{1}{20}s_1$,

$$L_1=126\cdot4\left(\left(\tfrac{5}{10}+log.\,nat.\,\tfrac{1}{2}\right)\frac{28961\times20}{60\times60}-\left(4\cdot120+1\cdot47+\frac{25}{21}\right)\left(\tfrac{1}{8}\right)^2\pi\frac{24\times2\times5}{60}\right)$$

$$=126\cdot4\left((0\cdot8333+1\cdot2528)\frac{28961}{180}-6\cdot78\times\frac{49}{16}\pi\right)$$

$$=126\cdot4\left(\frac{2\cdot0861\times28961}{180}-6\cdot78\times3\cdot0625\,\pi\right)$$

$$=126\cdot4\,(335\cdot64-65\cdot22)=34181\text{ ft. lbs.}=62\cdot14\text{ horse-power.}$$

The tension of the steam in the boiler is here unknown. That in the cylinder, however, is, before expansion, since the steam volume used per second is

$$Q=\frac{s+\sigma}{s_1}Fv=0\cdot3\times\frac{49\,\pi}{16}=2\cdot886\text{ cubic ft.}$$

$$p=\frac{\alpha M}{Q}-\beta=\frac{28961}{180\times2\cdot886}-4\cdot120=55\cdot750-4\cdot120=51\cdot630\text{ lbs.}$$

2. What quantity of water is used per second, in order that the mean piston pressure may be 7500 lbs.?

Since

$$v=\frac{24\times2\times5}{60}=4\text{ ft.,}$$

the delivery will be

$$L_1=4\times7500=30000\text{ ft. lbs.}$$

If we put, therefore, in the formula,

$$M = \frac{1 \cdot 14\, L_1 + 144\,(\beta + q + r)\, Fv}{144\, \alpha \left[\frac{s}{s+\sigma} + log.\ nat. \left(\frac{s_1 + \sigma}{s + \sigma} \right) \right]} \text{ cubic ft.},$$

this value for L_1, we have

$$M = \frac{1 \cdot 14 \times 30000 + 144 \times 65 \cdot 22}{144 \times 28961 \times 2 \cdot 0861} = \frac{34200 + 9391}{28961 \times 300 \cdot 39} = 0 \cdot 00501 \text{ cubic ft.},$$

or hourly, $3600 \times 0 \cdot 00501 = 18$ cubic ft.

§ **505.—Arrangement of a Steam Engine.**—Having now in the preceding deduced the more important rules for the calculation of the delivery of a steam engine, the inverse problem remains, viz., for a given delivery to determine the principal dimensions.

The first element to be determined is the steam quantity. We obtain this by inverting the formula for the delivery. If we take Morin-Poncelet's theory and put therefore the useful delivery,

$$\text{I.} \quad L_1 = \eta \times 144\, Q\, p_0 \left(1 + log.\ nat. \frac{F_1 s_1}{F s} - \frac{q_0}{p_1} \right) \text{ ft. lbs.},$$

we obtain the steam quantity,

$$\text{II.} \quad Q = \frac{L_1}{\eta \cdot 144\, p_0 \left(1 + log.\ nat. \frac{F_1 s_1}{F s} - \frac{q_0}{p_1} \right)} \text{ cubic ft.}$$

where, besides the delivery L_1, we need to know only the tensions p_0 and q_0, the expansion ratio

$$\varepsilon = \frac{F_1 s_1}{F s},$$

and the efficiency η. As a rule, machinists take η somewhat less than given by experiment, wherefore the effective delivery is generally greater than the nominal.

From the results of experiments given above (§ 501 and § 502), as well as from many others, it appears that the efficiency of an engine increases with its size, and thus approaches a certain limit. We have thus,

$$\eta = \frac{L_1}{L} = \frac{\mu \sqrt{L}}{1 + \nu \sqrt{L}},$$

where μ and ν are experimental coefficients, and L is the theoretical delivery in horse-powers.

1. For low-pressure engines, we may take with tolerable accuracy for $L = 4$ horse-power, $\eta = 0{\cdot}40$, and for $L =$ 100 horse-power $\eta = 0{\cdot}50$; hence

$$0{\cdot}4 = \frac{2\mu}{1 + 2\nu} \text{ and } 0{\cdot}5 = \frac{10\mu}{1 + 10\nu},$$

or

$$\mu = 0{\cdot}2 + 0{\cdot}4\nu \quad \text{and} \quad \mu = 0{\cdot}05 + 0{\cdot}5\nu,$$

so that

$$\nu = 1{\cdot}5 \quad \text{and} \quad \mu = 0{\cdot}8,$$

and the efficiency is

$$\eta = \frac{0{\cdot}8 \sqrt{L}}{1 + 1{\cdot}5 \sqrt{L}}.$$

We have, therefore, for

$L =$	1	4	9	16	25	36	49	64	81	100	144	225
$\eta =$	0·32	0·40	0·44	0·46	0·47	0·48	0·49	0·493	0·49	0·50	0·51	0·51

2. For Woolf engines or medium pressure with two cylinders, we have, according to Morin, for $L = 4$, $\eta = 0{\cdot}30$, and for $L = 100$, $\eta = 0{\cdot}566$; hence

$$\eta = \frac{0{\cdot}255 \sqrt{L}}{1 + 0{\cdot}351 \sqrt{L}},$$

and we obtain the following:

$L=$	1	4	9	16	25	36	49	64	81	100	144	225
$\eta=$	0·19	0·30	0·37	0·42	0·46	0·49	0·52	0·54	0·55	0·565	0·585	0·61

3. For high-pressure condensing engines, for $L = 4$, $\eta = 0{\cdot}34$, and for $L = 100$, $\eta = 0{\cdot}465$; whence

$$\eta = \frac{0{\cdot}506\sqrt{L}}{1 + 0{\cdot}988\sqrt{L}},$$

and for

$L=$	1	4	9	16	25	36	49	64	81	100	144	225
$\eta=$	0·25	0·34	0·38	0·41	0·43	0·44	0·45	0·45	0·46	0·465	0·47	0·48

4. For high-pressure non-condensing engines, for $L = 4$, $\eta = 0{\cdot}35$, and for $L = 100$, $\eta = 0{\cdot}517$; hence

$$\eta = \frac{0{\cdot}433\sqrt{L}}{1 + 0{\cdot}738\sqrt{L}};$$

hence for

$L=$	1	4	9	16	25	36	49	64	81	100	144	225
$\eta=$	0·25	0·35	0·39	0·43	0·46	0·48	0·49	0·50	0·51	0·515	0·525	0·535

§ **506.**—If the steam quantity Q is given, or has been calculated from the preceding formula, we may next fix the *mean piston velocity* v, and then determine the necessary area F of the piston.

In order to attain smooth motion and to diminish as much as possible the resistances, especially the losses of tension in the steam passages, we adopt a moderate velocity. According to Watt, the mean piston velocity should be 3·5 feet, 3 feet for small and 4 feet for large engines. The

increase of the velocity with the size of the engine affords the advantage that large engines require relatively smaller dimensions, smaller fly wheels, etc., than small ones. Watt's scale for piston velocities is as follows:

$L_1 =$	4 to 8	8 to 15	15 to 25	25 to 40	40 to 60	60 to 100 horse-power.
$v =$	34	37	40	43	46	50 inches.
$=$	2·83	3·08	3·33	3·58	3·83	4·17 feet.

Since the mean velocity has also a certain limit, we can put again

$$v = \frac{\mu \sqrt{L_1}}{1 + \nu \sqrt{L_1}}$$

where μ and ν are to be determined.

For $L_1 = 4$, $v = 34$ inches, and for $L_1 = 100$, $v = 50$ inches, therefore $34 = \frac{2\mu}{1 + 2\nu}$ and $50 = \frac{10\mu}{1 + 10\nu}$, hence for low-pressure steam engines,

$$\text{Ia.} \quad v = \frac{42{\cdot}5 \sqrt{L_1}}{1 + 0{\cdot}75 \sqrt{L_1}} \text{ inches.}$$

If we put in this formula $L_1 = \infty$, we have the greatest value for the mean piston velocity,

$$v = \frac{42{\cdot}5}{0{\cdot}75} = 57 \text{ inches} = 4{\cdot}75 \text{ feet.}$$

From the formula, we obtain the following scale:

$L_1 =$	1	4	9	16	25	36	49	64	81	100	144	225 horse-power.
$v =$	24	34	39	42·5	45	46	47	48	49	50	51	52 inches.
$=$	2·00	2·83	3·25	3·54	3·75	3·83	3·92	4·00	4·08	4·17	4·25	4·33 feet.

Medium and high pressure engines are often worked at higher velocities. Here, for $L_1 = 4$, $v = 40$ and for $L_1 = 100$, $v = 56$ inches, whence

$$\text{I b.} \quad v = \frac{56\sqrt{L_1}}{1 + 0{\cdot}9\sqrt{L_1}},$$

and the maximum value of v is

$$v = \frac{56}{0{\cdot}9} = 62 \text{ inches} = 5{\cdot}17 \text{ feet}.$$

In practice, a mean velocity of 6 feet is regarded as the utmost allowable, and as even too great for beam engines.

By means of the formula, we obtain the following scale:

$L_1 =$	1	4	9	16	25	36	49	64	81	100	144	225 horse-power.
$v =$	30	40	46	49	51	53	54	55	55·5	56	57	58 inches.
	2·50	3·33	3·83	4·08	4·25	4·42	4·50	4·57	4·62	4·67	4·75	4·83 feet.

The velocities in the last table are properly maxima, since in most cases the velocities for medium and high pressure engines lie between the values in the last two tables. According to Morin, indeed, high-pressure engines should have only the same velocity as low-pressure.

From the steam quantity Q and the mean piston velocity v, we can now determine, by means of the expansion ratio, $\varepsilon = \frac{s_1}{s}$, or, more exactly, $\varepsilon = \frac{s_1 + \sigma}{s + \sigma}$, the *piston area*,

$$\text{II.} \quad F = \varepsilon \frac{144\,Q}{v} \text{ square inches},$$

and hence the *cylinder diameter*,

$$\text{III.} \quad d = \sqrt{\frac{4F}{\pi}} = 1{\cdot}128\sqrt{F} = 13{\cdot}54\sqrt{\frac{\varepsilon Q}{v}} \text{ inches}.$$

§ 507.—Dimensions of the Steam Engine.—In order to determine the *length of stroke* as well as the other elements of an engine, it is necessary to know the number of revolutions per minute. For existing engines, this varies between 16 and 38, and there is therefore but little variation. According to Morin, we have the number n of revolutions as follows:

	Size of Engine in Effective Horse-Powers.					
	4—8	8—15	15—25	25—40	40—60	60—100
1. For Watt's engine	28	25	22	20	18	16
2. Woolf's engine	30	27	25	23	21	19
3. Single-cylinder high-pressure condensing:						
a. Without beam	38	34	30	28	26	25
b. With beam or oscillating cylinder	30	25	22	19	17	16
4. High-pressure engine non-condensing	38	34	30	28	26	24

Taking from this table the number n of revolutions per minute, we can, by means of the formulæ

$$s_1 = \frac{30\ v'}{n} \qquad \text{and} \qquad s = \frac{s_1}{\varepsilon} = \frac{30\ v'}{\varepsilon\ n},$$

determine the entire stroke as well as that portion of the stroke before the steam is cut off.

Since the ratio $\frac{s_1}{d}$ of the entire stroke s_1 to the piston diameter d is, for stationary engines of medium size, generally within the limits 2 and $2\frac{3}{4}$, and these limits are somewhat exceeded only for very small and very great stationary engines, it is preferable to determine before-hand the various values of $\frac{s_1}{d}$ for different systems and diameters, and thus find the stroke s_1 as well as the number of revolutions

$$n = \frac{30\ v'}{s_1}.$$

The number of revolutions is less for large engines than for small, and hence the first have a relatively less stroke than the last. We may put therefore,

$$\frac{s_1}{d} = \frac{\phi}{1 + \phi d}.$$

1. For low-pressure engines, we have generally

$$\text{for } d = 12 \text{ inches, } \frac{s_1}{d} = 2{\cdot}7, \text{ and}$$

$$\text{for } d = 48 \text{ inches, } \frac{s_1}{d} = 2{\cdot}0;$$

hence,

$$\frac{s_1}{d} = \frac{3{\cdot}058}{1 + 0{\cdot}01106\, d}.$$

We obtain from this formula the following table:

$d =$	6	12	18	24	30	36	42	48	54	60 in.
$\frac{s_1}{d} =$	2·87	2·70	2·56	2·42	2·30	2·19	2·09	2·00	1·91	1·84

2. For Woolf's engine or double-cylinder medium-pressure engines, we can use the same ratio, only here

$$\frac{s_1}{d_1} = \frac{3{\cdot}058}{1 + 0{\cdot}01106\, d_1},$$

where s_1 and d_1 are the stroke ard diameter of the piston in the large cylinder.

3. For high-pressure condensing engines, we distinguish those without and with balance beams. The first can make more revolutions than the last, and have therefore a smaller stroke.

(*a*) For high-pressure engines without beam, we have

$$\text{for } d = 12 \text{ inches}, \frac{s_1}{d} = 2{\cdot}50,$$

$$\text{for } d = 36 \text{ inches}, \frac{s_1}{d} = 1{\cdot}75;$$

hence

$$\frac{s_1}{d} = \frac{3{\cdot}182}{1 + 0{\cdot}02273\, d}.$$

We thus obtain the following table:

$d =$	6	12	18	24	30	36	42	48	54	60 in.
$\frac{s_1}{d} =$	2·80	2·50	2·25	2·06	1·89	1·75	1·63	1·52	1·43	1·35

(*b*) For high-pressure beam engines, we have,

$$\text{for } d = 12 \text{ inches}, \frac{s_1}{d} = 3{\cdot}25,$$

$$\text{for } d = 36 \text{ inches}, \frac{s_1}{d} = 2{\cdot}70;$$

hence,

$$\frac{s_1}{d} = \frac{3{\cdot}618}{1 + 0{\cdot}00945\, d}.$$

We thus obtain the following table:

$d =$	6	12	18	24	30	36	42	48	54	60 in.
$\frac{s_1}{d} =$	3·42	3·25	3·09	2·95	2·82	2·70	2·59	2·49	2·40	2·31

4. High-pressure non-condensing engines require for equal delivery, on the average, a piston diameter 8 per cent greater than condensing engines. Since for both, the

stroke $s_1 = \frac{30\,v'}{n}$, the ratio $\frac{s_1}{d}$ must be less in this case. We have, therefore,

(*a*) For high-pressure non-condensing engines, without beam,

$$\frac{s_1}{d} = \frac{3\cdot182\,(1-0\cdot08)}{1+0\cdot02273\,(1-0\cdot08)\,d} = \frac{2\cdot927}{1+0\cdot02091\,d},$$

and thus the table:

$d =$	6	12	18	24	30	36	42	48	54	60 in.
$\frac{s_1}{d} =$	2·60	2·34	2·13	1·95	1·80	1·67	1·56	1·46	1·37	1·30

Finally,

(*b*.) For high-pressure non-condensing engines *with* beam,

$$\frac{s_1}{d} = \frac{3\cdot618\,(1-0\cdot08)}{1+0\cdot00945\,(1-0\cdot08)\,d} = \frac{3\cdot3285}{1+0\cdot00869\,d}.$$

Hence the table:

$d =$	6	12	18	24	30	36	42	48	54	60 in.
$\frac{s_1}{s} =$	3·16	3·01	2·88	2·76	2·64	2·54	2·44	2·35	2·27	2·19

§ **508.**—For non-expanding engines, we have, of course, $s = s_1$; in a double-cylinder or Woolf's engine we must, however, distinguish between the stroke s_1 in the large or expansion cylinder from the stroke s in the small cylinder. In beam engines, the cylinders are placed one behind the other, so that the small one is nearer the axis of the beam than the large one, and s is about $\frac{3}{4}s_1$. We can therefore always regard the ratio $v = \frac{s_1}{s}$ as known, and have only to

find the ratio between F and F_1. A rule given in a former article serves to determine the velocity v of the piston in the large cylinder, and the following formula will serve to determine the area F of the piston in the small cylinder. Since the expansion ratio

$$\varepsilon = \frac{F_1 s_1}{F s}$$

may be considered as known, the area F_1 of the large piston is

$$\text{IV.} \quad F_1 = \varepsilon \frac{F s}{s_1} = \frac{\varepsilon}{\nu} F,$$

and the diameter of the large piston,

$$\text{V.} \quad d_1 = 1 \cdot 128 \sqrt{\frac{\varepsilon F}{\nu}}.$$

If, as sometimes happens, there is a certain expansion of steam in the small cylinder also, so that the steam is cut off at the end of the distance s_0, we have the expansion ratio,

$$\varepsilon = \frac{F_1 s_1}{F s_0},$$

or, denoting the expansion ratio of the steam in the small cylinder by ε_0, therefore, $s = \varepsilon_0 s_0$,

$$\varepsilon = \varepsilon_0 \frac{F_1 s_1}{F s}.$$

Accordingly, the small piston area F and its diameter are given by

$$\text{IV. } a. \quad F = \varepsilon_0 \frac{144\ Q}{v'} \text{ square inches.}$$

$$\text{V. } a. \quad d = \sqrt{\frac{4F}{\pi}} = 1 \cdot 128 \sqrt{F} = 13 \cdot 54 \sqrt{\frac{\varepsilon_0 Q}{v'}} \text{ inches,}$$

and the large piston area F_1 and its diameter d_1 are given by

IV. *b.* $$F_1 = \frac{\varepsilon}{\varepsilon_0}\frac{Fs}{s_1} = \frac{\varepsilon}{\varepsilon_0}\cdot\frac{F}{v} = \frac{\varepsilon}{v}\cdot\frac{144\,Q}{v'} \text{ square inches.}$$

V. *b.* $$d_1 = \sqrt{\frac{4F_1}{\pi}} = 1\cdot128\sqrt{F_1} = 13\cdot54\sqrt{\frac{\varepsilon\,Q}{v\,v'}} \text{ inches.}$$

If we take from the table, § 507, the number of revolutions n, we can find s and s_1 by the formulæ,

VI. *a.* $$s = \frac{30\,v'}{n}$$

VII. *a.* $$s_1 = v\,s = v\,\frac{30\,v'}{n}.$$

Also by means of the ratio in § 507,

$$\frac{s_1}{d_1} = \frac{\phi}{1+\phi\,d},$$

we can determine s_1 and s directly by

VI. *b.* $$s_1 = \frac{s_1}{d_1}\,s,$$

VII. *b.* $$s = \frac{s_1}{v} = \frac{30\,v'}{v\,n}.$$

EXAMPLE.—It is required to construct a Woolf engine of 25 horse-power useful delivery. Assume $p_0 = 3\cdot6$, $p_1 = 0\cdot6$, $q_0 = 0\cdot1$ atmosphere, also $\varepsilon_0 = \frac{3}{4}$. We have then

$$\varepsilon = \frac{F_1 s_1}{F s_0} = \frac{3\cdot6}{0\cdot6} = 6,$$

and for the quantity of steam per second, since $\eta = 0\cdot48$,

$$Q = \frac{25\times550}{0\cdot48\times144\times3\cdot6\times14\cdot7\left(1+\textit{log. nat.}\,6-\frac{0\cdot1}{0\cdot6}\right)}$$

$$= \frac{13750}{3658\,(1+1\cdot7918-0\cdot1666)} = \frac{13750}{3658\times2\cdot652}$$

$$= 1\cdot417 \text{ cubic feet.}$$

If we take the velocity v_1 of the large piston,

$$v_1 = 48 \text{ inches} = 4 \text{ ft.},$$

we have for that of the small piston,

$$v = \frac{s}{s_1} v_1 \frac{\tau'}{\nu} = \tfrac{3}{4} v_1 = 36 \text{ inches} = 3 \text{ ft.};$$

hence the area of this piston is

$$F = \varepsilon_0 \frac{144\ Q}{v} = \tfrac{3}{2} \frac{1 \cdot 417}{3} = 0 \cdot 7085 \text{ sq. ft.} = 102 \text{ sq. inches},$$

and the diameter is

$$d = 1 \cdot 128 \sqrt{102} = 11 \cdot 4 \text{ inches}.$$

Further, the area of the large piston is

$$F_1 = \frac{\varepsilon}{\varepsilon_0} \frac{F}{\nu} = \frac{6}{\tfrac{3}{2}} \cdot \frac{0 \cdot 7085}{\tfrac{4}{3}} = 3 \times 0 \cdot 7085 = 2 \cdot 1255 \text{ sq. ft.} = 306 \text{ sq. inches},$$

and hence its diameter is

$$d_1 = 1 \cdot 128 \sqrt{306} = 19 \cdot 7 \text{ inches, or about 20 inches.}$$

If we take $\frac{s_1}{d_1} = 2 \cdot 40$ (see Table, § 507), we have the stroke of the large piston,

$$s_1 = 2 \cdot 40 \times 20 = 48 \text{ inches} = 4 \text{ ft.},$$

and of the small piston,

$$s = \frac{s_1}{\nu} = \tfrac{3}{4} \times 48 = 36 \text{ inches} = 3 \text{ ft.}$$

Finally, the number of revolutions per minute is

$$n = \frac{30\ v_1}{s_1} = \frac{30\ v'}{s} = \frac{30 \times 4}{4} = 30.$$

§ 509.—Injection Water.—In condensing engines, the condenser and its pump require a special calculation. First, we have to determine the injection water M_1.

From the quantity of steam Q condensed, or

$$62\cdot5\,M = \frac{62\cdot5\,Q}{\mu} = \frac{62\cdot5}{28444}(1\cdot706+p)\,Q = \frac{(1\cdot706+p)\,Q}{455}\text{ lbs.},$$

as also from the temperature t_0 of the injection water, and the temperature t_2 in the interior of the condenser, we have, according to Watt's rule, the quantity M_1 of injection water by placing the amount of heat,

$$62\cdot5\,M_1\,(t_2 - t_0),$$

absorbed by M_1, equal to the quantity of heat,

$$62\cdot5\,M\,(640 - t_2),$$

which the steam loses when converted into water at t_2. Hence,

$$(t_2 - t_0)\,M_1 = (640 - t_2)\,M,$$

or

$$M_1 = \left(\frac{640 - t_2}{t_2 - t_0}\right)M = \left(\frac{640 - t_2}{t_2 - t_0}\right)\frac{Q}{\mu}\text{ cubic ft.}$$

According to Regnault (§ 380), we have

$$(t_2 - t_0)\,M_1 = (606\cdot5 + 0\cdot305\,t - t_2)\,M,$$

because the total heat of the steam at $t°$ temperature is

$$606\cdot5 + 0\cdot305\,t.$$

The amount of water necessary for condensation, therefore, is

$$M_1 = \left(\frac{606\cdot5 + 0\cdot305\,t - t_2}{t_2 - t_0}\right)M,$$

or the ratio of the injection to the feed water is,

$$\frac{M_1}{M} = \frac{606\cdot5 + 0\cdot305\,t - t_2}{t_2 - t_0}.$$

If we take the temperature of the injection water = 12° and that of the condenser = 35°, we have, by the first expression above,

$$\frac{M_1}{M} = \frac{640 - 35}{35 - 12} = \frac{605}{23} = 26{\cdot}3,$$

or by the second, if we take $t = 105°$,

$$\frac{M_1}{M} = \frac{606{\cdot}5 + 32 - 35}{35 - 12} = \frac{603{\cdot}5}{23} = 26{\cdot}2,$$

or almost precisely the same.

The difference comes out somewhat greater for engines of medium pressure. If, for example, we take $p = 4$ atmospheres, and the corresponding temperature $t = 144°$, we have, according to the second formula,

$$\frac{M_1}{M} = \frac{606{\cdot}5 + 43{\cdot}9 - 35}{35 - 12} = \frac{615{\cdot}4}{23} = 26{\cdot}8,$$

while the first formula gives, again,

$$\frac{M_1}{M} = 26{\cdot}3.$$

Since, then, the quantity of injection water is over 26 times as great as the feed water, we see at once that condensing engines are not always possible in all situations.

§ 510.—**Cold Water and Feed Pumps.**—From the quantity of injection water M_1, we can now determine the dimensions of the cold-water pump. We have

$$M_1 = \left(\frac{640 - t_2}{t_2 - t_0}\right) M = \left(\frac{640 - t_2}{t_2 - t_0}\right) \frac{Q}{\mu}.$$

If, now, we put $\frac{640 - t_2}{t_2 - t_0} = 26$, and for low pressure take $\mu = 1390$, while for medium pressure $p = 4$ atmospheres,

we take $\mu = 448$, we have the injection water for low-pressure engines,

$$M_1 = \frac{26\,Q}{1390} = 0{\cdot}0187\,Q,$$

and, on the other hand, for moderate pressure, of 4 atmospheres,

$$M_1 = \frac{26\,Q}{448} = 0{\cdot}0580\,Q.$$

If the cold-water pump is single acting, the product V_1 of the area and stroke of its piston will give the volume of water raised by it per revolution. The steam quantity per stroke is $2V = 2Fs$; hence we have

$$\frac{V_1}{2V} = \frac{M_1}{Q} = \frac{M_1}{\mu M}$$

We thus obtain, $\frac{V_1}{V} = \frac{2M_1}{\mu M} = \frac{2(640 - t_2)}{\mu(t_2 - t_0)} = 0{\cdot}0375$ for low-pressure, and $0{\cdot}1160$ for medium-pressure engines.

Since, however, some water always runs back, we must increase V_1 by at least 10 per cent, so that for low-pressure engines we have for the contents of the cold-water pump,

$$V_1 = 0{\cdot}041\,V.$$

According to Watt,

$$V_1 = \tfrac{1}{24}\,V,$$

and according to others,

$$V_1 = \tfrac{1}{18}\,V.$$

For medium pressure, also adding 10 per cent, we have,

$$V_1 = 0{\cdot}128\,V.$$

In practice, it is common to take $V_1 = \frac{1}{8}$ to $\frac{1}{6}$ of the cylinder space V, which is filled with fresh steam.

From the quantity of feed-water $M = \frac{Q}{\mu}$, we can find very easily the volume V_2 of the feed pump, or the product of the area and stroke of its piston. We have

$$\frac{V_2}{2\,V} = \frac{M}{Q} = \frac{1}{\mu},$$

or

$$V_2 = \frac{2}{\mu}\,V.$$

For low-pressure engines, tension 1·2 atmospheres, $\mu = 1390$, we have

$$V_2 = \frac{2}{1390}\,V = \frac{V}{695} = 0{\cdot}00144\,V,$$

while for engines of 4 atmospheres tension, where $\mu = 448$,

$$V_2 = \frac{2}{448}\,V = \frac{V}{224} = 0{\cdot}00446\,V.$$

To ensure rapid feed, however, this space is taken from three to six times as great as given by the formula.

§ 511.—**Air and Hot-Water Pump.**—The hot-water or air pump must have a certain size, since it has to pump off the water formed by the condensation of the steam, the injection water and the uncondensed steam of about $\frac{1}{10}$ atmosphere tension, as well as the air which is brought in by the injection water. The water to be pumped out is, per second, $M + M_1$, or about 28 M. Since, however, the injection water contains about $\frac{1}{14}$ of its volume of air, and this passes from the tension of 1 atmosphere to $\frac{1}{10}$ of an atmosphere, and from a temperature of 12° to 35°, this air occupies the

$\frac{10}{14}\,[1 + 0{\cdot}00367\,(35 - 12)] = \frac{5}{7}\,(1 + 0{\cdot}00367 \times 23) = 0{\cdot}775^{\text{th}}$ part of the space occupied by the water. Since, further, this air is mixed with steam of the same tension and tem-

perature, we have an almost equal volume of steam (§ 394), and therefore the volume of water, air, and steam lifted per second by the air pump is

$$M + M_1 (1 + 2 \times 0{\cdot}775) = M + 2{\cdot}55\ M_1,$$

or about

$$(1 + 2{\cdot}55 \times 26)\ M = 67\ M.$$

If we denote the capacity of the air pump by V_3, we have, as before,

$$\frac{V_3}{2V} = \frac{67\ M}{Q},$$

or the *capacity of the air pump* is

$$V_3 = \frac{134}{\mu} V.$$

For low-pressure engines, $\mu = 1390$, and

$$V_3 = \tfrac{134}{1390} V = \tfrac{1}{10} V.$$

For medium pressure, or 4 atmospheres, $\mu = 448$, and

$$V_3 = \tfrac{134}{448} V = \tfrac{3}{10} V.$$

According to Watt, we double this for safety. In Watt's engines, also, the stroke of the air pump is $\frac{1}{2}$ that of the steam piston, and the diameter is $\frac{2}{3}$, so that we have

$$V_3 = \tfrac{1}{2}(\tfrac{2}{3})^2 V = \tfrac{2}{9} V,$$

or fully double of the theoretical capacity.

Finally, as to the condenser, we give this a capacity

$$V_4 = \frac{V}{4} \text{ to } \frac{V}{3}.$$

§ 512.—**Dimensions of Steam Engine.**—From the steam quantity $V = Fs$, we can find the dimensions of the other parts of the engine. In order to make the cross-section of

the steam passage $\frac{1}{25}$ of the piston area, we make its width $d_1 = \frac{1}{5}$ of the piston diameter d. In high-pressure engines with little expansion, as, for instance, locomotives, this ratio of cross-section, as well as the exit pipe, may be $\frac{2}{25}$, and we therefore take the diameter $d_1 = \frac{2}{5} d$.

The principal dimensions of the boiler and furnace also depend upon the steam quantity Q or the heat W. The general rules will be found in § 404, *et al.*, so that it is only necessary here to point out the more essential relations.

The capacity of the boiler is made 15 to 20 times the quantity of water 3600 W evaporated per hour, and therefore equal to from 54000 W to 72000 W, of which 0·4 is steam and 0·6 water space (§ 405). The principal part of a boiler is the heating surface. We have already (§ 404) stated that we may estimate 4 pounds of steam per hour per square foot of heating surface. Assuming this, we have for the heating surface necessary to evaporate W cubic feet of water per hour,

$$F = \tfrac{6.6}{4} \times 3600\, W = 32400\, W \text{ square feet.}$$

According to the experiments of Wicksteed, the quantity of water evaporated per hour by 1 square foot of heating surface, for the Cornish trunk boiler, is

0·09 cubic ft. = 5·62 pounds,

and for Cornish cylindrical boilers, with interior furnace, which have a slow combustion, only

0·0143 cubic ft. = 0·89 pounds.

For marine and locomotive boilers, the combustion is much more rapid, and the steam quantity is two to three times as great as for the ordinary stationary steam boiler of equal heating surface.

Finally, as to the consumption of fuel necessary for the evaporation of the amount of water $M = \frac{Q}{\mu}$, this of course depends upon its quality. According to Wicksteed, as also

numerous recent experiments, 1 pound of good English hard coal gives from 7 to 8 pounds of steam, hence My pounds of steam require

$$K = \frac{My}{8} \text{ to } \frac{My}{7} \text{ pounds of coal.}$$

In Watt's engines without expansion, 10 to 13 pounds of good hard coal per hour are counted upon for each horse-power; in high-pressure non-condensing engines, only 8 to 11 pounds; condensing, 5 to 7 pounds; and, finally, for high-pressure non-condensing engines, without expansion, 17 to 20 pounds.

REMARK.—Further and more special information will be found in the "Ingenieur."

The other parts of the steam engine, such as the crank, fly-wheel, governor, etc., will be treated of in the third volume of this work. The theory of the slide-valve and of link and valve motions will also be fully treated there.

If the performance of a proposed engine could be certainly foretold, in advance of its construction, it would, of course, be easy to proportion it so that any desired conditions could be fulfilled. In practice, however, it is necessary to make certain assumptions, but if these are based on the records of reliable experiments they give results in general whose close approximation to absolute accuracy leaves little to be desired. The present note furnishes examples, rules, and tables for determining the proper proportion of steam engines under most circumstances that occur in practice.

In designing an engine for any given purpose, the principal points to be determined are:

The dimensions of the cylinder.

The consumption of steam.

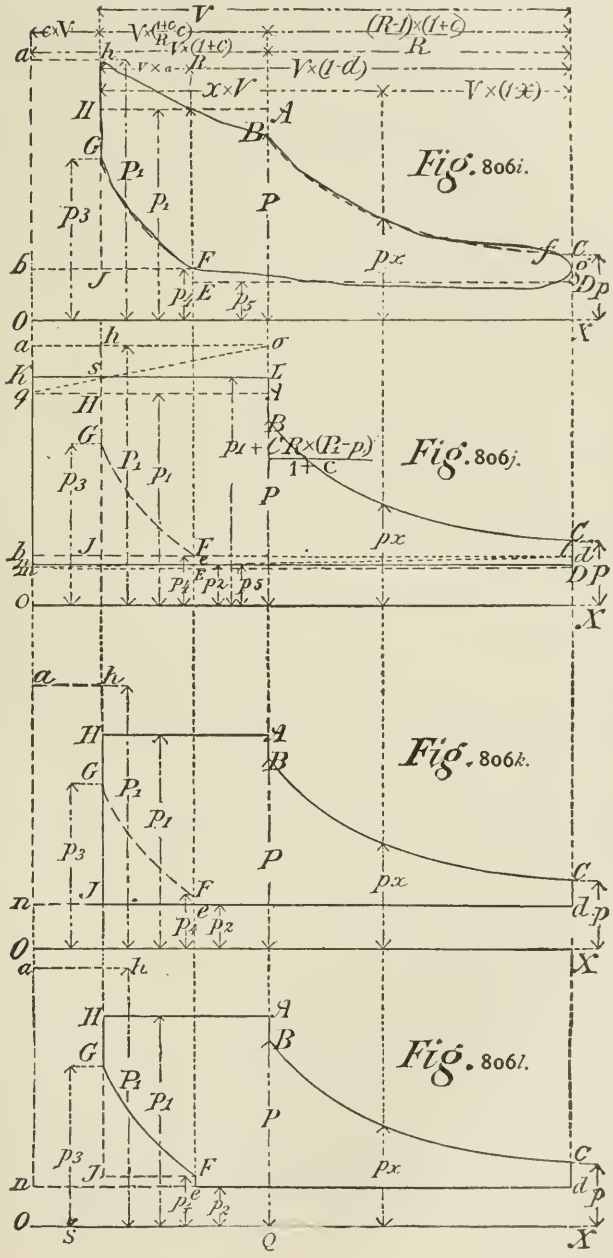
As the problem is usually presented, it is required to determine the diameter and length of stroke of an engine that will develop a given horse-power, the following elements being given:

Initial pressure of steam.

Point of cut-off.

Revolutions per minute, or piston speed in feet per minute.

Now it is evident that if the mean effective pressure of the steam at each stroke can be determined, or if the indicator diagram that will be taken can be laid down in advance of the construction of the engine, the dimensions of the steam cylinder can be obtained by a simple calculation. The diagram in full lines, Fig. 806*i*, is such as would be given by an engine of good design, with the exception that some of the peculiarities are exaggerated for the sake of clearness.


V
(R-1)×(1+c)
R
V×(1-d)
x×V
V×(1-x)
Fig. 806i.
Fig. 806j.
Fig. 806k.
Fig. 806l.
P
P1
P2
P3
P5
px
S
Q
X
Y

In this figure, $O\,X$ is the zero line to which all pressures are referred, and by following the diagram through a complete revolution of the engine, the elements that are known and those that must be assumed can readily be traced.

1. It will be seen that the steam acts upon the piston at the commencement of the stroke with the initial pressure P_1, which pressure has diminished to P at the point of cut-off, B, where the steam valve closes.

Exactly how much the pressure will fall between the points of admission and cut-off, in any particular case, depends upon the area of port and piston speed; and by using the dimensions given in Art. 461, a difference of five pounds between P_1 and P will be an ample allowance. When this difference is assumed, an equivalent line, $H\,A$, can be drawn, representing mean pressure up to point of cut-off $= \dfrac{P_1 + P}{2}$.

2. When the steam valve has closed, the steam in the cylinder expands from B to f, in the curve shown, and at f the exhaust valve opens.

In general it is sufficiently accurate to assume that the steam expands in an isothermal curve, the pressure varying inversely as the volume. In cases where the steam is dry, or nearly so, as when steam jackets are used, it is somewhat more accurate to assume that the pressure of the steam varies inversely as the $\frac{17}{16}$th power of the volume, this being a close approximation to the actual relation between the pressure and volume of dry saturated steam, for ordinary pressures and ratios of expansion. If, then, the ratio of expansion is known, the curve of expansion can be laid out in accordance with either of the principles just stated. It has already been explained, in Art. 461, that the points at which cut-off, release, and cushion commence can be fixed at pleasure, so that any assumptions in regard to these points can be verified in a proposed engine. But the fraction of stroke completed up to point of cut-off is only the *apparent cut-off*, since all steam cylinders have clearance space in which steam also expands, and if c has the same ratio to the stroke of piston as the clearance space at one end of the cylinder has to the space swept through by the piston per stroke, it is evident that the *real cut-off* is

$$\frac{\text{distance from commencement of stroke to point of cut-off} + c}{\text{stroke of piston} + c.}$$

Hence the volume of clearance must be known before the ratio of expansion and the corresponding curve of expansion can be determined. This can be calculated with precision for any particular case, and in general practice it is found to vary between 5 and 10 per cent of the volume of the cylinder, or space swept through by the piston per stroke—being nearer the former figure, in the best practice. The clearance for a proposed design can be safely assumed, in the preliminary calculation, between these figures. As to the point of cut-off that is most desirable, this varies somewhat with steam pressure, size of cylinder, and piston speed, and can be best determined by a reference to the practical examples in another part of this note.

3. On the return stroke of the piston, the exhaust valve remains open

until cushion commences, with a slightly varying back pressure, which increases to p 4 at the point F, where the exhaust valve closes.

With exhaust ports of ample size, it will be safe to assume the mean back pressure, exclusive of cushion, between one and two pounds above the atmosphere per square inch of piston area in non-condensing, and between four and five pounds above zero in condensing engines. There are many engines giving much better results but the foregoing figures probably represent a fair average.

4. When the exhaust valve closes, the steam remaining in the cylinder is compressed in the curve $F\,G$, its pressure increasing from p 4 to p 3, and at G steam is again admitted to the cylinder, changing the pressure from p 3 to P_1.

The compression curve is ordinarily assumed to follow the same law as the curve of expansion. The amount of compression required in any given case depends upon the initial pressure, clearance, and weight and velocity of the reciprocating parts of the engine. It is not desirable to compress the steam to a higher point than the initial pressure, and it is a good plan, if possible, to make the mean total compression pressure equal to the mean pressure due to the retardation of the reciprocating parts during the fraction of stroke in which cushion occurs. The most economical ratio of compression is generally the same as the ratio of expansion, but the most advantageous ratio in a practical point of view, as already noticed, depends upon the energy of the reciprocating parts.

It is not necessary, in general, to lay down the diagram of a proposed engine on paper, if rules for making the necessary calculations are furnished, and the process of calculating the several parts of the diagram will be somewhat as follows:

a. Fig. 806*i*, as already stated, shows the general form of the diagram that it is assumed will be produced. Find $H\,A$, the mean pressure, exclusive of clearance, up to point of cut-off $= \dfrac{P_1 + P}{2}$.

b. In Fig, 806*j*, add to the length of the diagram a distance representing the fraction of clearance reduced to equivalent length of stroke on the scale of the diagram, supposing the original length of the diagram to represent the stroke of the piston. Then find the line $K\,L$, representing mean pressure, clearance included, up to the point of cut-off, and lay off the expansion curve $B\,C$, in accordance with the assumed law of expansion. Also find a line $n\,D$ to represent the mean back pressure, independent of cushion, but including clearance.

c. In Fig. 806*k*, lay off the curve of compression, $F\,G$, and subtracting the back pressure due to compression, there remains a diagram $H\,A\,B\,C\,d\,e\,F\,G$, Fig. 806*l*, which is equivalent in area to the one which it is assumed the engine will produce.

Table *A* contains a series of formulas for obtaining the area of this diagram, when the proper assumptions are made, and in the succeeding tables will be found constants for use in the calculations.

Table *B* gives the real ratio of expansion for any apparent ratio, when the fraction of clearance is known.

For example, in the case of an engine cutting off steam at one fourth of the stroke, with 8 per cent clearance, the *apparent cut-off* is of course 0·25, but the *real cut-off*, as given by the table, is 0·306, and the *real ratio of expansion* $\frac{1}{0\cdot306} = 3\cdot268$.

Notation for Tables A, B, and C.

A = area of piston, in square inches.
S = length of stroke, in feet.
s = distance from commencement of stroke to point of release, in feet.
D = distance, in feet, from commencement of stroke to point of cut-off.
c = fraction of clearance.
P_1 = initial pressure of steam.
P = pressure of steam at point of cut-off.
p_1 = mean pressure up to point of cut-off, clearance space not included.
p_2 = mean back pressure, exclusive of cushion.
p_4 = back pressure at point where cushion commences.
t = temperature of steam, on Fahrenheit scale, at pressure P.
t_x = temperature of steam, on Fahrenheit scale, at pressure $p\ x$.
w_x = weight, in pounds, of a cubic foot of steam, at pressure x.

All pressures, when not otherwise stated, are total pressures (above a vacuum), in pounds per square inch.

Table A.—Formulas for the Expansion of Steam.

Quantity.	Symbol.	General Formula.	Formula for isothermal curve of expansion.	Approximate formula for dry saturated steam.	Number of formula.
Space swept through by piston per stroke, in cubic feet—represented on an indicator diagram by the length of the diagram (see Fig. 806*i*).	V	$\frac{A \times S}{144}$			1
Space swept through by piston up to point of cut-off, in cubic feet—represented on anindicator diagram by that portion of the length from commencement to cut-off (see Fig. 806*i*).	v	$V \times \left(\frac{1+c}{R} - c\right)$			2
Volume of clearance at one end of cylinder, in cubic feet—represented on an indicator diagram by an addition to the length of the diagram (see Fig. 806*i*).	C	$c \times V$			3
Apparent ratio of expansion.	r	$\frac{S}{D}$			4
Apparent cut-off.	$\frac{1}{r}$	$\frac{D}{S}$			5
Real ratio of expansion.	R	$\frac{1+c}{\frac{1}{r}+c}$	$\frac{P}{p}$	$\left(\frac{P}{p}\right)^{\frac{10}{17}}$	6
Real cut-off.	$\frac{1}{R}$	$\frac{\frac{1}{r}+c}{1+c}$	$\frac{p}{P}$	$\left(\frac{p}{P}\right)^{\frac{10}{17}}$	7
Terminal pressure.	p		$P \times \frac{1}{R}$	$P \times \left(\frac{1}{R}\right)^{\frac{17}{10}}$	8

TABLE A.—FORMULAS FOR THE EXPANSION OF STEAM—(*Continued*).

QUANTITY.	SYMBOL.	GENERAL FORMULA.	Formula for isothermal curve of expansion.	Approximate formula for dry saturated steam.	Number of formula.
Pressure at point of cut-off.	P		$p \times R$	$p \times R^{\frac{17}{16}}$	9
Pressure at point distant $x \times S$ from commencement of stroke (x being a proper fraction).	p_x		$P \times \frac{\frac{1}{r}+c}{x+c}$	$P \times \left(\frac{\frac{1}{r}+c}{x+c}\right)^{\frac{17}{16}}$	10
Mean pressure up to point of cut-off, clearance space included (see Fig. 806*f*).	P_i	$p_1 + \frac{c\,R \times (P_1 - p_1)}{1+c}$			11
Mean effective pressure, uncorrected for clearance and cushion (being the mean altitude of the diagram *K L B C D n*, Fig. 806*f*).	P_m		$\frac{P_i + P \times hyp.\ log.\ R}{R} - p_2$	$\frac{P_i + 16\,P \times \left[1 - \left(\frac{1}{R}\right)^{\frac{1}{16}}\right]}{R} - p_2$	12
Mean effective pressure, corrected for clearance, no cushion (being the mean altitude of the diagram *H A B C d J*, Fig. 806*k*).	p_m	$P_m - c \times (P_1 - P\,m)$			13
Fraction of stroke uncompleted when cushion commences.	d		$c \times \left(\frac{p_3}{p_4} - 1\right)$	$c \times \left[\left(\frac{p_3}{p_4}\right)^{\frac{16}{17}} - 1\right]$	14
Ratio of compression for cushion.	N	$\frac{d+c}{c}$	$\frac{p_3}{p_4}$	$\left(\frac{p_3}{p_4}\right)^{\frac{16}{17}}$	15
Final cushion pressure.	p_3		$p_4 \times N$	$p_4 \times N^{\frac{17}{16}}$	16

TABLE A.—FORMULAS FOR THE EXPANSION OF STEAM—(*Continued*).

QUANTITY.	SYMBOL.	GENERAL FORMULA.	Formula for isothermal curve of expansion.	Approximate formula for dry saturated steam.	Number of formula.
Mean cushion pressure (being the mean altitude of the diagram $GF\mathcal{F}$, Fig. 806*l*).	p_c		$p_3 \times \frac{hyp.\ log.\ N}{N-1}$	$p_3 \times \frac{16 \times \left[1-\left(\frac{1}{N}\right)^{\frac{1}{16}}\right]}{N-1}$	17
Mean effective pressure, corrected for clearance and cushion (being the mean altitude of the diagram $HABCdeFG$, Fig. 806*l*).	P_c	$p_m - c \times (N-1) \times (p_c - p_4)$			18
Mean total pressure during expansion (being the mean altitude of the diagram $BCXQ$, Fig. 806*l*).	p_r		$P \times \frac{hyp.\ log.\ R}{R-1}$	$P \times \frac{16 \times \left[1-\left(\frac{1}{R}\right)^{\frac{1}{16}}\right]}{R-1}$	19
Portion of mean total pressure due to expansion (being area of diagram $BCXQ$, Fig. 806*l*, divided by total length of diagram, OX).	P_2		$P \times \frac{hyp.\ log.\ R}{R}$	$P \times \frac{16 \times \left[1-\left(\frac{1}{R}\right)^{\frac{1}{16}}\right]}{R}$	20
Mean total pressure, when pressure at point of cut-off is equal to initial pressure, uncorrected for clearance and cushion.	P_t		$P_1 \times \frac{1+hyp.\ log.\ R}{R}$	$P_1 \times \left[17 \times \frac{1}{R} - 16 \times \left(\frac{1}{R}\right)^{\frac{17}{16}}\right]$	21
Ratio of mechanical effect before expansion, to total mechanical effect (being area of diagram $HAQS$, Fig. 806*l*, divided by area of diagram $HABCXS$).	M	$\frac{p_1}{r \times (p_m + p_2)}$			22

Table A.—Formulas for the Expansion of Steam—(*Continued*).

Quantity.	Symbol.	General Formula.	Formula for isothermal curve of expansion.	Approximate formula for dry saturated steam.	Number of formula.
Ratio of mechanical effect during expansion, to total mechanical effect (being area of diagram $BCXQ$, Fig. 806, divided by area of diagram $HABCXS$).	m	$\frac{p_r \times (r-1)}{r \times (p_m + p_2)}$			23
Units of heat converted into work, at each stroke of piston.	H	$\frac{(p_m + p_2) \times m \times A \times S}{772}$			24
Pounds of steam condensed for work, at each stroke of piston.	W	$\frac{H}{\text{Latent heat in a lb. of steam at terminal pressure}}$			25
Temperature of steam at any point of expansion curve.	t_x			$(t+100) \times \left(\frac{p_x}{P}\right)^{\frac{1}{6}} - 100$	26
Pounds of steam used per stroke, by terminal pressure.	W_t	$\left(C + \frac{A \times S}{144}\right) \times w_p$			27
Pounds of steam saved per stroke, by cushioning.	W_c	$\left(C + \frac{d \times A \times S}{144}\right) \times w p_4$			28
Pounds of steam per stroke accounted for by indicator.	W_i	$W_t + W - W_c$			29

TABLE B.

REAL CUT-OFF $\frac{1}{R}$ CORRESPONDING TO APPARENT CUT-OFF $\frac{1}{r}$ FOR DIFFERENT FRACTIONS OF CLEARANCE.

Apparent Cut-off $\frac{1}{r}$.	Real Cut-off $\frac{1}{R}$ for Fraction of Clearance of										Apparent Cut-off $\frac{1}{r}$.
	·01	·02	·03	·04	·05	·06	·07	·08	·09	·1	
·01	·020	·029	·039	·048	·057	·066	·075	·083	·092	·100	·01
·02	·030	·039	·049	·058	·067	·075	·084	·093	·101	·109	·02
·03	·040	·049	·058	·067	·076	·085	·093	·102	·110	·118	·03
·04	·050	·059	·068	·077	·086	·094	·103	·111	·119	·127	·04
·05	·059	·069	·078	·087	·095	·104	·112	·120	·128	·136	·05
·06	·069	·078	·087	·096	·105	·113	·122	·130	·138	·145	·06
·07	·079	·088	·097	·106	·114	·123	·131	·139	·147	·155	·07
·08	·080	·098	·107	·115	·124	·132	·140	·148	·156	·164	·08
·09	·099	·108	·117	·125	·133	·142	·150	·157	·165	·173	·09
·10	·109	·118	·126	·135	·143	·151	·159	·167	·174	·182	·10
·11	·119	·127	·136	·144	·152	·160	·168	·176	·183	·191	·11
·12	·129	·137	·146	·154	·162	·170	·178	·185	·193	·200	·12
·13	·139	·147	·155	·163	·171	·179	·187	·194	·202	·209	·13
·14	·149	·157	·165	·173	·181	·189	·196	·204	·211	·218	·14
·15	·158	·167	·175	·183	·190	·198	·206	·213	·220	·227	·15
·16	·168	·176	·184	·192	·200	·208	·215	·222	·229	·236	·16
·17	·178	·186	·194	·202	·210	·217	·224	·231	·239	·245	·17
·18	·188	·196	·204	·212	·219	·226	·234	·241	·248	·255	·18
·19	·198	·206	·214	·221	·229	·236	·243	·250	·257	·264	·19
·20	·208	·216	·223	·231	·238	·245	·252	·259	·266	·273	·20
·21	·218	·225	·233	·240	·248	·255	·262	·269	·275	·282	·21
·22	·228	·235	·243	·250	·257	·264	·271	·278	·284	·291	·22
·23	·238	·245	·252	·260	·267	·274	·280	·287	·294	·300	·23
·24	·248	·255	·262	·269	·276	·283	·290	·296	·303	·309	·24
·25	·257	·265	·272	·279	·286	·292	·299	·306	·312	·318	·25
·26	·267	·275	·282	·288	·295	·302	·308	·315	·321	·327	·26
·27	·277	·284	·291	·298	·305	·311	·318	·324	·330	·336	·27
·28	·287	·294	·301	·308	·314	·321	·327	·333	·339	·345	·28
·29	·297	·304	·311	·317	·324	·330	·336	·343	·349	·355	·29
·30	·307	·314	·320	·327	·333	·340	·346	·352	·358	·364	·30
·31	·317	·324	·330	·337	·343	·349	·355	·361	·367	·373	·31
·32	·327	·333	·340	·346	·352	·358	·364	·370	·376	·382	·32
·33	·337	·343	·350	·356	·362	·368	·374	·380	·385	·391	·33
·34	·347	·353	·359	·365	·371	·377	·383	·389	·395	·400	·34
·35	·356	·363	·369	·375	·381	·387	·393	·398	·404	·409	·35
·36	·366	·373	·379	·385	·390	·396	·402	·407	·413	·418	·36
·37	·376	·382	·383	·394	·400	·406	·411	·417	·422	·427	·37
·38	·386	·392	·398	·404	·410	·415	·421	·426	·431	·436	·38
·39	·396	·402	·408	·413	·419	·425	·430	·435	·440	·445	·39
·40	·406	·412	·417	·423	·429	·434	·439	·444	·450	·455	·40
·41	·416	·422	·427	·433	·438	·443	·449	·454	·459	·463	·41
·42	·426	·431	·437	·442	·448	·453	·458	·463	·468	·473	·42
·43	·436	·441	·447	·452	·457	·462	·467	·472	·477	·482	·43
·44	·446	·451	·456	·462	·467	·472	·477	·481	·486	·491	·44
·45	·455	·461	·466	·471	·476	·481	·486	·491	·495	·500	·45
·46	·465	·471	·476	·481	·486	·491	·495	·500	·505	·509	·46
·47	·475	·480	·485	·490	·495	·500	·505	·509	·514	·518	·47
·48	·485	·490	·495	·500	·505	·509	·514	·519	·523	·527	·48
·49	·495	·500	·505	·510	·514	·519	·523	·528	·532	·536	·49

TABLE B.

REAL CUT-OFF $\frac{1}{R}$ CORRESPONDING TO APPARENT CUT-OFF $\frac{1}{r}$ FOR DIFFERENT FRACTIONS OF CLEARANCE—(*Continued*).

Apparnt Cut-off $\frac{1}{r}$.	REAL CUT-OFF $\frac{1}{R}$ FOR FRACTION OF CLEARANCE OF										Apparnt Cut-off $\frac{1}{r}$.
	·01	·02	·03	·04	·05	·06	·07	·08	·09	·1	
·50	·503	·510	·515	·519	·524	·528	·533	·537	·541	·545	·50
·51	·515	·520	·524	·529	·533	·538	·542	·546	·550	·554	·51
·52	·525	·529	·534	·538	·543	·547	·551	·556	·560	·564	·52
·53	·535	·539	·544	·548	·552	·557	·561	·565	·569	·573	·53
·54	·545	·549	·533	·558	·562	·566	·570	·574	·578	·582	·54
·55	·554	·559	·563	·567	·571	·575	·579	·583	·587	·591	·55
·56	·564	·569	·573	·577	·581	·585	·589	·593	·596	·600	·56
·57	·574	·578	·583	·587	·590	·594	·598	·602	·606	·609	·57
·58	·584	·588	·592	·596	·600	·604	·607	·611	·615	·618	·58
·59	·594	·598	·602	·606	·610	·613	·617	·620	·624	·627	·59
·60	·604	·608	·612	·615	·619	·623	·626	·630	·633	·636	·60
·61	·614	·618	·621	·625	·629	·632	·636	·639	·642	·645	·61
·62	·624	·627	·631	·635	·638	·642	·645	·648	·651	·655	·62
·63	·634	·637	·641	·644	·648	·651	·654	·657	·661	·664	·63
·64	·644	·647	·650	·654	·657	·660	·664	·667	·670	·673	·64
·65	·653	·657	·660	·663	·667	·670	·673	·676	·679	·682	·65
·66	·663	·667	·670	·673	·676	·679	·682	·685	·688	·691	·66
·67	·673	·676	·680	·683	·686	·689	·692	·694	·697	·700	·67
·68	·683	·686	·689	·692	·695	·698	·701	·704	·706	·709	·68
·69	·693	·696	·699	·702	·705	·708	·710	·713	·716	·718	·69
·70	·703	·706	·709	·712	·714	·717	·720	·722	·725	·727	·70
·71	·713	·716	·718	·721	·724	·726	·729	·731	·734	·736	·71
·72	·723	·725	·728	·731	·733	·736	·738	·741	·743	·745	·72
·73	·733	·735	·738	·740	·743	·745	·748	·750	·752	·755	·73
·74	·743	·745	·748	·750	·752	·755	·757	·759	·761	·764	·74
·75	·752	·755	·757	·760	·762	·764	·766	·769	·771	·773	·75
·76	·762	·765	·767	·769	·771	·774	·776	·778	·780	·782	·76
·77	·772	·775	·777	·779	·781	·783	·785	·787	·789	·791	·77
·78	·782	·784	·786	·788	·790	·792	·794	·796	·798	·800	·78
·79	·792	·794	·796	·798	·800	·802	·804	·806	·807	·809	·79
·80	·802	·804	·806	·808	·810	·811	·813	·815	·817	·818	·80
·81	·812	·814	·816	·817	·819	·821	·822	·824	·826	·827	·81
·82	·822	·824	·825	·827	·829	·830	·832	·833	·835	·836	·82
·83	·832	·833	·835	·837	·838	·840	·841	·843	·844	·845	·83
·84	·842	·843	·845	·846	·848	·849	·850	·852	·853	·855	·84
·85	·851	·853	·854	·856	·857	·858	·860	·861	·862	·864	·85
·86	·861	·863	·864	·865	·867	·868	·869	·870	·8 2	·873	·86
·87	·871	·873	·874	·875	·876	·877	·879	·880	·881	·882	·87
·88	·881	·882	·884	·885	·886	·887	·888	·889	·890	·891	·88
·89	·891	·892	·893	·894	·895	·896	·897	·898	·899	·900	·89
·90	·901	·902	·903	·904	·905	·906	·907	·907	·908	·909	·90
·91	·911	·912	·913	·913	·914	·915	·917	·917	·917	·918	·91
·92	·921	·922	·922	·923	·924	·925	·925	·926	·927	·927	·92
·93	·931	·931	·932	·933	·933	·934	·935	·935	·936	·936	·93
·94	·941	·941	·942	·942	·943	·943	·944	·944	·945	·945	·94
·95	·950	·951	·951	·952	·952	·953	·953	·954	·954	·955	·95
·96	·960	·961	·961	·962	·962	·962	·963	·963	·963	·964	·96
·97	·970	·971	·971	·971	·971	·972	·972	·972	·972	·973	·97
·98	·980	·980	·981	·981	·981	·981	·981	·981	·982	·982	·98
·99	·990	·990	·990	·990	·990	·991	·991	·991	·991	·991	·99

TABLE C.

Constants for the Expansion of Steam.

Argument, given ratio, volumes or pressures.		Real cut-off for given ratio of pressures. $\left(\frac{p}{P}\right)^{\frac{10}{17}}$	Ratio of terminal pressure to pressure at point of cut-off, for given ratio of volumes. $\left(\frac{1}{R}\right)^{\frac{10}{10}}$	Ratio of mean total pressure during expansion, to pressure at point of cut-off, for given ratio of volumes.		Ratio of mean total to initial pressure, when pressure at point of cut-off is equal to initial, for given ratio of volumes.		Ratio of portion of mean total pressure due to expansion, to pressure at point of cut-off, for given ratio of volumes.		Ratio of terminal to initial temperature (Fahrenheit Scale + 100), for given ratio of pressures. $\left(\frac{p_x}{P}\right)^{\frac{1}{n}}$	Argument, given ratio, volumes, or pressures.
				Isothermal curve. $\frac{hyp.\ log.\ R}{R-1}$	Dry saturated steam. $16\times\left[1-\left(\frac{1}{R}\right)^{\frac{1}{16}}\right]\div(R-1)$	Isothermal curve. $\frac{1+hyp.\ log.\ R}{R}$	Dry saturated steam. $17\times\frac{1}{R}-16\times\left(\frac{1}{R}\right)^{\frac{17}{16}}$	Isothermal curve. $\frac{hyp.\ log.\ R}{R}$	Dry saturated steam. $16\times\left[1-\left(\frac{1}{R}\right)^{\frac{1}{16}}\right]\div R$		
$\frac{1}{R}$, $\frac{p}{P}$	R, $\frac{P}{p}$	$\frac{1}{R}$	$\frac{p}{P}$	$\frac{p_r}{P}$		$\frac{P_t}{P_1}$		$\frac{P_2}{P}$		$\frac{t_x+100}{T+100}$	$\frac{1}{R}$, $\frac{p}{P}$
1	2	3	4	5	6	7	8	9	10	11	12
·01	100·	·0131	·0075	·0463	·0404	·0561	·0500	·0461	·0400	·4642	·01
·02	50·	·0252	·0157	·0798	·0708	·0982	·0894	·0782	·0694	·5210	·02
·03	33·333	·0369	·0241	·1085	·0974	·1352	·1245	·1032	·0945	·5574	·03
·04	25·	·0483	·0327	·1341	·1215	·1688	·1566	·1288	·1166	·5848	·04
·05	20·	·0596	·0415	·1576	·1438	·1998	·1866	·1498	·1366	·6070	·05
·06	16·667	·0708	·0503	·1796	·1647	·2288	·2148	·1688	·1548	·6257	·06
·07	14·286	·0819	·0593	·2002	·1844	·2562	·2415	·1862	·1715	·6420	·07
·08	12·5	·0928	·0683	·2196	·2032	·2821	·2669	·2021	·1869	·6564	·08
·09	11·111	·1037	·0774	·2382	·2211	·3067	·2912	·2167	·2012	·6694	·09
·10	10·	·1145	·0866	·2558	·2383	·3303	·3145	·2303	·2145	·6813	·10
·11	9·091	·1253	·0958	·2728	·2548	·3528	·3368	·2428	·2268	·6922	·11
·12	8·333	·1359	·1051	·2891	·2708	·3744	·3583	·2544	·2383	·7023	·12
·13	7·692	·1466	·1144	·3049	·2862	·3952	·3790	·2652	·2490	·7117	·13
·14	7·143	·1572	·1238	·3201	·3012	·4153	·3990	·2753	·2590	·7206	·14
·15	6·667	·1677	·1332	·3348	·3157	·4346	·4183	·2846	·2683	·7289	·15
·16	6·25	·1782	·1427	·3491	·3298	·4532	·4370	·2932	·2770	·7368	·16
·17	5·882	·1887	·1522	·3629	·3436	·4712	·4552	·3012	·2852	·7443	·17
·18	5·556	·1991	·1617	·3764	·3570	·4887	·4727	·3087	·2927	·7514	·18
·19	5·263	·2095	·1713	·3896	·3700	·5055	·4897	·3155	·2997	·7582	·19

TABLE C.

Constants for the Expansion of Steam—(*Continued*).

$\frac{1}{R}, \frac{p}{P}$	$R, \frac{P}{p}$	$\frac{1}{R}$	$\frac{p}{P}$	$\frac{p_r}{P}$		$\frac{P_t}{P_1}$		$\frac{P_2}{P}$		$\frac{t_x + 100}{T + 100}$	$\frac{1}{R}, \frac{p}{P}$
				Isothermal curve.	Dry saturated steam.	Isothermal curve.	Dry saturated steam.	Isothermal curve.	Dry saturated steam.		
1	2	3	4	5	6	7	8	9	10	11	12
·20	5·	·2199	·1809	·4024	·3828	·5219	·5062	·3219	·3062	·7647	·20
·21	4·762	·2302	·1905	·4149	·3953	·5377	·5223	·3277	·3123	·7710	·21
·22	4·545	·2405	·2001	·4271	·4075	·5531	·5378	·3331	·3178	·7770	·22
·23	4·348	·2508	·2098	·4390	·4194	·5680	·5530	·3380	·3230	·7828	·23
·24	4·167	·2610	·2193	·4507	·4312	·5825	·5677	·3425	·3277	·7883	·24
·25	4·	·2712	·2293	·4621	·4426	·5966	·5820	·3466	·3320	·7937	·25
·26	3·846	·2814	·2390	·4733	·4539	·6102	·5959	·3502	·3359	·7989	·26
·27	3·704	·2916	·2488	·4843	·4650	·6235	·6094	·3535	·3394	·8040	·27
·28	3·571	·3018	·2586	·4950	·4759	·6304	·6226	·3504	·3426	·8088	·28
·29	3·448	·3119	·2684	·5056	·4866	·6490	·6355	·3590	·3455	·8136	·29
·30	3·333	·3220	·2783	·5160	·4971	·6612	·6479	·3612	·3479	·8182	·30
·31	3·226	·3321	·2881	·5262	·5074	·6731	·6601	·3631	·3501	·8227	·31
·32	3·125	·3422	·2980	·5362	·5176	·6846	·6719	·3646	·3519	·8270	·32
·33	3·03	·3522	·8079	·5461	·5276	·6959	·6835	·3659	·3535	·8313	·33
·34	2·941	·3623	·3178	·5553	·5374	·7068	·6947	·3668	·3547	·8354	·34
·35	2·857	·3723	·3278	·5653	·5471	·7174	·7056	·3674	·3556	·8395	·35
·36	2·778	·3823	·3377	·5747	·5567	·7278	·7163	·3678	·3563	·8434	·36
·37	2·703	·3923	·3477	·5739	·5662	·7379	·7267	·3679	·3567	·8473	·37
·38	2·632	·4023	·3577	·5930	·5755	·7477	·7368	·3677	·3568	·8511	·38
·39	2·564	·4122	·3677	·6020	·5846	·7572	·7466	·3672	·3566	·8548	·39
·40	2·5	·4222	·3777	·6109	·5937	·7665	·7562	·3665	·3562	·8584	·40
·41	2·439	·4321	·3878	·6196	·6026	7756	·7656	·3656	·3556	·8619	·41
·42	2·381	·4420	·3978	·6282	·6115	·7844	·7747	·3644	·3547	·8654	·42
·43	2·326	·4519	·4079	·6367	·6200	·7929	·7835	·3629	·3535	·8688	·43
·44	2·273	·4618	·4180	·6451	·6288	·8012	·7921	·3612	·3521	·8721	·44
·45	2·222	·4716	·4281	·6533	·6373	·8093	·8005	·3593	·3505	·8754	·45
·46	2·174	·4815	·4392	·6615	·6457	·8172	·8087	·3572	·3487	·8786	·46
·47	2·128	·4913	·4483	·6696	·6540	·8249	·8166	·3549	·3466	·8818	·47
·48	2·083	·5012	·4585	·6775	·6622	·8323	·8244	·3523	·3444	·8849	·48
·49	2·041	·5110	·4686	·6854	·6703	·8395	·8319	·3495	·3419	·8879	·49

TABLE C.

Constants for the Expansion of Steam—(*Continued*).

$\frac{1}{R}, \frac{p}{P}$	$R, \frac{P}{p}$	$\frac{1}{R}$	$\frac{p}{P}$	$\frac{p_r}{P}$		$\frac{P_t}{P_1}$		$\frac{P_2}{P}$		$\frac{t_r + 100}{T + 100}$	$\frac{1}{R}, \frac{p}{P}$
				Isothermal curve.	Dry saturated steam.	Isothermal curve.	Dry saturated steam.	Isothermal curve.	Dry saturated steam.		
1	2	3	4	5	6	7	8	9	10	11	12
·50	2·	·5208	·4788	·6932	·6784	·8466	·8392	·3466	·3392	·8909	·50
·51	1·961	·5306	·4890	·7008	·6863	·8534	·8463	·3434	·3363	·8938	·51
·52	1·923	·5404	·4992	·7084	·6941	·8600	·8532	·3400	·3332	·8967	·52
·53	1·887	·5502	·5094	·7159	·7019	·8665	·8599	·3365	·3299	·8996	·53
·54	1·852	·5599	·5196	·7234	·7096	·8727	·8664	·3327	·3264	·9024	·54
·55	1·818	·5697	·5298	·7307	·7172	·8788	·8727	·3288	·3227	·9052	·55
·56	1·786	·5794	·5401	·7380	·7247	·8847	·8789	·3247	·3189	·9079	·56
·57	1·754	·5892	·5503	·7451	·7322	·8904	·8848	·3204	·3148	·9106	·57
·58	1·724	·5989	·5606	·7522	·7396	·8959	·8906	·3159	·3106	·9132	·58
·59	1·695	·6086	·5709	·7593	·7469	·9013	·8962	·3113	·3062	·9158	·59
·60	1·667	·6183	·5812	·7662	·7541	·9065	·9017	·3065	·3017	·9184	·60
·61	1·639	·6280	·5914	·7731	·7613	·9115	·9069	·3015	·2969	·9209	·61
·62	1·613	·6377	·6018	·7800	·7684	·9164	·9120	·2964	·2920	·9234	·62
·63	1·587	·6474	·6121	·78 7	·7755	·9211	·9169	·2911	·2869	·9259	·63
·64	1·563	·6570	·6224	·7934	·7824	·9256	·9217	·2856	·2817	·9283	·64
·65	1·533	·6667	·6327	·8000	·7894	·9300	·9263	·2800	·2763	·9307	·65
·66	1·515	·6763	·6431	·8066	·7962	·9342	·9307	·2742	·2707	·9331	·66
·67	1·493	·6360	·6534	·8131	·8030	·9383	·9350	·2683	·2650	·9354	·67
·68	1·471	·6956	·6638	·8195	·8097	·9423	·9391	·2623	·2591	·9378	·68
·69	1·449	·7052	·6742	·8259	·8164	·9460	·9431	·2560	·2531	·9400	·69
·70	1·429	·7148	·6846	·8322	·8230	·9497	·9469	·2497	·2469	·9423	·70
·71	1·408	·7245	·6950	·8385	·8296	·9532	·9506	·2432	·2406	·9445	·71
·72	1·389	·7341	·7054	·8447	·8361	·9565	·9541	·2365	·2341	·9467	·72
·73	1·37	·7436	·7158	·8509	·8426	·9597	·9575	·2297	·2275	·9489	·73
·74	1·351	·7532	·7262	·8600	·8490	·9628	·9607	·2228	·2207	·9511	·74
·75	1·333	·7628	·7366	·8631	·8553	·9658	·9638	·2158	·2138	·9532	·75
·76	1·316	·7724	·7471	·8691	·8616	·9686	·9668	·2086	·2068	·9553	·76
·77	1·299	·7819	·7575	·8750	·8679	·9713	·9696	·2013	·1996	·9574	·77
·78	1·282	·7915	·7680	·8809	·8741	·9738	·9723	·1938	·1923	·9594	·78
·79	1·266	·8010	·7785	·8868	·8803	·9762	·9749	·1862	·1849	·9015	·79

TABLE C.

Constants for the Expansion of Steam—(*Continued*).

$\frac{1}{K}, \frac{p}{P}$	$K, \frac{P}{p}$	$\frac{1}{K}$	$\frac{p}{P}$	$\frac{p_r}{P}$		$\frac{P_t}{P_1}$		$\frac{P_2}{P}$		$\frac{t_2+100}{T+100}$	$\frac{1}{K}, \frac{p}{P}$
				Isothermal curve.	Dry saturated steam.	Isothermal curve.	Dry saturated steam.	Isothermal curve.	Dry saturated steam.		
1	2	3	4	5	6	7	8	9	10	11	12
·80	1·25	·8106	·7889	·8926	·8864	·9785	·9773	·1785	·1773	·9635	·80
·81	1·235	·8201	·7994	·8983	·8925	·9807	·9796	·1707	·1696	·9655	·81
·82	1·22	·8296	·8099	·9041	·8985	·9827	·9817	·1627	·1617	·9675	·82
·83	1·205	·8392	·8204	·9097	·9045	·9847	·9838	·1547	·1538	·9694	·84
·84	1·19	·8487	·8309	·9154	·9104	·9865	·9857	·1465	·1457	·9714	83
·85	1·176	·8582	·8414	·9209	·9162	·9881	·9874	·1381	·1374	·9733	·85
·86	1·163	·8577	·8519	·9265	·9221	·9897	·9891	·1297	·1291	·9752	·86
·87	1·149	·8772	·8625	·9320	·9279	·9912	·9906	·1212	·1206	·9771	·87
·88	1·136	·8866	·8730	·9375	·9337	·9925	·9920	·1125	·1120	·9789	·88
·89	1·124	·8961	·8835	·9429	·9394	·9937	·9933	·1037	·1033	·9808	·89
·90	1·111	·9056	·8941	·9482	·9451	·9948	·9945	·0948	·0945	·9826	·90
·91	1·099	·9151	·9047	·9536	·9508	·9958	·9956	·0858	·0856	·9844	·91
·92	1·087	·9245	·9152	·9589	·9564	·9967	·9965	·0767	·0765	·9862	·92
·93	1·075	·9340	·9258	·9642	·9620	·9975	·9973	·0675	·0673	·9880	·93
·94	1·064	·9434	·9364	·9694	·9675	·9982	·9981	·0582	·0581	·9897	·94
·95	1·053	·9529	·9470	·9746	·9730	·9987	·9987	·0487	·0487	·9915	·95
·96	1·042	·9623	·9576	·9797	·9785	·9992	·9991	·0392	·0391	·9932	·96
·97	1·031	·9417	·9682	·9849	·9840	·9996	·9995	·0296	·0295	·9949	·97
·98	1·02	·9812	·9788	·9899	·9895	·9998	·9998	·0198	·0198	·9966	·98
·99	1·01	·9906	·9894	·9950	·9947	·9999	·9999	·0100	·0100	·9983	·99

Table C contains constants for substitution in the formulas of Table A. The use of these three tables can be best illustrated by some examples.

1. If steam having an initial pressure (P) of 80 pounds per square inch is allowed to expand until its pressure is reduced to 25 pounds (p), find the pressures at three intermediate equidistant points and the mean pressure during expansion, supposing that the pressure varies inversely as the $\frac{17}{16}$th power of the volume.

By formula (7), Table A, the real cut-off is $\left(\frac{25}{80}\right)^{\frac{16}{17}} = 0{\cdot}31^{\frac{16}{17}}$, which is found in column 3 of Table C to be 0·3321, so that the ratio of expansion is $\frac{1}{0{\cdot}3321} = 3{\cdot}01$, or the volume of the steam is increased from 1 to 3·01. Hence the several ratios of expansion for the three equidistant intermediate points are 1·5, 2, and 2·5. Entering Table C with these quantities as arguments in column 2, the pressures at these intermediate points are obtained from column 4, as follows:

1st intermediate point, pressure $= 80 \times 0{\cdot}6534 = 52{\cdot}3$
2d " " " $= 80 \times 0{\cdot}4788 = 38{\cdot}3$
3d " " " $= 80 \times 0{\cdot}3777 = 30{\cdot}2$.

The mean pressure during expansion is given by column 6 of Table C, using the argument 0·33 in column 1, so that the mean pressure is $80 \times 0{\cdot}5276 = 42{\cdot}2$ pounds per square inch. In solving the above example, it will be observed that the nearest arguments in the table have been used. It would, of course, be easy if greater accuracy were required to find the constants for the given arguments by interpolation; but this is not ordinarily necessary.

2. The cylinder of a certain steam-engine is 24 inches in diameter, and the stroke of piston is 4 feet. The clearance space at each end of the cylinder is $\frac{8}{10}$ of a cubic foot. The engine makes 60 revolutions per minute, the steam is cut off when the piston has moved 16 inches from commencement of stroke, and cushion commences when the portion of stroke uncompleted is 12 inches. The steam expands and is compressed in accordance with the law that the pressure varies inversely as the volume, and the absolute pressures at different parts of the stroke are as follows:

Initial pressure, P_1, 70; pressure at point of cut-off, P, 66; mean back pressure, exclusive of cushion, p_2, 5; back pressure at point where cushion commences, p_4, 6. From the above data it is required to find the indicated horse-power of the engine.

Using Tables A to C for the appropriate formulas and constants, it is found that the space swept through by the piston per stroke,

$$V = 4 \times 0{\cdot}7854 \times 4 = 12{\cdot}5664 \text{ cubic feet.}$$

The fraction of clearance at each end of cylinder,

$$C = \frac{12{\cdot}5664}{0{\cdot}8} = 0{\cdot}064.$$

The apparent cut-off,

$$\frac{1}{r} = \frac{16}{48} = 0{\cdot}33.$$

The real cut-off,

$$\frac{1}{R} = 0{\cdot}37.$$

The real ratio of expansion,

$$R = 2{\cdot}703.$$

The terminal pressure, if the release occurs at the end of stroke

$$p = 66 \times 0{\cdot}37 = 24{\cdot}4 \text{ pounds per square inch.}$$

The mean pressure up to point of cut-off, clearance space not included,

$$p_1 = \frac{70 + 66}{2} = 68 \text{ pounds per square inch.}$$

The mean pressure up to point of cut-off, clearance space included,

$$P_i = 68 + \frac{0{\cdot}064 \times 2{\cdot}703 \times (70 - 68)}{1{\cdot}064} = 68{\cdot}3 \text{ pounds per square inch.}$$

The mean effective pressure, uncorrected for clearance and cushion,

$$P_m = \frac{68{\cdot}3}{2{\cdot}703} + 66 \times 0{\cdot}3679 - 5 = 44{\cdot}4 \text{ pounds per square inch.}$$

The mean effective pressure, corrected for clearance,

$$p_m = 44{\cdot}4 - 0{\cdot}064 \times (70 - 44{\cdot}4) = 42{\cdot}8 \text{ pounds per square inch.}$$

The fraction of stroke uncompleted when cushion commences,

$$d = \frac{12}{48} = 0{\cdot}25.$$

The ratio of compression for cushion,

$$N = \frac{0{\cdot}25 + 0{\cdot}064}{0{\cdot}064} = 4{\cdot}9.$$

The final cushion pressure,

$$p_3 = 6 \times 4{\cdot}9 = 29{\cdot}4 \text{ pounds per square inch.}$$

The mean cushion pressure,

$$p_c = 29{\cdot}4 \times 0{\cdot}4024 = 11{\cdot}8 \text{ pounds per square inch.}$$

The mean effective pressure, corrected for clearance and cushion,

$$P_e = 42{\cdot}8 - 0{\cdot}064 \times 3{\cdot}9 \times (11{\cdot}8 - 6) = 41{\cdot}4 \text{ pounds per square inch.}$$

Hence the indicated horse-power,

$$I.H.P. = \frac{41{\cdot}4 \times (24)^2 \times 0{\cdot}7854 \times 480}{33000} = 272.$$

3. Suppose, in the engine whose dimensions are given in the last example, release occurs when the piston has completed $\frac{95}{100}$ of the stroke—what is the amount of steam used per hour, as accounted for by the indicator?

The pressure at the point where release commences is

$$p_z = 66 \times \frac{0{\cdot}33 + 0{\cdot}064}{0{\cdot}95 + 0{\cdot}064} = 26{\cdot}3.$$

The engine makes $2 \times 60 \times 60 = 7200$ strokes per hour, and by the steam table in Art. 391 the weight of a cubic foot of steam at a pressure of 26·3 pounds per square inch is 0·065572 pounds; so that the weight of steam used per hour, by terminal pressure, is

$$7200 \times (0{\cdot}8 + 0{\cdot}95 \times 12{\cdot}5664) \times 0{\cdot}065572 = 6014 \text{ pounds.}$$

By the steam table in Art. 391, the weight of a cubic foot of steam at the pressure when cushion commences, 6, is 0·016357, so that the weight of steam saved by cushioning per hour is

$$7200 \times (0{\cdot}8 + 0{\cdot}25 \times 12{\cdot}5664) \times 0{\cdot}016357 = 420 \text{ pounds.}$$

The mean total pressure during expansion,

$$p_r = 66 \times 0{\cdot}5839 = 38{\cdot}5.$$

The ratio of mechanical effect during expansion to total mechanical effect,

$$m = \frac{38{\cdot}5 \times 2}{3 \times (42{\cdot}8 + 5)} = 0{\cdot}537.$$

The units of heat converted into work, per hour, are

$$\frac{7200 \times (42{\cdot}8 + 5) \times 0{\cdot}537 \times 452{\cdot}39 \times 4}{772} = 432876.$$

By the steam table in Art. 391, the latent heat of a pound of steam at the terminal pressure, 26·3, is 944·282 thermal units, so that the pounds of steam condensed for work, per hour, are

$$\frac{432876}{944\cdot282} = 578.$$

Hence the pounds of steam accounted for by the indicator, per hour, are

$$6014 + 578 - 420 = 6172.$$

There is always more steam used by an engine than the amount determined by the above calculation, being the steam condensed on account of the variation in temperature of the interior surfaces of the cylinder and that condensed by radiation from external surfaces. In the case of a well-clothed cylinder, experiment seems to show that the amount of steam unaccounted for depends upon the area of internal surface; and though these experiments have not been sufficiently extended to determine the constants for condensation with precision, it can be stated that, in general, in the case of non-condensing engines, with cylinders not steam-jacketed but well clothed, the condensation per hour will be between 20 and 30 pounds per square foot of internal condensing surface, from 10 to 20 pounds per square foot of internal surface, for condensing engines under the same circumstances, and about half the above amounts for steam-jacketed cylinders. The exact value of the constant for any particular case depends upon the size of the cylinder and the range of temperature during a stroke; but it is impossible, in the absence of further experiments, to give an exact formula. In the case of the engine under consideration, it may be assumed that the condensation is 15 pounds per hour per square foot of internal surface—this surface being the whole interior of the cylinder, including the ports, together with the two sides of the piston and the surface of the piston-rod. Supposing the interior surface of each port to be 6 sq. ft., the length of the interior surface of the cylinder 5 feet, and the diameter of the piston-rod 3½ inches, the internal surface on which condensation may take place at each stroke of the piston is distributed about as follows:

Ports	12·0 sq. ft.
Sides of cylinder	31·4 " "
Cylinder-heads and sides of piston	12·6 " "
Piston-rod	4·6 " "
Total	60·6 " "

So that the steam unaccounted for by the indicator per hour is

$$60\cdot6 \times 15 = 909 \text{ pounds},$$

and the total steam used by the engine per hour is

$$6172 + 909 = 7081 \text{ pounds,}$$

or,

$$\frac{7081}{272} = 26\cdot \text{ pounds of steam per indicated horse-power per hour.}$$

Having found the probable amount of steam that will be used by a proposed engine, a boiler can be designed to furnish the required amount by the principles explained in Art. 438.

The importance of determining a rule by which the amount of steam unaccounted for by the indicator can be definitely fixed for any given case, should induce future experimenters to give the necessary data in the records of their tests. It is not uncommon to give the amount of water unaccounted for as a certain per cent of the total water used, but it seems probable that it would be more correct to charge it as the condensation per square foot of internal surface.

4. The power utilized by an engine, or the *net horse-power*, is of more importance to the steam-user than the *indicated horse-power*, which is the power due to the mean pressure of steam in the cylinder, including that required to overcome prejudicial resistances. In well-designed engines the net horse-power averages from 80 to 90 per cent of the indicated horse-power; and assuming that it is 85 per cent of the indicated power in the case under consideration, the effective power of the engine will be

$$272 \times 0\cdot 85 = 231,$$

and this will require $\frac{7081}{231} = 30\cdot 6$ pounds of steam per net horse-power per hour.

Some examples of the relation between indicated and net horse-power are given in the tables that follow.

5. In the case of the engine considered in the preceding examples no account has been taken of the effect of the piston-rod in reducing the effective piston area, but attention should be given to this in practice. It is evident that the piston area on one side is reduced by the area of the piston-rod, so that the effective area, which should be used in all calculations, is

$$\frac{2 \times \text{piston area} - \text{area of piston-rod}}{2}$$

which would give for the effective area in the case under consideration,

$$\frac{2 \times 452\cdot 39 - 9\cdot 62}{2} = 447\cdot 58 \text{ square inches,}$$

and would reduce the indicated horse-power to

$$\frac{41{\cdot}4 \times 447{\cdot}58 \times 480}{33000} = 270,$$

and the net horse-power to

$$270 \times 0{\cdot}85 = 229.$$

6. Suppose that the mean effective pressure in a particular case, calculated as in example 2, is 35 pounds per square inch, and that it is required to find the dimensions of an engine that shall exert 350 net horse-power, with a piston speed of 500 feet a minute, it being assumed that the net horse-power is 82 per cent of the indicated.

Under these circumstances, the net pressure will be

$$35 \times 0{\cdot}82 = 28{\cdot}7 \text{ pounds per square inch,}$$

and each square inch of effective piston area will exert

$$\frac{28{\cdot}7 \times 500}{33000} = 0{\cdot}43485 \text{ net horse-power,}$$

and the effective piston area required will be

$$\frac{350}{0{\cdot}43485} = 804{\cdot}88 \text{ square inches.}$$

This corresponds to a cylinder about 32 inches in diameter, and if it is found that a piston-rod $4\frac{1}{2}$ inches in diameter is required, the total piston area should be

$$\frac{2 \times 804{\cdot}88 + 15{\cdot}90}{2} = 812{\cdot}83 \text{ square inches,}$$

and the corresponding diameter of cylinder is

$$\sqrt{\frac{812{\cdot}83}{0{\cdot}7854}} = 32{\cdot}17 \text{ inches.}$$

By assuming the number of revolutions per minute, the piston stroke can readily be determined. Thus, if the speed is fixed at 50 revolutions per minute the piston must move

$$\frac{500}{50} = 10 \text{ feet per revolution,}$$

so that the stroke is

$$\frac{12}{2} = 5 \text{ feet.}$$

7. Formula (26), Table A, in connection with column 11, Table C, is useful for the approximate determination of the temperature of steam at different absolute pressures. To find, for example, the temperature of steam at a pressure of 50 pounds per square inch, we know that the temperature at 14·7 pounds per square inch is 212° Fahrenheit, so that the ratio of the pressures is

$$\frac{14\cdot7}{50} = 0\cdot294.$$

Hence the approximate temperature, by column 11, Table C, is

$$\frac{312^\circ}{0\cdot8154} - 100^\circ = 282^\circ\cdot6.$$

By the steam table in Art. 391 it will be seen that the temperature corresponding to this pressure is, more accurately, 280°·9.

The preceding rules and examples, while showing what performance may reasonably be expected from steam-engines under certain assumed conditions, do not fix these conditions for all cases that may arise, and can properly be supplemented by examples drawn from approved practice. In designing an engine, for instance, it is necessary to fix the point of cut-off or ratio of expansion. This can be done by the aid of the preceding principles, but the operation will be quite laborious. Thus, the designer may fix the steam pressure, piston speed, and ratio of net to indicated horse-power, and then calculate the size of engine required to develop a given net horse-power and the probable consumption of steam for various grades of expansion, finally selecting the case that gives the most economical result, and this course is often to be recommended. The designer can also frequently find examples of successful practice, which he may follow with advantage.

In regard to the question of the best point of cut-off there has been much discussion; and while the matter is by no means absolutely settled, it is believed that the majority of engineers will agree in fixing the best point of cut-off in simple engines with initial pressure at from 60 to 100 pounds per square inch, somewhere between quarter and half stroke. A very elaborate series of experiments was made by the United States Government, extending over three years, for the purpose of settling this question. Unfortunately, however, the experiments were never completed, and although they are quite decisive for the circumstances under which they were made, these circumstances were not such as obtain generally in modern practice—the initial pressure and piston speed being ordinarily greater than in these experiments. Still, the experiments are of great value and will repay careful study. A record is presented in Table D, by the kind permission of Chief-Engineer B. F. Isherwood, U. S. N., and this is believed to be the first complete account that has been published. This table, besides showing the best conditions under the circumstances of the tests, is useful as indicating the data that should be recorded in a complete experiment, and can be studied to advantage by future experimenters.

The experiments recorded in Table D were supplemented by a series conducted by the proprietors of the Novelty Iron Works, New York, for the purpose of determining the proper proportions for engines of various powers and the steam required for the different cases. These determinations are contained in a work entitled "Tables and Diagrams Relating to Non-Condensing Engines and Boilers," New York, 1872, and Table E, compiled from this work, gives the proportions specially recommended for engines up to 150 net horse-power.

Number for reference.	D MEN	HORSE-PO Total.	Indicated.	N	ERATURE, Feed.	REMARKS.	Number for reference.
1		24	25		43	48	49
1	May	31·04	21·73	I	56·7	ction) — 5·1 Non-condensing. Coal wet.	1
2	April	31·01	21·77	I	52·8	re.) " "	2
3	Marc	22·78	20·37	I	36·6	sq. ft. Cylinder refitted before commencing.	3
4	"	21·3	19·18	I	38·1	" Steam throttled.	4
5	"	13·08	10·57		40·4	"	5
6	April	10·37	8·13		43·1	"	6
7	Septe	32·01	30·48	2	69·0	sq. ft. Independent cut-off. Cylinder rebored and entirely refitted.	7
8	"	31·65	29·56	2	69·0	luced by ashes on back.	8
9	"	29·01	27·61	2	68·1	" " " " "	9
10	"	29·72	27·79	2	69·0	....	10
11	"	26·8	25·4	2	67·8	luced by ashes on back.	11
12	"	24·35	23·12	2	67·5	" " " " "	12
13	Octo	21·75	20·54	I	65·8	" " " " "	13
14	"	16·92	15·77	I	64·3	" " " " "	14
15	Octo	22·52	21·05	I	58·0	on of experiment 13.	15
16	"	18·37	16·92	I	60·3	" " 18.	16
17	"	18·11	17·0	I	63·0	packed before commencing experiment.	17
18	"	9·19	7·89		62·5	d piston and fitted new follower before commencing. G. S. reduced.	18
19	"	12·35	11·27		62·6	luced by ashes on back.	19
20	June	37·47	24·92	2	68·3	ction) — 3·1 Non-condensing.	20
21	May	36·35	24·21	2	60·2	re.) " "	21
22	Nove	13·14	10·04		50·8	sq. ft.	22
23	"	16·44	13·33	I	52·6	"	23
24	Octo	27·4	24·86	2	57·2	" Steam throttled.	24
25	"	27·41	24·56	2	59·0	....	25
26	Dece	26·55	23·57	2	40·6	sq. ft. Steam throttled. Raining on boiler.	26
27	Nove	11·66	8·67		51·3	"	27
28	Dece	25·33	21·98	1	45·3	"	28
29	Nove	14·72	11·69		47·5	"	29
30	Dece	22·23	19·29	I	38·6	very cold.	30
31	Nove	14·01	10·93		48·7	sq. ft.	31
32	Janu	21·96	19·05	1	35·1	" Steam throttled. Exhaust valves not tight.	32
33	Dece	21·87	19·18	1	35·1	"	33
34	Nove	10·49	7·62		51·3	"	34
35	Augu	34·89	20·02	1	72·3	densing. New indicator.	35
36	July	40·76	26·2	2	73·2	"	36
37	Augu	43·73	28·99	2	75·0	"	37
38	"	44·86	30·37	2	72·7	"	38
39	Janu	30·13	26·24	2	35·0	sq. ft.	39
40	Febr	14·36	10·68		36·7	"	40
41	"	29·67	26·27	2	35·7	" Steam throttled.	41
42	"	29·43	26·37	2	35·8	... .	42
43	"	18·54	14·56	I	36·0	....	43
44	"	18·63	14·81	I	36·0	r spring broke.	44
45	March	17·73	13·9	I	36·5	sq. ft.	45
46	"	24·66	20·64	1	36·7	"	46
47	Nove	51·75	23·97	1	49·2	team-chest took fire. Non-condensing.	47
48	"	57·37	28·2	2	48·0	densing.	48
49	June	46·65	36·24	3	69·0	d to new location after completing preceding experiment	49
50	May	38·45	30·98	2	52·9	compare with experiment 57. Greater lead.	50
51	"	38·53	31·74	2	55·0	ink motion.	51
52	April	22·74	15·55	1	52·6	compare with experiment 56.	52
53	June	43·04	34·44	2	63·2	lent cut-off.	53
54	April	17·96	10·93		52·1	sq. ft.	54
55	Septe	36·48	28·66	2	69·6	"	55
56	"	22·24	14·61	1	70·0	"	56
57	"	36·92	28·68	2	67·6	" Coal wet.	57
58	"	17·61	10·95		71·3	"	58
59	Augu	24·02	11·78		69·8	....	59
60	"	45·33	31·87	2	71·5	....	60
61		29·26	17·06	1	70·1	sq. ft.	61

TABLE D.

Government Expansion Experiments made at New York, 1865–1867.

TABLE E.

Dimensions of Non-condensing Engines from 5 to 150 Net Horse-Power.

Pressure above atmosphere in pounds per square inch.	Point of cut-off in fraction of stroke.	Revolutions per minute.	Pounds of water per hour. Per indicated horse-power.	Pounds of water per hour. Per net horse-power.	Revolutions per minute.	Pounds of water per hour. Per indicated horse-power.	Pounds of water per hour. Per net horse-power.	Revolutions per minute.	Pounds of water per hour. Per indicated horse-power.	Pounds of water per hour. Per net horse-power.	Revolutions per minute.	Pounds of water per hour. Per indicated horse-power.	Pounds of water per hour. Per net horse-power.	Revolutions per minute.	Pounds of water per hour. Per indicated horse-power.	Pounds of water per hour. Per net horse-power.	Revolutions per minute.	Pounds of water per hour. Per indicated horse-power.	Pounds of water per hour. Per net horse-power.
Net horse-power.		5						10						15					
Diameter and stroke, in inches.		Long-stroke engine, 5 × 12			Short-stroke engine, 5 × 9.			Long-stroke engine, 6 × 16.			Short-stroke engine, 6 × 9.			Long-stroke engine, 7 × 20.			Short-stroke engine, 7 × 12.		
100	¼	65	30·4	38·0	129	28·9	36·6	100	29·0	36·2	179	26·7	33·8	87	29·0	35·8	145	27·1	33·9
	½	65	37·6	47·0	87	35·9	45·4	68	35·8	44·8	121	33·2	42·0	58	35·7	44·0	98	33·6	42·0
	¾	..	..	..	..	..	..	59	42·2	52·8	106	39·5	50·0	..	..	..	..	..	..
80	¼	123	31·4	39·2	167	30·0	38·0	128	30·2	37·7	231	28·5	36·1	112	30·2	37·3	189	28·1	35·1
	½	81	39·0	48·8	110	37·0	46·8	85	37·2	46·5	153	34·3	43·4	74	37·1	45·8	129	34·4	43·0
	¾	71	45·4	56·8	95	43·7	52·8	74	43·7	54·6	132	40·8	51·6	65	43·4	53·4	108	41·0	51·3
60	¼	173	33·3	41·8	233	31·6	40·0	..	..	..	..	..	..	158	32·1	39·6	266	29·8	37·3
	½	111	40·9	51·2	149	38·9	49·0	116	39·4	49·2	207	35·8	45·3	100	39·3	48·5	169	36·3	45·4
	¾	101	46·9	59·4	128	45·5	57·6	100	45·9	57·4	179	42·6	53·9	86	45·9	56·7	146	42·9	53·6

TABLE E.

Dimensions of Non-condensing Engines from 5 to 150 Net Horse-Power—(*Continued*).

Pressure above atmosphere in pounds per square inch.	Point of cut-off in fraction of stroke.	20						25						30					
Net horse-power.		Long-stroke engine, 8 × 20.			Short-stroke engine, 8 × 12.			Long-stroke engine, 9 × 24.			Short-stroke engine, 9 × 15.			Long-stroke engine, 10 × 24.			Short-stroke engine, 10 × 15.		
Diameter and stroke, in inches.		Revolutions per minute.	Pounds of water per hour. Per indicated horse-power.	Pounds of water per hour. Per net horse-power.	Revolutions per minute.	Pounds of water per hour. Per indicated horse-power.	Pounds of water per hour. Per net horse-power.	Revolutions per minute.	Pounds of water per hour. Per indicated horse-power.	Pounds of water per hour. Per net horse-power.	Revolutions per minute.	Pounds of water per hour. Per indicated horse-power.	Pounds of water per hour. Per net horse-power.	Revolutions per minute.	Pounds of water per hour. Per indicated horse-power.	Pounds of water per hour. Per net horse-power.	Revolutions per minute.	Pounds of water per hour. Per indicated horse-power.	Pounds of water per hour. Per net horse-power.
100	¼	88	28·3	35·0	149	26·7	33·4	72	28·5	34·8	116	27·0	33·3	70	28·2	34·4	113	26·8	33·1
	½	60	36·3	44·8	101	32·8	41·0	..	..	..	..	..	..	..	..	..	..	.	..
	¾	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
80	¼	114	29·7	36·7	193	27·8	34·8	93	30·0	36·6	150	28·3	34·9	90	29·8	36·3	146	28·1	34·7
	½	75	36·3	44·8	128	33·9	42·4	61	30·6	44·6	99	34·6	42·7	60	36·2	44·1	97	35·4	43·7
	¾	66	42·6	52·6	111	40·4	50·5	53	42·9	52·3	87	40·9	50·5	..	..	..	..	..	..
60	¼	..	..	..	..	..	..	131	32·0	39·1	212	30·1	37·2	..	..	..	..	..	..
	½	102	38·5	47·1	173	35·7	44·7	83	39·2	47·8	135	36·6	45·2	81	38·8	47·3	131	36·4	44·9
	¾	88	45·1	55·7	149	42·2	52·8	72	45·4	55·4	116	43·2	53·3	70	45·0	54·9	114	42·9	53·0

TABLE E.

Dimensions of Non-condensing Engines from 5 to 150 Net Horse-Power—(*Continued*).

Pressure above atmosphere in pounds per square inch.	Point of cut-off in fraction of stroke.	40						50						60					
		Revolutions per minute.	Pounds of water per hour.		Revolutions per minute.	Pounds of water per hour.		Revolutions per minute.	Pounds of water per hour.		Revolutions per minute.	Pounds of water per hour.		Revolutions per minute.	Pounds of water per hour.		Revolutions per minute.	Pounds of water per hour.	
			Per indicated horse-power.	Per net horse-power.		Per indicated horse-power.	Per net horse-power.		Per indicated horse-power.	Per net horse-power.		Per indicated horse-power.	Per net horse-power.		Per indicated horse-power.	Per net horse-power.		Per indicated horse-power.	Per net horse-power.
Net horse-power.																			
Diameter and stroke, in inches.		Long-stroke engine, 11 × 30.			Short-stroke engine, 11 × 18.			Long-stroke engine, 12 × 30.			Short-stroke engine, 12 × 18.			Long-stroke engine, 13 × 36.			Short-stroke engine, 13 × 21.	
100	¼	61	28·4	34·2	103	26·7	32·6	64	27·9	33·6	108	26·3	32·1	54	28·0	33·3	93	26·6	32·1
80	¼	79	29·9	36·0	133	28·1	34·3	83	29·4	35·4	139	27·7	33·8	69	29·6	35·2	121	27·9	33·6
	½	..	..	..	..	..	..	54	35·8	43·1	92	33·9	41·3	..	..	..	..	..	..
60	¼	111	32·2	38·8	187	29·9	36·5	116	31·6	38·1	191	29·7	36·2	..	..	..	..	..	..
	½	70	39·0	47·0	119	36·2	44·2	74	38·3	46·3	125	35·7	43·5	62	38·6	46·0	108	36·2	43·6
	¾	61	45·4	54·7	102	42·8	52·2	64	44·5	53·6	108	42·2	51·5	54	44·8	53·3	93	42·7	51·5

TABLE E.

Dimensions of Non-condensing Engines from 5 to 150 Net Horse-Power—(*Continued*).

Pressure above atmosphere in pounds per square inch.	Point of cut-off in fraction of stroke.	70						80						90					
Diameter and stroke, in inches.		Long-stroke engine, 14 × 36.			Short-stroke engine, 14 × 21.			Long-stroke engine, 15 × 36.			Short-stroke engine, 15 × 24.			Long-stroke engine, 16 × 42.			Short-stroke engine, 16 × 24.		
		Revolutions per minute.	Pounds of water per hour.		Revolutions per minute.	Pounds of water per hour.		Revolutions per minute.	Pounds of water per hour.		Revolutions per minute.	Pounds of water per hour.		Revolutions per minute.	Pounds of water per hour.		Revolutions per minute.	Pounds of water per hour.	
			Per indicated horse-power.	Per net horse-power.		Per indicated horse-power.	Per net horse-power.		Per indicated horse-power.	Per net horse-power.		Per indicated horse-power.	Per net horse-power.		Per indicated horse-power.	Per net horse-power.		Per indicated horse-power.	Per net horse-power.
100	¼	54	27·7	33·0	94	26·3	31·7	54	27·5	32·7	82	26·5	31·9	45	27·7	32·6	80	26·4	31·4
80	¼	70	29·3	34·9	121	27·7	33·3	70	29·1	34·6	106	27·9	33·6	58	29·4	34·6	103	27·9	33·2
60	¼	..	..	..	..	.	..	..	..	..	..	..	..	82	31·9	37·5	145	30·0	35·7
	½	63	38·0	45·2	108	36·0	43·4	62	38·0	45·2	95	36·2	43·6	52	38·4	45·2	93	36·1	43·0
	¾	54	44·4	52·9	93	42·4	51·1	..	..	..	..	..	..	45	44·7	52·6	80	42·5	50·6

TABLE E.

Dimensions of Non-condensing Engines from 5 to 150 Net Horse-Power—(*Continued*).

Pressure above atmosphere in pounds per square inch.	Point of cut-off in fraction of stroke.	Net horse-power.	Diameter and stroke, in inches.	Revolutions per minute.	Pounds of water per hour.		Revolutions per minute.	Pounds of water per hour.	
					Per indicated horse-power.	Per net horse-power.		Per indicated horse-power.	Per net horse-power.
100	¼	100	Long-stroke engine, 17 × 42.	45	27·5	32·4	61	26·9	32·0
80	¼	100	Long-stroke engine, 17 × 42.	57	29·3	34·9	79	28·5	33·9
60	¼	100	Long-stroke engine, 17 × 42.	81	31·8	37·4	112	30·7	36·6
	½	100	Long-stroke engine, 17 × 42.	51	38·2	44·9	71	37·1	44·2
	¾	100	Long-stroke engine, 17 × 42.	44	44·4	52·2	61	43·4	51·4
100	¼	125	Long-stroke engine, 19 × 48.	..	..	..	..	..	..
80	¼	125	Long-stroke engine, 19 × 48.	50	29·4	34·2	80	28·1	33·0
60	¼	125	Long-stroke engine, 19 × 48.	60	31·9	37·1	113	30·4	35·8
	½	125	Long-stroke engine, 19 × 48.	45	38·3	44·5	72	36·5	43·0
	¾	125	Long-stroke engine, 19 × 48.	..	..	..	..	..	..
100	¼	150	Long-stroke engine, 21 × 48.	40	27·1	31·4	61	26·4	31·1
80	¼	150	Long-stroke engine, 21 × 48.	52	28·9	33·6	79	28·0	32·9
60	¼	150	Long-stroke engine, 21 × 48.	73	31·4	36·5	112	30·2	35·5
	½	150	Long-stroke engine, 21 × 48.	46	37·8	44·0	71	36·3	42·7
	¾	150	Long-stroke engine, 21 × 48.	40	44·0	51·8	61	42·8	50·4

In Tables F and G are presented the most important dimensions of portable and stationary engines, which have been chosen with a view of fairly representing the practice of American builders, although the lists are quite small in comparison with the number of manufacturers. The several columns are sufficiently detailed to require no further explanation.

TABLE F.

Dimensions of Portable and Semi-Portable Engines.

Number for reference.	Name.	Boiler.			Furnace.			Thickness of Boiler Sheets.			Stay Bolts.		Tubes.			Grate surface.	Heating Surface.		Number for reference.
		Vertical or horizontal.	Height or length.	Diameter of shell.	Length.	Width.	Height.	Shell.	Crown sheet.	Tube heads.	Diameter.	Distance between centres.	Number.	External diameter.	Length.		In fire-box.	Total.	
			In.	In.	In.	In.	In.	In.	In.	In.	In.	Inches.		In.	In.	Sq. ft.	Sq. ft.	Sq. ft.	
1	J. C. Hoadley Co.	Horizontal.	109·75	27·0	36·5	23·0	28·9	·25	·25	·375	·875	4·5	29	2·25	54·0	5·75	28·0	108·0	1
2	Fitchburgh Steam Engine Co.	Vertical.	84·0	36·0	30·0 dia.	..	30·0	·25	·3125	·3125	·75	10·0	52	2·25	54·0	4·59	24·5	156·0	2
3	Erie City Iron Works.	Horizontal.	138·0	30·0	38·0	24·0	36·0	·25	·3125	·375	·875	5·5	22	3·0	78·0	6·32	30·0	120·0	3
4	Baxter	Vertical.	51·0	32·0	22·0 dia.	..	23·0	·26	·3125	·3125	..	....	24	2·0	18·0	2·5	..	58·0	4
5	Whitman & Burrell.	Horizontal.	102·0	30·0	30·0	24·0	20·0	·25	·3125	·3125	·625	6·0	15	3·0	89·0	5·0	10·0	120·0	5
6	Gaar, Scott & Co.		101·0	27·5	32·0	23·5	34·0	·25	·3125	·375	·875	5·0	27	2·5	54·0	5·29	20·0	85·7	6
7	Sample, McElroy & Co.	Vertical.	66·0	28·0	25·0 dia.	..	30·0	·25	·375	·375	·875	9·0	37	2·0	36·0	3·4	17·2	69·5	7
8	N. Y. Safety Steam Power Co.	"	75·0	35·0	30·0 dia.	..	21·0	·3125	·375	·375	·75	9·0	64	2·0	48·0	4·9	19·2	153·1	8
9	Lane & Bodley.	Horizontal.	102·75	28·0	37·0	17·8	28·8	·25	·25	·3125	·75	6·75 × 5·5	30	2·25	49·6	4·56	24·3	95·1	9
10	Niles Tool Works.	Vertical.	60·0	30·0	25·5 dia	..	22·0	·25	·3125	·3125	..	....	40	2·75	30·0	3·48	13·9	82·6	10
11	Emory W. Mills.	Horizontal.	110·0	19·0	16·0	27·5	27·0	·25	·25	·3125	·75	3·5	31	1·5	66·0	3·0	18·0	86·0	11
12	Snyder's Little Giant.	Vertical.	48·0	10·0	15·0 dia.	..	.	·3125	·375	....	..	....	32	1·1	16·0	0·92	..	14·0	12

TABLE F.

Dimensions of Portable and Semi-Portable Engines—(*Continued*).

Number for reference.	Height of chimney above grate.	Water room in boiler.	Steam space in boiler.	Draught Areas.				Ratio of		Diameter of Boiler Connections.				Engine Vertical or Horizontal.	Arrangement of Cut-off.	Steam Cylinder.		Number for reference.
				Opening through ash-pit doors.	Air spaces in grate bars.	Cross-section of tubes.	Cross-section of chimney.	Heating to grate surface.	Cross-section of tubes to grate surface.	Feed pipe.	Blow-off pipe.	Steam pipe.	Safety valve.			Diameter.	Stroke of piston.	
	Ft.	Cub. ft.	Cub. ft.	Sq. ft.	Sq. ft.	Sq. ft.	Sq. ft.			In.	In.	In.				In.	In.	
1	8·5	11·2	7·5	1·28	1·92	0·671	0·545	18·8	·117	0·75	1·0	1·5	2·0	Horizontal.	Controlled by governor.	7·5	10·0	1
2	...	18·0	11·0	0·486	2·43	1·2	0·55	34·0	·261	1·0	1·0	1·5	1 5	Vertical.	Fixed.	7·0	7·0	2
3	2·20	...	...	0·59	....	0·929	1·07	19·0	·169	1·0	1·5	1·5	2·0	Horizontal.	"	7·0	10·0	3
4	20·0	5·25	5·1	....	...	0·429	0·349	23·2	·172	1·0	0·75	....	1·0	Vertical.	"	5·0	5·0	4
5	11·5	15·13	8·75	0·521	1·77	0·633	0·417	24·0	·127	1·0	1·0	1·25	1·25	Horizontal.	Adjustable.	7·0	10·0	5
6	7·5	15·97	7·27	0·563	1·85	0·767	0·545	16·2	·145	0·75	1·5	1·5	1·5	"	Fixed.	6·5	13·0	6
7	....	7·25	5·2	0·375	0·792	0·661	0·785	20·4	·194	0·75	1·5	1·25	1·0	Vertical.	"	5·0	6·0	7
8	....	19·53	2·68	1·24	2·45	1·144	0·78	31·1	·234	0·75	1·25	1·25	2·0	"	"	7·0	9·0	8
9	16·4	17·53	6·67	1·24	1·86	0·694	0·327	20·8	·152	1·0	1·0	2·0	1·5	Horizontal.	"	7·0	12·0	9
10	....	6·29	3·78	0·66	1·16	1·4	0·54	23·4	·402	0·5	0 75	1·25	2·5	"	"	5·0	7·0	10
11	7·0	8·0	4·0	1·0	1·56	0·301	0·087	28·7	·1	1·0	1·0	1·75	1·25	"	Adjustable by governor.	6·0	6·0	11
12	....	0·9	0·46	0·122	0·458	...	0·165	15·3	..	0·375	0·375	....	....	"	Fixed.	2·75	4·5	12

TABLE F.

Dimensions of Portable and Semi-Portable Engines—(*Continued*).

Number for reference.	Port Area.		Clearance in per cent of piston displacement.	Kind of Valve.	Diameter of valve stem.	Stroke of valve.	Diameter of piston rod.	Diameter of exhaust pipe.	Feed Pump.		Fly-Wheels.						Number for reference.
	Steam port. Sq. in.	Exhaust port. Sq. in.			In.	In.	In.	In.	Diameter. In.	Stroke. In.	Number.	Diameter. Inches. 1st.	Diameter. Inches. 2d.	Width of face. Inches. 1st.	Width of face. Inches. 2d.	Weight. Pounds. 1st.	Weight. Pounds. 2d.	
1	3·93	4·13	10·0	Piston.	·625	$1\frac{1}{16}$ to $2\frac{1}{16}$	1·25	2 × 2	0·875	10·0	2	36	48·0	9·0	7·0	150	150	1
2	3·5	3·5	8·0	"	·5	2·5	1·38	2·0	1·75	3·0	1	42	....	8·0	....	650	....	2
3	4·0	7·63	6·25	Plain slide.	·151	2·25	1·19	2·0	1·125	10·0	2	30	44·0	8·5	10·5	120	350	3
4	1·0	1·75	6·6	" "	·625	1·0	0·88	1·5	1·0	1·5	2	24	14·0	5·0	6·5	160	48	4
5	3·75	15·0	9·0	" "	·938	....	1·06	2·0	2·0	....	1	48	...	8 25	....	320	....	5
6	3·0	6·5	9·23	" "	·875	2·25	1·25	2·0	0·75	13·0	1	42	....	8·0	....	370	....	6
7	2·5	5·0	19·5	" "	·563	1·75	0·94	2·0	1·625	3·0	1	26	....	6·5	....	200	....	7
8	3·13	5·0	6·86	" "	·75	2·5	2·13	1·5	1·75	2·5	1	42	....	7·5	....	565	...	8
9	3·62	6·87	9·07	" "	·938	1·88	1·31	1·81	0·75	12 0	2	40	15·9	7·88	4·38	345	35	9
10	1·25	2·0	3·86	Double end slide.	·625	1·25	1·0	1·5	1·75	1·5	1	24	....	6·5	....	238	....	10
11	4·1	11·0	11·75	Piston.	·5	3·5	1·0	2·0	1·625	1·0	1	36	....	6·0	....	260	....	11
12	0·38	0·69	7·67	Plain slide.	·5	0·63	...	1·0	1·0	....	2	16	6·0	1·75	2·0	35	...	12

TABLE F.

Dimensions of Portable and Semi-Portable Engines—(*Continued*).

Number for reference.	Engine attached to Boiler or independent.	Engine mounted on Truck Wheels or not.	Diameter of Truck Wheels.		Weight of Machine.		Dimensions over all.			Working conditions, as rated by Maker.				Number for reference.
			Front. In.	Rear. In.	Light. Lbs.	With water and fire. Lbs.	Length. In.	Width. In.	Height without stack. In.	Steam pressure above atmosphere. Lbs. per sq. in.	Point of cut-off from commencement of stroke. In.	Revolutions per minute.	Net Horse-power.	
1	Attached.	Mounted.	42·0	54	7505	8202	125	72·0	101·0	120	1·7	240	18·0	1
2	Independent.	Unmounted.	....	.. .	5480	6600	84	48·0	114·0	60	4·3	220	14·5	2
3	Attached.	Mounted.	44·0	54	6000	6500	144	72·0	90·0	.. .	...	175	10·0	3
4	"	Unmounted.	....	....	3100	3400	42	42·0	90·0	70	3·5	240	6·0	4
5	"	Mounted.	36·0	48	4800	5360	126	69·0	108·0	75	5·25	140	12·0	5
6	"	"	40·0	48	5225	6110	127	78·0	84·0	60	8·67	190	10·0	6
7	Independent.	Unmounted.	....	....	2400	2900	55	36·0	87·0	100	3·5	400	6·0	7
8	"	"	...	....	3542	4802	77	42·0	126·0	45	5·25	160	10·0	8
9	Attached.	Mounted.	33·1	41	5514	6600	148	79·5	80·5	75	4·9	200	15·0	9
10	Independent.	Unmounted.	....	....	2700	3200	54	34·0	65·5	60	5·25	225	7·0	10
11	Attached.	Mounted.	39·0	43	4250	4850	129	62·5	86·0	100	4·5	250	10·0	11
12	Independent.	Unmounted.	....	....	535	630	50	54·0	48·0	60	1·38	300	1·0	12

TABLE G.

Dimensions of Horizontal Stationary Engines, Non-condensing.

Designation of Line.		Erie City Iron Works.	Jerome Wheelock.	Lane & Bodley.	Providence Steam-Engine Co.		Charles T. Porter.	Watts, Campbell & Co.
					No. 1.	No. 2.		
Steam cylinder.	Diameter.....In.	14·0	14·0	9·0	12·0	26·0	18·0	20·0
	Stroke of piston..... "	18·0	42·0	15·6	30·0	48·0	30·0	48·0
Port area.	Steam..... Sq. in.	12·375	16 (both in one).	6·2	7·5	35·0	17·5	20·81
	Exhaust..... "	14·625		11·5	9·0	40·5	24·75	31·5
Clearance.	Volume.....Cub. in.	155·0	60·0	62·0	95·0	695·0	509·0	314.0
	Equivalent length of stroke.....In.	1·0	·39	·97	·84	1·31	2·0	1·0
	In per cent of piston displacement per stroke.	5·56	·93	6·25	2·8	2·7	6·67	2·08
Cut-off fixed or variable.		Fixed.	Variable.	Fixed.	Variable.	Variable.	Variable.	Variable.
Valves.	Description.....	Plain slide.	4 plug cocks.	Plain slide.	4 slides.	4 slides.	Balanced slide.	Corliss.
	Diameter of steam valve stem..In.	1·25	1·25	·938	1·375	2·0	2, each ·75	1·875
	" " exhaust " " ..... "				·875	1·0	1·0	1·875
	Stroke of steam valve..... "	4·125	10 in. on 8 in. crank.	2·25	Variable.	Variable.	Variable.	....
	" " exhaust " ..... "				1·125	2·5	2·25	....
Diameter of pipes.	Steam.....In.	3·5	4·5	3·0	4·0	7·0	7·0	5·0
	Exhaust..... "	5·0	6·0	4·0	5·0	8·0	7·0	6·5
Fly Wheel.	Diameter.....In	110·0	144·0	79·0	96·0	216·0	108·0	192·0
	Width of face..... "	7·0	24·0	4·5	17·0	32·0	30·0	30·0
	Weight.....Lbs.	2500	10000	1832	3500	23000	5000	20000
Working conditions, as rated by maker.	Steam pressure above atmosphere. Lbs. per sq. in.	....	60·0	60·0	75·0	75·0	80·0	80·0
	Cut-off from commencement of stroke.....In.	....	....	10·9	....	...	7·5	12·0
	Exhaust opens from " " " ..... "	....	....	14·875	....	...	2·5	2·75
	" closes " " " " ..... "	....	....	14·5	....	...	2·5	3·0
	Revolutions per minute.....	125·0	75·0	200·0	70·0	65·0	140·0	65·0
	Net horse-power.....	50·0	75·0	34·0	40·0	240·0	200·0	165·0
Dimensions, over all.	Length.....In.	123·0	264·0	110·0	201·0	354·0	237·0	345·0
	Width..... "	24·0	120·0	25·0	96·0	156·0	110·0	144·0
	Height..... "	32·0	120·0	23·0	....	....	....	126·0
Total weight of engine.....Lbs.		9500	20000	3592	11000	56000	17000	26900

Particulars of the trials of a number of stationary and portable engines are contained in Table H. Experiments 1 to 10 were made, with one or two exceptions, with what were called "racing engines"—that is, engines built with far more care than was used for the general market, and fired by carefully-trained experts—while the remaining experiments were made with engines of the ordinary commercial type, under ordinary management. It will be seen, however, by a reference to experiment 16, that an American engine, as ordinarily built for the market, gave results but little inferior to those obtained with the "racers;" while a comparison of experiments 17 and 18, which were made with the same engine, with no change except that which was due to the experience gained in the first trial, shows how much the result may be affected by the management, other things being equal.

Although compound engines have been generally adopted for ocean steamers within the last few years, producing greater economy than was usual in the case of the simple engines formerly employed, yet as the change was made simultaneously with the increase of steam pressure and piston speed, there are many engineers who deny that there is any economy in the compound system. Although this question has been vigorously discussed for several years, theoretical considerations have been advanced more frequently than experimental proof, and, in fact, there are scarcely any complete records of comparative tests of simple and compound engines. Table I. contains a summary of some experiments made on United States revenue steamers with simple and compound engines. These experiments were made with engines of rather small size, were generally of very short duration, and there was no attempt to determine experimentally the relation between the net and indicated horse-power. They constitute, however, almost the only experimental record bearing on this question that is worthy of consideration. Table I. is compiled from a paper on "Compound and Non-Compound Engines," published in the "Transactions of the American Society of Civil Engineers," iii., 368, and the "Report of the Trial of the Steam Machinery of the United States Revenue Steamer 'Gallatin,'" Washington, 1875. The principal dimensions of the engines, as given in the above papers (which also contain an interesting discussion of the subject), were as follows:

"Rush," compound engine, 2 vertical cylinders with intermediate receiver, both cylinders steam-jacketed. Diameter of cylinders, 24 and 38 inches respectively; stroke of pistons, 27 inches.

"Bache," compound engine, two vertical cylinders with intermediate chamber, large cylinder only steam-jacketed. Diameter of cylinders, 15·98 and 25 inches; stroke of pistons, 24 inches; per cent of clearance, 4·86 high-pressure and 4·05 low-pressure cylinder. In the experiments with the engines of the "Bache," marked "single," only the large cylinder was used.

"Dexter," single vertical cylinder, not steam-jacketed. Diameter, 26 inches; stroke of piston, 36 inches.

"Dallas," single vertical cylinder, not steam-jacketed. Diameter, 36 inches; stroke of piston, 30 inches.

"Gallatin," single vertical cylinder, steam-jacketed. Diameter, 34·1 inches; stroke of piston, 30 inches; per cent of clearance, 6·59.

TABLE H.

Summary of Tests of Non-condensing Portable and Stationary Engines.

Number for reference.	Name.	Length of trial. Hrs	Diameter of cylinder. In.	Stroke of piston. In.	Revolutions per minute.	Point of cut-off from commencement of stroke. In.	Pressure above atmosphere, in pounds per square inch. In boiler.	Initial.	Mean effective.	Horse-power. Indicated.	Net.	Ratio of net to indicated.	Pounds of water per hour. Per indicated horse-power.	Per net horse-power.	Pounds of coal per hour. Per indicated horse-power.	Per net horse-power.	Number for reference.
	Portable Engines at Royal Agricultural Society's Show, Cardiff, 1872.																
1	Marshall, Sons & Co.	4·15	8·5	12·0	168·8	2·04	80·0	77·0	31·25	18·0	14·3	·797	23·99	30·11	2·62	3·3	1
2	Clayton & Shuttleworth,	4·84	9·0	12·0	113·9	2·76	80·0	71·5	32·1	..	14·26	....	....	27·49	....	2·84	2
3	E. Hayes	1·48	9·0	12·0	122·6	7·2	63·0	41·6	19·6	9·1	7·55	·823	22·27	26·85	8·3	10·0	3
4	Davey, Paxman & Co.	4·33	8·63	12·0	114·2	1·92	80·0	73·0	33·9	13·6	11·94	·878	28·44	32·4	2·85	3·25	4
5	Brown & May.	4·13	7·25	12·0	139·1	2.7	80·5	73·0	29·2	10·1	9·27	·92	27·55	29·97	3·02	3·29	5
6	Tasker & Sons.	2·75	9·0	12·0	123·7	4·56	59·5	52·0	29·72	14·0	12·37	·88	34·16	38·66	4·36	4·94	6
7	Reading Iron Works	4·93	8·5	14·0	138·2	3·52	80·3	72·5	37·0	20·32	16·77	·826	23·59	28·49	2·38	2·88	7
8	E. R. & F. Turner.	3·87	9·0	12·0	179·2	3·84	80·0	77·2	36·24	24·8	20·0	·803	24·07	29·85	2·9	3·63	8
9	Barrows & Stewart.	2·5	9·5	13·0	116·1	7·8	70·0	47·2	25·8	13·8	11·63	·84	34·64	41·1	4·87	5·78	9
10	Ashby, Jeffery & Luke.	1·8	9·5	14·0	125·0	2·24	80·0	63·0	20·4	12·6	8·33	·66	40·56	61·36	4·94	7·47	10
	Portable Engines at Cincinnati Industrial Exposition, 1876.																
11	Lane & Bodley.	5·0	7·0	12·0	203·3	....	72·3	....	....	18·46	15·06	·816	40·81	50·01	6·56	8·04	11
12	Woodsum Machine Co.	5·0	6·5	12·5	208·3	....	75·3	...	....	12·53	10·47	·836	71·46	85·22	11·43	13·67	12
13	Robinson Machine Works.	5·0	6·5	12·0	205·8	....	74·6	....	....	14·95	11·58	·775	53·13	68·7	9·22	11·9	13
14	Gaar, Scott & Co.	5·0	6·5	13·0	188·4	....	74·4	....	....	15·36	12·34	·803	64·6	68·01	8·89	11·07	14
15	Brownell & Kielmeier Manufacturing Co	5·0	6·0	10·0	197·6	....	74·2	....	....	9·87	8·34	·845	61·1	72·31	9·99	11·82	15
	Portable Engine at Centennial Exposition, 1876.																
16	J. C. Hoadley Co.	6·03	14·56	20·0	126·0	3·53	120·0	117·6	38·49	80·3	72·7	·905	25·61	28·27	3·35	3·69	16

TABLE H.

Summary of Tests of Non-condensing Portable and Stationary Engines—(*Continued*).

Number for reference.	Name.	Length of trial. Hrs	Diameter of cylinder. In.	Stroke of piston. In.	Revolutions per minute.	Point of cut-off from commencement of stroke. In.	Pressure above atmosphere, in pounds per square inch. In boiler.	Pressure above atmosphere, in pounds per square inch. Initial.	Pressure above atmosphere, in pounds per square inch. Mean effective.	Horse-power. Indicated.	Horse-power. Net.	Horse-power. Ratio of net to indicated.	Pounds of water per hour. Per indicated horse-power.	Pounds of water per hour. Per net horse-power.	Pounds of coal per hour. Per indicated horse-power.	Pounds of coal per hour. Per net horse-power.	Number for reference.
	Portable Engines at N. Y. State Agricultural Fair, 1877.																
17	Porter Manufacturing Co., Limited	4·0	7·0	10·0	141·3	....	72·4	...	....	....	7·39	....	....	70·16	....	13·02	17
18	" " " "	4·0	7·0	10·0	138·4	....	79·1	....	....	....	9·25	....	.. .	43·22	....	5·54	18
19	" " " "	2·0	5·5	7·0	190·9	....	91·8	...	....	....	5·22	....	....	61·61	....	10·06	19
20	Fishkill Landing Machine Co	4·0	6·0	6·0	238·6	....	119·1	....	....	....	8·88	...	....	45·1	....	6·33	20
21	Mansfield Machine Works	4·0	7·0	10·0	162·9	....	59·9	...	...	....	7·39	....	....	76·61	....	12·69	21
22	G. Westinghouse & Co	4·0	6·0	7·0	260·1	....	69·9	....	....	....	7·95	....	....	63·78	....	9·02	22
23	E. M. Birdsall & Co	4·0	7·0	9·0	204·4	....	65·8	....	. .	....	8·29	....	....	63·31	.	10·04	23
24	Watertown Steam-Engine Co	4·0	6·0	12·0	184·4	....	89·6	....	...	....	7·13	....	....	62·45	....	9·23	24
25	Frick & Co	4·0	7·0	10·0	178·5	....	89·5	....	....	....	6·28	....	...	70·32	...	9·68	25
26	Oneida Iron Works	4·0	7·0	10·0	182·8	....	85·3	...	....	...	6·64	....	....	76·18	....	10·51	26
27	B. W. Payne & Sons	2·0	4·0	5·0	227·1	....	91·1	....	....	. .	3·27	....	....	61·26	....	7·66	27
	Automatic Cut-Off Engines at American Institute Exhibition, 1869.																
28	Babcock & Wilcox	8·0	16·0	42·0	60·3	7·94	81·7	76·1	31·06	78·79	68·7	·872	25·48	29·23	....	....	28
29	Harris-Corliss	8·0	16·13	42·0	60·3	9·49	80·5	70·9	29·73	76·58	69·1	·901	26·06	28·88	....	....	29
	Automatic Cut-Off Engines at Cincinnati Industrial Exposition, 1874.																
30	Harris-Corliss	8·0	16·06	48·0	60·1	9·94	70·5	63·3	25·38	74·93	65·89	·879	29·18	33·19	....	....	30
31	Cooper	8·0	16·0	30·0	84·3	7·83	70·3	70·1	29·18	74·94	66·59	·889	30·56	33·95	....	...	31
	Slide-Valve Engines, Regulated by Throttle, at Cincinnati Industrial Exposition, 1875.																
32	Buckeye	5·0	10·0	14·0	209·8	8·75	79·0	58·4	35·15	40·95	36·69	·896	43·15	48·16	....	....	32
33	Lane & Bodley	5·0	9·0	15·6	194·7	13·57	77·9	58·9	39·06	38·12	33·75	·885	44·86	50·67	....	....	33

TABLE I.

Summary of Experiments with U. S. Revenue Steamers "Rush," "Bache," "Dexter," "Dallas," and "Gallatin," 1874, 1875.

Number for reference.	Duration of experiment in hours.	Ratio of expansion.	Revolutions per minute.	Pressure in Pounds per Square Inch.				Indicated Horse-power.		Pounds of water per indicated horse-power per hour.	Remarks.	Number for reference.
				Initial, above zero.		Indicated.						
				Small cylinder.	Large or single cylinder.	Small cylinder.	Large or single cylinder.	Small cylinder.	Both or single cylinder.			
1	55·0	6·22	70·8	82·3	23·5	29·7	12·7	127·9	266·5	18·4	"Rush," compound, steam jackets in use	1
2	6·0	4·03	55·5	50·2	22·0	27·1	15·7	63·7	168·7	22·1	" " " " "	2
3	1·93	6·98	53·2	93·5	21·1	42·3	14·5	54·1	99·2	20·3	"Bache," compound, steam jackets in use.	3
4	1·98	5·73	56·3	90·3	23·6	42·9	15·9	58·1	110·5	20·4	" " " " "	4
5	2·07	9·19	48·2	92·0	18·9	35·4	13·0	40·9	77·5	20·7	" " " " "	5
6	2·0	4·24	60·6	87·3	28·5	45·3	19·3	65·9	134·5	21·2	" " " " "	6
7	7·07	5·71	55·6	90·0	23·7	45·4	14·0	60·6	106·0	22·0	" " " " "	7
8	15·23	5·1	53·6	88·8	22·8	43·1	14·9	55·4	102·3	22·4	" " " " "	8
9	1·73	16·85	38·9	90·6	15·9	24·5	10·3	22·9	46·4	25·1	" " " " "	9
10	2·07	6·66	47·7	91·1	17·2	43·5	17·8	40·9	77·1	23·0	" " " " not in use.	10
11	2·13	5·63	49·3	87·7	18·6	45·1	11·3	53·3	85·8	23·2	" " " " " "	11
12	1·83	9·15	42·6	89·7	15·7	34·8	8·2	35·6	55·9	23·8	" " " " " "	12
13	2·12	5·11	53·8	....	90·7	....	36·9	...	116·0	23·2	" single, " " in use.	13
14	1·68	8·57	46·2	...	90·1	....	27·5	...	74·6	24·1	" " " " "	14
15	2·1	12 62	39·9	....	88·8	....	23·4	....	54·8	27·1	" " " " "	15
16	1·88	2·18	45·3	....	41·5	....	25·2	....	66·7	34·0	" " " " "	16
17	2·05	5·32	47·1	....	87·4	....	32·3	....	89·1	26·2	" " " " not in use.	17
18	1·98	7·62	44·9	....	90·7	...	27·3	...	71·8	29·6	" " " " " "	18
19	1·8	11·82	37·3	....	88·9	....	21·6	....	47·2	35·1	" " " " " "	19
20	2·92	4·46	56·5	....	80·4	....	34·4	....	185·9	23·9	"Dexter," single, not jacketed.	20
21	34·5	3·49	61·1	....	79·2	...	37·5	....	219·0	23·9	" " " "	21

TABLE I.

Summary of Experiments with U. S. Revenue Steamers "Rush," "Bache," "Dexter," "Dallas," and "Gallatin," 1874, 1875—(*Continued*).

Number for reference.	Duration of experiment in hours.	Ratio of expansion.	Revolutions per minute.	Pressure in Pounds per Square Inch. Initial, above zero. Small cylinder.	Initial, above zero. Large or single cylinder.	Indicated. Small cylinder.	Indicated. Large or single cylinder.	Indicated Horse-power. Small cylinder.	Indicated Horse-power. Both or single cylinder.	Pounds of water per indicated horse-power per hour.	Remarks.	Number for reference.
22	1·42	3·67	64·3	....	79·3	.. .	37·1	....	228·1	24·1	"Dexter," single, not jacketed.	22
23	0·65	2·72	72·8	....	77·0	....	42·0	....	292·4	24·3	" " " "	23
24	1·32	3·34	50·8	....	52·3	....	25·6	....	124·3	28·8	" " " "	24
25	1·2	2·42	55·3	....	51·4	....	30·7	...	161·8	28·9	" " " "	25
26	0·92	2·08	60·7	....	53·6	....	33·8	....	196·2	31·8	" " " "	26
27	1·52	5·07	48·7	....	46·9	....	18·5	...	138·0	26·7	"Dallas," single, not jacketed.	27
28	31·0	3·13	61·5	....	46·6	....	23·5	....	221·4	26·9	" " " "	28
29	1·55	3·89	56·9	....	47·0	....	21·4	....	186·8	27·0	" " " "	29
30	1·6	2·91	64·5	....	45·9	....	24·6	....	242·8	28·9	" " " "	30
31	1·53	2·32	63·5	....	39·4	....	24·1	....	234·3	31·0	" " " "	31
32	2·22	7·31	51·1	...	85·0	....	26·1	....	197·0	20·5	"Gallatin," single, steam jacket in use.	32
33	1·93	4·19	68·7	.. .	80·7	....	36·9	..	348·0	21·5	" " " " "	33
34	2·5	4·91	62·2	....	82·4	....	35·0	....	289·2	21·7	" " " " not in use.	34
35	2·02	4·94	59·9	....	81·7	....	34·0	....	279·6	21·9	" " " " " "	35
36	23·98	4·5	61·5	....	77·8	....	33·4	....	281·6	22·0	" " " " in use.	36
37	2·2	4·82	45·9	....	54·6	....	21·7	....	136·4	22·4	" " " " "	37
38	2·02	6·08	44·3	....	58·0	....	19·9	....	121·1	22·9	" " " " "	38
39	2·07	3·45	58·6	....	70·2	....	32·1	....	258·5	23·0	" " " " "	39
40	2·03	3·71	49·2	. ..	54·7	....	24·6	....	166·2	23·2	" " " " "	40
41	1·87	4·87	58·5	....	82·6	....	31·8	....	255·3	23·5	" " " " "	41
42	2·05	5·63	56·0	. ..	80·8	....	30·1	....	231·4	23·8	" " " " not in use.	42

TABLE I.

Summary of Experiments with U. S. Revenue Steamers "Rush," "Bache," "Dexter," "Dallas," and "Gallatin," 1874, 1875—(*Continued*).

Number for reference.	Duration of experiment in hours.	Ratio of expansion.	Revolutions per minute	Pressure in Pounds per Square Inch.				Indicated Horse-power.		Pounds of water per indicated horse-power per hour.	Remarks.	Number for reference.
				Initial, above zero.		Indicated.						
				Small cylinder.	Large or single cylinder.	Small cylinder.	Large or single cylinder.	Small cylinder.	Both or single cylinder.			
43	2·05	3·73	50·8	....	55·8	...	26·2	....	182·2	24·0	"Gallatin," single, steam jacket in use.	43
44	2·05	5·07	46·1	...	53·9	....	21·7	....	137·5	24·0	" " " " not in use.	44
45	1·95	2·37	60·9	...	63·6	...	34·0	....	284·2	24·2	" " " " in use.	45
46	23·95	4·46	60·5	....	76·2	....	32·4	....	268·6	24·4	" " " " not in use.	46
47	2·32	2·72	56·0	....	52·3	....	30·8	....	236·9	24·5	" " " " " "	47
48	2·22	4·49	50·3	....	54·7	....	23·6	....	163·3	24·8	" " " " in use.	48
49	1·92	2·47	62·0	....	66·9	....	35·0	....	297·8	25·2	" " " " not in use.	49
50	2·08	2·4	54·5	....	49·3	....	28·5	....	213·0	25·3	" " " " in use.	50
51	2·03	3·83	61·6	...	78·1	....	33·4	....	282·5	25·3	" " " " "	51
52	3·75	3·32	51·2	....	52·4	....	26·4	....	185·2	25·7	" " " " "	52
53	2·05	4·07	49·5	....	82·6	....	27·9	..	189·6	25·9	" " " " " Condensing with-	53
54	1·77	5·92	43·0	....	56·3	....	20·8	....	122·8	26·0	" " " " not in use. [out vacuum.	54
55	2·1	3·16	50·1	....	51·8	....	26 5	....	182·4	26·3	" " " " " "	55
56	2·3	2·21	58·2	....	50·4	....	31·9	....	2·5·0	26·5	" " " " in use.	56
57	1·98	5·21	44·2	....	52·8	....	21·0	....	127·2	26·7	" " " " not in use.	57
58	2·12	3·52	43·8	....	78·8	....	45·9	...	212·2	27·3	" " " " in use. Condensing with-	58
59	2·17	2·23	55·8	....	50·3	....	30·9	....	236·5	28·1	" " " " not in use. [out vacuum.	59
60	2·22	3·48	51·7	....	79·1	....	28·8	....	204·8	29·4	" " " " " " " "	60
61	2·2	4·37	46·7	....	83·7	....	26·5	....	169·6	30·0	" " " " " " " "	61
62	2·1	2·0	41·3	....	26·3	...	16·8	....	95·3	33·3	" " " " in use.	62
63	2·2	1·54	42·3	....	25·0	....	17·7	....	102·9	34·9	" " " " "	63
64	2·13	1·49	42·5	...	24·1	....	16·8	....	97·9	37·4	" " " " "	64
65	1·92	2·02	40·1	...	25·3	....	15·8	....	87·0	40·4	" " " " not in use.	65
66	2·25	1·51	40·9	....	23·7	....	16·1	....	90·2	44·2	" " " " " "	66

Table I. contains, as will be seen, some comparative experiments as to the value of a steam-jacket; but these are in some cases contradictory, and a more complete series of experiments on steam-jackets is summarized in Table J, which is compiled from a paper on "Experiments, by Henry Watermann, 1864," in the "Annual Report of the Chief of the Bureau of Steam Engineering for the Year 1875."

TABLE J.

Summary of Experiments with Watermann's Engine.

Designation of experiments.	Ratio of expansion.	Revolutions per minute.	Pressure in Pounds per Square Inch. Initial, above zero.	Pressure in Pounds per Square Inch. Mean net.	Net horse-power.	Pounds of water per net horse-power per hour.	Remarks.
A	3·62	43·1	50·6	21·25	8·66	44·75	No steam in jacket.
B	1·93	43·1	37·7	21·83	8·9	54·19	" " "
C	1·47	43·1	33·3	20·75	8·46	58·62	" " "
D	1·11	43·1	30·5	20·87	8·51	64·64	" " "
E	3·62	61·4	21·0	4·28	2·48	129·56	" " "
F	1·11	61·5	15·2	6·15	3·51	117·02	" " "
G	3·62	43·2	50·2	21·77	8·88	35·25	Steam of boiler pressure in jacket.
H	3·08	43·1	44·3	21·82	8·9	36·2	" " "
I	1·93	43·2	34·4	20·87	8·52	39·03	" " "
J	1·93	43·1	36·1	22·07	8·99	38·27	" " "
K	1·32	43·1	31·4	21·18	8·63	43·75	" " "
L	1·11	43·2	28·6	20·34	8·64	49·52	" " "
M	1·11	61·4	13·6	5·92	3·43	80·97	" " "
N	3·08	43·2	47·1	20·82	8·49	32·28	Pressure of steam in jacket, 129 lbs. above zero.
O	1·93	43·1	34·4	20·33	8·3	36·49	" " " 133 " " "
P	1·32	43·0	31·9	21·73	8·43	40·84	" " " 142 " " "
Q	3·62	43·1	47·6	20·58	8·39	40·56	Steam of boiler pressure in jacket. Jacket not drained.
R	1·93	43·1	36·0	22·31	9·09	38·56	" " " " except upper cylinder head jacket.
S	1·32	43·1	33·2	21·98	8·95	43·57	" " " " lower " "
T	3·62	43·1	46·0	20·23	8·24	39·17	" " " " piston and piston rod " "

The principal dimensions of the engine, and other particulars, are appended.

Condensing engine, single vertical cylinder, steam-jacketed on sides, heads, in piston and piston-rod. Diameter of cylinder, 10·05 inches; stroke of piston, 24 inches; displacement of piston per stroke, 1·0828 cub. ft.; clearance at each end of cylinder, ·0222 cub. ft. Each experiment was continued for 30 hours.]

§ 513.—Principle of the Caloric or Hot-air Engine.— If a volume of air V expands and passes from the pressure p to p_1, without changing its temperature, it performs an amount of work

$$L = Vp \log. nat. \left(\frac{p_1}{p}\right) \quad \text{(Compare Vol. I., § 388.)}$$

If, however, it is expanded by heating to V_1, for instance, $V_1 = 2\,V$, or double its original volume, then the work for the same tensions is

$$L_1 = V_1 p \log. nat. \left(\frac{p_1}{p}\right),$$

or, in the special case above,

$$L_1 = 2\,Vp \log. nat. \left(\frac{p_1}{p}\right),$$

or twice as great as before the heating.

In general, the increase of energy due to the increase of volume $V_1 - V$, is

$$L_1 - L = (V_1 - V) p \log. nat. \left(\frac{p_1}{p}\right).$$

If t is the initial temperature and t_1 the temperature of the air after heating, we have

$$\frac{V_1}{V} = \frac{1 + \delta t_1}{1 + \delta t} \quad \text{(Vol. I., § 392);}$$

hence

$$\delta t_1 = (1 + \delta t)\frac{V_1}{V} - 1,$$

and the increase of temperature is

$$t_1 - t = \frac{(V_1 - V)(1 + \delta t)}{\delta V}.$$

If the specific heat for constant pressure,

$$\omega = \kappa \omega_1 = 1{\cdot}41 \times 0{\cdot}2375 = 0{\cdot}335,$$

the heat necessary for this rise of temperature is

$$W = \omega(t_1 - t)\, V\gamma = 0{\cdot}335\,(V_1 - V)\left(\frac{1 + \delta t}{\delta}\right)\gamma,$$

where γ is the density of the given quantity of air V.

If we take, finally, the mechanical equivalent of heat = 1390 foot pounds (§ 379), we have for the ratio of the work performed to the corresponding expenditure of heat,

$$\eta = \frac{L_1 - L}{1390\, W} = \frac{(V_1 - V)\, p\, log.\, nat.\left(\frac{p_1}{p}\right)}{0{\cdot}335 \times 1390\,(V_1 - V)(1 + \delta t)\frac{\gamma}{\delta}}$$

$$= \frac{\delta}{456} \cdot \frac{p}{\gamma(1 + \delta t)} \cdot log.\, nat.\left(\frac{p_1}{p}\right).$$

Since $\delta = 0{\cdot}00367$ and $\gamma = \frac{0{\cdot}005672\, p}{144(1 + \delta t)}$,

or

$$\frac{p}{(1 + \delta t)\gamma} = \frac{144}{0{\cdot}005672},$$

we have more simply, the efficiency

$$\eta = \frac{144 \times 0{\cdot}00367}{0{\cdot}005672 \times 456} log.\, nat.\left(\frac{p_1}{p}\right) = 0{\cdot}2043\, log.\, nat.\left(\frac{p_1}{p}\right).$$

If, for example, the ratio $\frac{p_1}{p} = 2$, we have

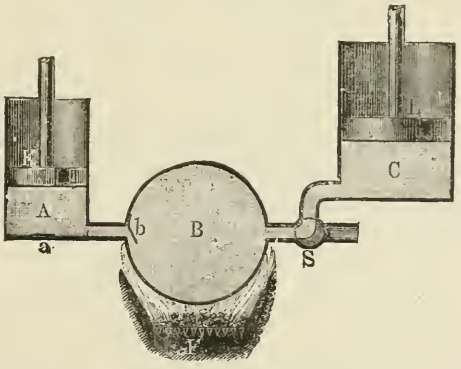
$$\eta = 0 \cdot 2043 \, log. \, nat. \, 2 = 0 \cdot 2043 \times 0 \cdot 6931 = 0 \cdot 1416,$$

or about $\frac{1}{7}$.

There is therefore only about $\frac{1}{7}$ of the entire work corresponding to the heat imparted, performed. According to § 490, this is the theoretical efficiency of a steam engine under the most favorable circumstances, and only for very high tensions.

§ **514.—Caloric Engine.**—Fig. 806 gives an ideal representation of a caloric or heat engine. *A* is the small cylinder whose piston *K* sucks in air through the valve *a* when rising, and, descending, forces it through *b* into the reser-

Fig. 807.



voir *B*. At *F* is a grate, by which the air in *B* is heated before it enters the large cylinder *C*. Finally, *S* is a valve, by which the entrance of the air from *B* to *C* and the exit of the same from *C* into the outer air is regulated.

If p is the tension of the outer air, and p_1 that in the reservoir *B*; s_0 the stroke of the piston *L* before expansion, and s_1 the entire stroke, then

$$\frac{s_0}{s_1} = \frac{p}{p_1}$$

or

$$s_0 = \left(\frac{p}{p_1}\right) s_1.$$

If, also, $V = Fs$ is the capacity of the force pump A, or the volume of air forced into the reservoir B at each stroke, measured under the outer pressure p, we have for the capacity of the working cylinder, and hence for the volume of air at the temperature t_1 used and discharged into the air at each stroke, measured under the outer pressure p,

$$V_1 = F_1 s_1 = \left(\frac{1 + \delta t_1}{1 + \delta t}\right) V.$$

At the beginning of expansion, this quantity occupies only the space

$$V_0 = F_1 s_0 = F_1 s_1 \left(\frac{p}{p_1}\right) = \frac{1 + \delta t_1}{1 + \delta t} \cdot \frac{p}{p_1} \cdot V.$$

If $\frac{1 + \delta t_1}{1 + \delta t} = \frac{p_1}{p}$, we have $V_0 = V$, or the space occupied by the heated air in the cylinder C, equal to the capacity of the cylinder A. In this case, we have for the corresponding temperature of the heated air,

$$t_1 - t = \frac{p_1 - p}{p} \left(\frac{1}{\delta} + t\right);$$

thus, for $\frac{p_1}{p} = 2$ and $t = 10°$,

$$t_1 - t = \frac{1}{0.00367} + 10 = 282.5.$$

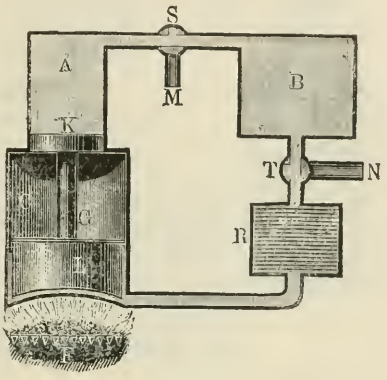
This high temperature is the chief practical hindrance to the introduction of the heat engine. In order to prevent drying of the piston L, it is made hollow and filled with some poor conductor of heat.

In order to economize as much as possible the heat of

the discharged air, and use it for heating the air for the next stroke, Ericsson causes the air before exit to pass through what he calls a "*regenerator*," which consists of a network of wires. Since this arrangement has not proved durable, it is omitted in recent engines.

The principal parts of Ericsson's air engine are shown in Fig. 808. The pistons K and L move in the cylinders A and C, and are united by the rod G. The fire grate is at F, directly under the working cylinder C. B is a special air reservoir and R the regenerator. S and T are two regulating valves, which regulate the entrance and exit of the air as well as its passage from A to B and from B to R. The space between the pistons K and L is a vacuum. As KL rises, the air previously sucked in through M is forced from A to B, then through R, where it becomes heated by the regenerator and finally below L. At a certain point of the stroke the valve T is turned so as to shut off communication between B and R, and thus during the rest of the stroke the air works expansively. At the end of the upward stroke, S and T are turned so that A communicates with the outer air by M and R by N, and the pistons sink by their own weight. Fresh air is thus drawn in at M, while the heated air escapes at N, imparting a portion of its heat to the regenerator R on the way. When the pistons are at the bottom of the stroke, S and T are again turned, so that air can pass from A to B, R and L, and a new stroke begins.

Fig. 808.

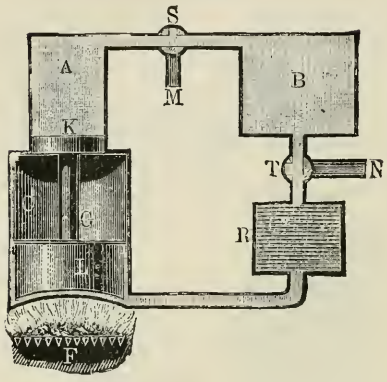


The work of this engine per stroke is therefore

$$L = (V_1 - V)\,p\ log.\ nat.\left(\frac{p_1}{p}\right),$$

or if, during the expansion of the air, there is no heat imparted,

Fig. 809.



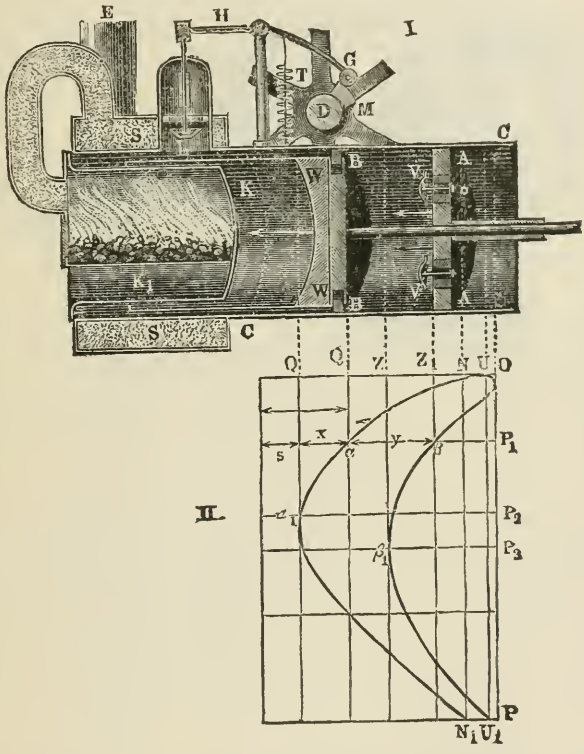
$$L = (V_1 - V)\, p \frac{\kappa - 1}{\kappa} \left[1 - \left(\frac{p_1}{p}\right)^{\frac{\kappa - 1}{\kappa}}\right],$$

where V is the space described by the piston K and V_1 that by L, p the pressure of the exterior air, and p_1 that of the heated air at the beginning of expansion, and κ the ratio of the specific heats $\frac{\omega}{\omega_1}$.

§ **515.**—In Fig. 810 we have represented the arrangement of a new air engine by Ericsson. The furnace F is in the interior of a cylinder which projects into the working cylinder $C\,C$, the products of combustion passing through a pipe R to the chamber S, and then into the chimney E. In the cylinder $C\,C$ we have two pistons, the working piston $A\,A$, and the feed piston $B\,B$, so arranged that during a stroke they first move away from each other and then afterwards approach, so that at the end of the stroke they are, as at the beginning, close together. Both pistons have valves. The valves $V\,V$ of the driving piston open in the direction of the axis, while the peculiar valve $W\,W$ of the feed or compression piston opens radially, so that it alternately presses against the sides of the cylinder and moves away from them, in the latter case opening communication between both sides of this piston. A peculiar arrangement of crank, rods, and levers, connects this piston with the fly-wheel D. As the pistons separate, both moving in the direction of the arrow or backwards, the valves $V\,V$ open, and $W\,W$ is shut. Fresh air thus

enters the space between the pistons, while the air in front of *B B* is forced out through the valve *L*. This valve is connected by means of the lever *H G* with a cam upon the fly-wheel and with a spiral spring *T*. During the backward motion of both pistons, the space *B B V V* between them, as also the space *W W K L* in front of the compress-

FIG. 810.

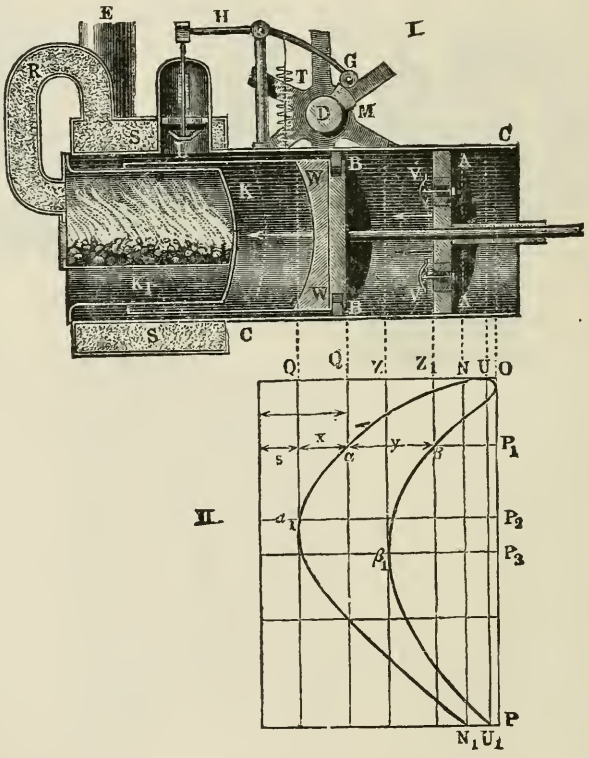


ing piston, is in communication with the outer air. The pressure upon both sides of both pistons is nearly the same, and the mechanical effect is zero.

During the forward motion of both pistons (opposite to the arrow), the valves *V V* and *L* are closed, and the ring valve *W W* is opened, and in both spaces *B B V* and *W W K* we have heated air. The piston *A A* is, therefore,

forced forwards by the difference of pressure between the heated air and the outer air, while the pressure of the heated air upon both sides of $B\,B$ is the same. The delivery of the engine per stroke is, therefore, the product of the effective pressure upon the piston $A\,A$ into the space through which it passes.

Fig. 811.



The diagram II. in Fig. 811 gives a graphical representation of the relation between the motions of both pistons and the space between them. The horizontal lines give the travel of the pistons, and the verticals give the travel of the crank on the fly-wheel D. While the crank makes a complete revolution, given by the line $O\,P = 2\,\pi\,r$, the piston $B\,B$ moves through the distance $N\,Q$ and back,

and $A\,A$ through $U\,Z$ and back. If the crank is at P_1, the first piston is at Q_1 and the other at Z_1; if at P_2, the corresponding position of $B\,B$ is at Q, and at P_3 the working piston $A\,A$ stands at the end Z of its stroke. While the two pistons at the beginning and end of their travel are distant $N\,U$, at the position of the crank P_1, their distance from each other is $Q_1\,Z_1$, etc. Projecting these points upon the horizontal through P_1, we obtain the points α and β of the curves $N\,\alpha\,\alpha_1\,N_1$ and $U\,\beta\,\beta_1\,U_1$, which give the relation between the piston distances and the motion of the crank.

§ 516.—**Theory of Ericsson's Heat Engine.**—By means of the mechanical theory of heat, we can determine the delivery of an Ericsson engine (according to Zeuner), as follows:

Let F be the area of piston, p the inner excess of pressure over the atmosphere, and $d\,s$ the elementary portion of the travel s of the working piston. Then the elementary work is

$$d\,L_1 = F\,p\,d\,s.$$

If T_1 is the absolute temperature of the air in the space between the furnace and $B\,B$, and T that in the space between the pistons, then, according to § 364, we have the corresponding specific volumes (per unit of weight),

$$v_1 = \frac{R\,T_1}{p} \quad \text{and} \quad v_2 = \frac{R\,T}{p},$$

hence

$$F\,d\,s = (v_1 - v_2)\,d\,G_1,$$

$$d\,L_1 = R\,(T_1 - T)\,d\,G_1,$$

where $d\,G_1$ is the quantity of air which, during the distance $d\,s$, passes from one side to the other of $B\,B$.

Further, the total quantity of air $G = G_1 + G_2$, in the engine, is the constant sum of the quantities upon both sides of $B\,B$; hence

$$d\,G_1 + d\,G_2 = 0,$$

or

$$d\,G_1 = -\,d\,G_2.$$

We have also $G_2\,v_2 = F\,y$, where y is the variable distance $\alpha\,\beta$ of the two pistons from each other; hence

$$G_2 = \frac{F\,y}{v_2} = \frac{F\,p\,y}{R\,T},$$

$$d\,G_1 = -\,d\,G_2 = -\,\frac{F}{R\,T}\,d\,(p\,y),$$

and

$$d\,L_1 = -\left(\frac{T_1 - T}{T}\right) F\,d\,(p\,y).$$

Integrating,

$$L_1 = -\left(\frac{T_1 - T}{T}\right) F\,p\,y + Const.$$

If the initial pressure is p_1 and piston distance y_1, we have

$$0 = -\,\frac{T_1 - T}{T}\,F\,p_1\,y_1 + Const.;$$

hence the delivery is

$$L_1 = \left(\frac{T_1 - T}{T}\right) F\,(p_1\,y_1 - p\,y).$$

Further,

$$G = G_1 + G_2 = \frac{F\,p}{R}\left(\frac{z}{T_1} + \frac{y}{T}\right) = \frac{F\,p_1}{R}\left(\frac{s}{T_1} + \frac{y_1}{T}\right),$$

where s is the length of the initial prism of air behind $B\,B$, and the distance $s - x$ is denoted by z; hence,

$$p = \frac{z\,T + y_1\,T_1}{z\,T + y\,T_1}\,p_1,$$

and

$$p_1\,y_1 - p\,y = \frac{(z\,y - s\,y)\,T\,p_1}{z\,T + y\,T_1}.$$

The delivery, therefore, is

$$L_1 = \frac{(T_1 - T)\,(z\,y_1 - s\,y)}{z\,T + y\,T_1}\,F\,p_1.$$

Taking into account the work of the outer back pressure,

$$L_2 = F\,p \cdot \overline{Z\,U} = F\,p\,(Q\,U - Q\,Z)$$
$$= F\,p\,(x + y - y_1) = F\,p\,(z - s + y - y_1),$$

we have the useful work,

$$L_0 = L_1 - L_2 = \left(\frac{(T_1 - T)\,(z\,y_1 - s\,y)}{z\,T + y\,T_1} - (x + y - y_1)\right) F\,p,$$

remaining, under the assumption that the inner tension p_1 at the end of piston travel is equal to the outer air pressure p.

If the machine makes n revolutions per minute, the weight of the air used per second is

$$G = Q\,\gamma = F\,(y_1 - y)\,\frac{p}{R\,T} \cdot \frac{n}{60},$$

and hence the delivery of the air engine per second is

$$L = \frac{n}{60}\,L_0 = \left(\frac{(T_1 - T)\,(z\,y_1 - s\,y)}{z\,T + y\,T_1} - (x + y - y_1)\right) \frac{R\,T}{y_1 - y}\,G.$$

In practice, the quantity of air used, G, is taken much greater than given by our formula above, and hence the delivery is less than that found by the last expression.

EXAMPLE.—In an Ericsson air engine, the piston diameter is $d = 2$ ft., the length of air space behind the feed piston at the beginning of backward motion is $s = \frac{1}{4}$ ft., the entire stroke of the working piston is $x = 1{\cdot}5$ ft., the initial distance between the two pistons is $y_1 = 1{\cdot}1$ ft. and the final distance $y = 0{\cdot}1$ ft., the mean temperature of the hot air during the stroke is $t_1 = 300°$, and that of the outer air $t = 10°$, hence $T_1 = 573$ and $T = 283°$. If the engine makes fifty revolutions per minute, and the outer pressure $p = 14{\cdot}7$ pounds per square inch, what is the delivery?

Here $T_1 - T = t_1 - t = 290°$, $F = \dfrac{\pi d^2}{4} = 3{\cdot}14$ sq. ft., $p = 14{\cdot}7 \times 144 = 2116$ lbs., $z_1 = s = 0{\cdot}25$, $z = s + x = 1{\cdot}75$,

$$z y_1 - s y = 1{\cdot}75 \times 1{\cdot}1 - 0{\cdot}25 \times 0{\cdot}1 = 1{\cdot}90,$$

$$z T + y T_1 = 1{\cdot}75 \times 283 + 0{\cdot}1 \times 573 = 552,$$

$$x + y - y_1 = 1{\cdot}5 + 0{\cdot}1 - 1{\cdot}1 = 0{\cdot}5 ;$$

hence the delivery per revolution is

$$L_0 = \left(\frac{290 \times 1{\cdot}9}{552} - 0{\cdot}5\right) 3{\cdot}14 \times 2116 = (0{\cdot}998 - 0{\cdot}5)\, 6644$$

$$= 0{\cdot}498 \times 6644 = 3308 \text{ ft. lbs.},$$

or, per second,

$$L = \frac{n}{60} L_0 = \tfrac{5}{6} \cdot 3308 = 2757 \text{ ft. lbs.},$$

while the quantity of air used per second is

$$G = F(y_1 - y) \frac{p}{R T} \times \frac{n}{60} = F(y_1 - y)\, \gamma \cdot \frac{n}{60}$$

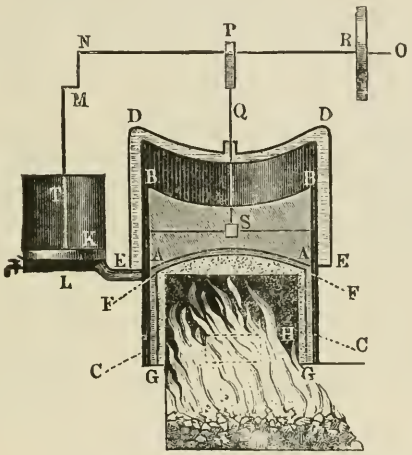
$$= 3{\cdot}14 \times 1 \times 0{\cdot}0807 \times \tfrac{5}{6} = 0{\cdot}211_1 \text{ lbs.}$$

§ **517.—Closed-Air Engine.** — While in the open-air engine of Ericsson a fresh supply of air is used at each revolution, closed engines use the same quantity over. Such are the air engines of Schwarzkopf and Laubereau, as also of Belou, etc.

In Fig. 812 we have represented Laubereau's air engine. The compressing piston *A A B B*, which here

also has a cylindrical addition $A\,C\,C\,A$, moves within a double cylinder $D\,E\,E\,D$, between whose walls cold water is pumped in at one side and drawn out at the other. The products of combustion strike the concave top $F\,F$

FIG. 812.



and pass round along $F\,G\,F\,G$ to the chimney, the opening of which is shown by the rectangle in broken lines. We see at once, that as the piston $A\,A\,B\,B$ rises, the cold air passes from the space $B\,B\,D\,D$ into the heated space $A\,A\,F\,F$, and inversely, as this piston descends, the heated air in $A\,A\,F\,F$ is forced into the cold space $B\,B\,D\,D$. During the ascent, the air passes from the cold to the warm space, expands, passes through the pipe $E\,L$ into the cylinder T and forces up the working piston K, which, by means of the rod and crank $K\,M\,N$, causes the shaft $N\,O$ to revolve. This shaft carries a fly-wheel (not shown) and a wheel R, and also an eccentric such as is shown in Fig. 777, which works in a frame at the end of the piston rod $Q\,S$, and causes the motion of the piston $A\,A\,B\,B$. As this piston descends, the air which passes above it is cooled by the walls $E\,D\,D\,E$, its tension diminishes, and the outer pressure upon K causes it to descend. If both pistons are below, the pressure in $A\,A\,F\,F$, owing to the fire, increases,

until it is greater than in *B B D D*, and the piston *A A B B* rises again and a new stroke is made.

The hot-air engine of Belou consists of two double-acting blast cylinders, one small, the feed cylinder, and the other large, the working cylinder, and a furnace between the two. Air is drawn in by the first cylinder and driven into the furnace, from which it enters the second cylinder heated and expanded, sets the piston there in motion, and thus causes a shaft with fly-wheel to revolve. The shaft works, by means of eccentrics, the two valves of the working cylinder, so that the heated air is admitted and exhausted above and below.

This engine, like all other hot-air engines, requires much more fuel than a steam engine for equal delivery.

See Tresca's article upon experiments with a Belou hot-air engine in the "Bulletin de la Société d'Encouragement," January, 1867; also Dingler's "Polytech. Journal," Bd. 185, Delabar's report upon the engines of Belou and Laubereau.

[Stirling's engine, represented in Figs. 812*a*, 812*b*, and 812*c*, is one of the most successful examples of a closed air engine, the arrangement being such that the temperature of the air is changed at constant volume, or nearly so. These Figures, together with the account of the performance of the engine, are taken from an "Account of Stirling's Air Engine," by Patrick Stirling, published in the "Transactions of the Institution of Engineers in Scotland," vol. iv.

Fig. 812*a* is a general view of the machine, partly in section; Fig. 812*b* an enlarged section of one of the air receivers; and Fig. 812*c* a section in plan of the economizer. The two receivers, *A*, *A*, made of cast-iron, are each placed in a furnace and communicate freely, the one with the upper and the other with the lower part of the working cylinder *B*. The following description is from an interesting article on "Air Engines," published in *Engineering*, 1875:

"Each of these receivers contains an inner one, the bottom of which is pierced with numerous small holes. A large plunger *C*, filled with non-conducting material, and similar in form to the air-vessel, fills up about four fifths of it, and is capable of being moved up and down through the other fifth. Between the inner and outer receiver is left an annular space about 2 inches wide, filled with an immense number of very thin metal plates, and this constitutes the economizer. In the upper part of the receiver, and surrounding the part of the cover which projects downward, are coils of tubing

Fig. 812*a*.

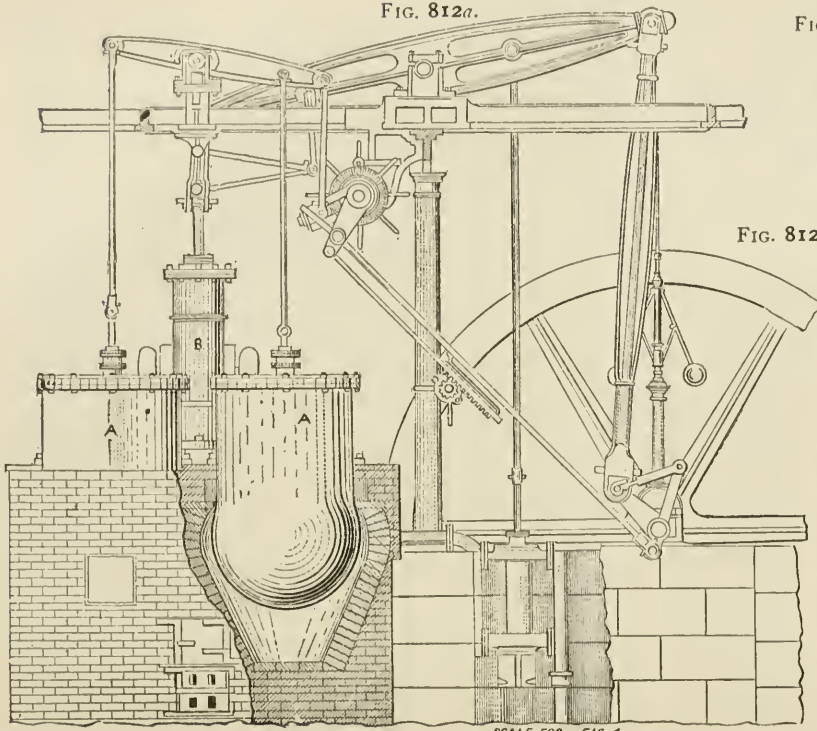
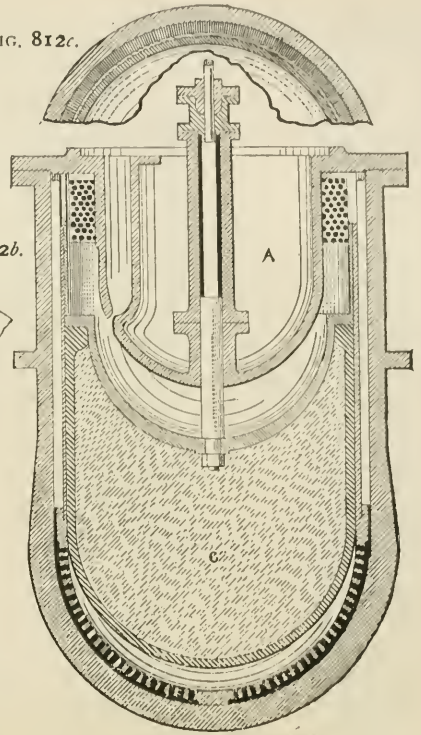


Fig. 812*b*.

Fig. 812*c*.



SCALE FOR FIG 1

ING. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 FT.

SCALE FOR FIG 2 3.

ING. 1 0 6 3 0 1 2 3 4

through which cold water is continually circulating; these constitute the refrigerator. The two air-vessels are arranged in precisely the same way, and the plunger-rods are connected to the two ends of a beam, to which a rocking motion is given by suitable mechanism. Thus when one plunger is at the top of its stroke the other is at the bottom, and *vice versa*. The receivers and cylinder are entirely shut off from communication with the atmosphere, with the exception that a small air-pump is employed to force sufficient air in to make up the loss by leakage.

"It will now be seen that each receiver, along with the end of the cylinder with which it communicates, contains a certain invariable quantity of air; that by the motion of the plunger a large portion of the air can be displaced from one end to the other of the receiver without alteration of volume; and that as the piston in the cylinder moves up and down the air must alternately expand and be compressed. In expanding it must do work upon the piston, and in being compressed the piston must do work upon it, and the useful effect of the engine is evidently the difference between these two quantities of work. It will be noticed that the air when at the bottom of the receiver will be heated by the furnace, and when at the top cooled by the refrigerator; and the object to be attained is that the expansion shall take place when the air is at the higher temperature, and the compression when it is at the lower, and in this way the energy exerted by the expanding air will always be greater than that required to compress the air on the opposite side of the piston, although the ratio of expansion is equal to that of compression, and the quantity of air is the same in both cases."

This engine was regularly in operation for four years at the Dundee foundry in Scotland, producing on an average 37 net horse-power, with a consumption of 1000 pounds of coal per day, or at the rate of 2·7 pounds of coal per net horse-power per hour, occasionally using as little as 2·5 pounds. The range of temperature was between about 150° and 600° Fahrenheit, and the range of pressure between 10 and 15½ atmospheres. The working cylinder was 16 inches in diameter, with a stroke of 4 feet, and the receivers were each 4 feet in diameter internally.

The persistent efforts of inventors have overcome many of the difficulties originally encountered in the operation of air engines, and these motors are rapidly gaining favor. Indeed, there are some engineers who anticipate that the steam engine will be superseded by the air engine in the course of a few years. Some rules and tables are added, which, in connection with those that have already been given, may be of service to designers of this class of engines.

Notation.

Let t and p = absolute temperature and pressure corresponding to volume v. T and P = absolute temperature and pressure corresponding to volume V. (Absolute temperature, Fahrenheit = temperature, Fahrenheit scale + 461°·2; absolute temperature, centigrade = temperature, centigrade scale + 274°.)

I. Relation between Temperature, Pressure, and Volume of Dry Air, for Arbitrary Changes of these Elements.

a. Volume constant.

$$P = p \times \frac{T}{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$T = t \times \frac{P}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

b. Temperature constant.

$$P = p \times \frac{v'}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$V = v' \times \frac{p}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

c. Pressure constant.

$$T = t \times \frac{v'}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$V = v' \times \frac{t}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

d. Temperature, pressure, and volume variable.

$$P = p \times \frac{v \times T}{V \times t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$T = t \times \frac{P \times V}{p \times v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$V = v' \times \frac{p \times T}{P \times t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

Examples.

1. If air at a pressure of 30 pounds per square inch and temperature of 100° Fahrenheit is heated to 300°, at constant volume, the pressure increases to

$$P = 30 \times \frac{300 + 461{\cdot}2}{100 + 461{\cdot}2} = 40{\cdot}7 \text{ pounds per square inch.}$$

2. If air at a pressure of 15 pounds per square inch is compressed to $\frac{1}{4}$ of its original volume, at constant temperature, the pressure increases to

$$P = 15 \times \frac{1}{0.25} = 60 \text{ pounds per square inch.}$$

3. If air at a pressure of 20 pounds per square inch, and temperature of 60° Fahrenheit, is compressed from a volume of 2 cubic feet to 1 cubic foot, and at the same time is heated to 150° Fahrenheit, the pressure is increased to

$$P = 20 \times \frac{2 \times (150 + 461 \cdot 2)}{1 \times (60 + 461 \cdot 2)} = 46 \cdot 9 \text{ pounds per square inch.}$$

II. Relation between Temperature, Pressure, and Volume of Dry Air, expanding without Gain or Loss of Heat (Adiabatic Curve of Expansion).

a. Change of temperature.

For given change of volume, or given cut-off, $\frac{1}{R}$,

$$T = t \times \left(\frac{v}{V}\right)^{0.408} = R^{0.408} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10).$$

For given change of pressure,

$$T = t \times \left(\frac{P}{p}\right)^{0.29} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11).$$

b. Change of pressure.

For given change of volume, or given cut-off, $\frac{1}{R}$,

$$P = p \times \left(\frac{v}{V}\right)^{1.408} = R^{1.408} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12).$$

For given change of temperature,

$$P = p \times \left(\frac{T}{t}\right)^{3.451} \quad . \quad . \quad (13).$$

c. Change of volume.

For given change of pressure,

$$V = v \times \left(\frac{p}{P}\right)^{0.71} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14).$$

For given change of temperature,

$$V = v \times \left(\frac{t}{T}\right)^{2.451} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15).$$

d. Mean total pressure during expansion, P being pressure at commencement of expansion, and R the real ratio of expansion.

$$p r = \frac{P \times \left[2 \cdot 451 - 2 \cdot 451 \times \left(\frac{1}{R}\right)^{0.408}\right]}{R - 1} \quad . \quad . \quad . \quad . \quad (16).$$

e. Mean total pressure during stroke, P i being mean pressure up to point of cut-off, clearance included, P pressure at point of cut-off, and R the real ratio of expansion.

$$P m = \frac{P i + P \times \left[2 \cdot 451 - 2 \cdot 451 \times \left(\frac{1}{R}\right)^{0.408}\right]}{R} \quad . \quad . \quad (17).$$

f. Mean total pressure during stroke, when pressure at point of cut-off, P, is equal to initial pressure; R being the real ratio of expansion.

$$P t = \frac{P \times \left[3 \cdot 451 - 2 \cdot 451 \times \left(\frac{1}{R}\right)^{0.408}\right]}{R} \quad . \quad . \quad . \quad . \quad (18).$$

g. Portion of mean total pressure during stroke, due to expansion—P being pressure at point of cut-off, and R the real ratio of expansion.

$$P_2 = \frac{P \times \left[2 \cdot 451 - 2 \cdot 451 \times \left(\frac{1}{R}\right)^{0.408}\right]}{R} \quad . \quad . \quad . \quad (19).$$

For air expanding at constant temperature (isothermal curve of expansion) and for general formulas, Tables A, B, and C, Art. 512, can be used. Tables A and B, in the present note, give constants for substitution in formulas (10) to (19) inclusive. In Table A, columns (3), (5), (7), (9), (11), and (13), contain the reciprocals of the corresponding

TABLE A.

Pressure, Temperature, and Volume of Air expanding without Gain or Loss of Heat.

Given ratio, temperature, pressure, or volume.	Ratio of absolute pressures, $\frac{p}{P}, \frac{P}{p}$				Ratio of absolute temperatures, $\frac{t}{T}, \frac{T}{t}$				Ratio of volumes, $\frac{V}{v}, \frac{v}{V}$				Given ratio, temperature, pressure, or volume.
	For given ratio of volumes.		For given ratio of absolute temperatures.		For given ratio of volumes.		For given ratio of pressures.		For given ratio of pressures.		For given ratio of absolute temperatures.		
$\frac{p}{P}, \frac{t}{T}, \frac{V}{v}$	$\left(\frac{V}{v}\right)^{1\cdot408}$	$\left(\frac{v}{V}\right)^{1\cdot408}$	$\left(\frac{t}{T}\right)^{3\cdot451}$	$\left(\frac{T}{t}\right)^{3\cdot451}$	$\left(\frac{V}{v}\right)^{0\cdot408}$	$\left(\frac{v}{V}\right)^{0\cdot408}$	$\left(\frac{p}{P}\right)^{0\cdot29}$	$\left(\frac{P}{p}\right)^{0\cdot29}$	$\left(\frac{p}{P}\right)^{0\cdot71}$	$\left(\frac{P}{p}\right)^{0\cdot71}$	$\left(\frac{t}{T}\right)^{2\cdot451}$	$\left(\frac{T}{t}\right)^{2\cdot451}$	$\frac{p}{P}, \frac{t}{T}, \frac{V}{v}$
1	2	3	4	5	6	7	8	9	10	11	12	13	14
·01	·0015	654·6	....	....	·1528	6·546	·2633	3·798	·0380	26·33	....	....	·01
·02	·0041	246·7	....	....	·2027	4·934	·3219	3·107	·0621	16·09	....	...	·02
·03	·0072	139·4	....	....	·2392	4·182	·3620	2·762	·0829	12·07	....	....	·03
·04	·0108	92·96	....	....	·2689	3·718	·3935	2·542	·1017	9·837	...	....	·04
·05	·0147	67·90	....	....	·2946	3·395	·4198	2·382	·1191	8·395	....	....	·05
·06	·0190	52·52	....	....	·3173	3·151	·4425	2·260	·1356	7·376	....	....	·06
·07	·0237	42·28	....	...	·3379	2·959	·4627	2·161	·1513	6·611	....	....	·07
·08	·0286	35·03	....	....	·3568	2·802	·4810	2·079	·1663	6·013	....	....	·08
·09	·0337	29·68	....	....	·3744	2·671	·4977	2·009	·1808	5·530	....	....	·09
·10	·0391	25·59	·00035	2825·0	·3908	2·559	·5131	1·949	·1949	5·131	·00354	282·5	·10
·11	·0447	22·37	·00049	2033·0	·4063	2·461	·5275	1·896	·2085	4·795	·00447	223·6	·11
·12	·0505	19·79	·00066	1506·0	·4210	2·375	·5410	1·849	·2218	4·508	·00553	180·7	·12
·13	·0566	17·68	·00088	1142·0	·4350	2·299	·5537	1·806	·2348	4·259	·00673	148·6	·13
·14	·0628	15·93	·00113	884·5	·4484	2·230	·5657	1·768	·2475	4·041	·00808	123·8	·14
·15	·0692	14·46	·00143	697·1	·4612	2·168	·5771	1·733	·2599	3·847	·00956	104·6	·15
·16	·0758	13·20	·00179	557·9	·4735	2·112	·5880	1·701	·2721	3·675	·01120	89·27	·16
·17	·0825	12·12	·00221	452·6	·4853	2·061	·5984	1·671	·2841	3·520	·01300	76·94	·17
·18	·0894	11·18	·00269	371·6	·4968	2·013	·6084	1·644	·2959	3·380	·01495	66·88	·18
·19	·0965	10·36	·00324	308·3	·5079	1·969	·6180	1·618	·3074	3·253	·01707	58·58	·19

TABLE A.

Pressure, Temperature, and Volume of Air expanding without Gain or Loss of Heat—(*Continued*).

$\frac{p}{P}$, $\frac{t}{T}$, $\frac{V}{v}$	$\left(\frac{V}{v}\right)^{1\cdot408}$	$\left(\frac{v}{V}\right)^{1\cdot408}$	$\left(\frac{t}{T}\right)^{3\cdot451}$	$\left(\frac{T}{t}\right)^{3\cdot451}$	$\left(\frac{V}{v}\right)^{0\cdot408}$	$\left(\frac{v}{V}\right)^{0\cdot408}$	$\left(\frac{p}{P}\right)^{0\cdot20}$	$\left(\frac{P}{p}\right)^{0\cdot29}$	$\left(\frac{p}{P}\right)^{0\cdot71}$	$\left(\frac{P}{p}\right)^{0\cdot71}$	$\left(\frac{t}{T}\right)^{2\cdot451}$	$\left(\frac{T}{t}\right)^{2\cdot451}$	$\frac{p}{P}$, $\frac{t}{T}$, $\frac{V}{v}$
1	2	3	4	5	6	7	8	9	10	11	12	13	14
·20	·1037	9·641	·00387	258·3	·5186	1·928	·6273	1·594	·3188	3·136	·01936	51·66	·20
·21	·1111	9·002	·00458	218·3	·5290	1·890	·6362	1·572	·3301	3·030	·02182	45·84	·21
·22	·1186	8·431	·00538	185·9	·5392	1·855	·6448	1·551	·3412	2·931	·02445	40·90	·22
·23	·1263	7·919	·00627	159·5	·5490	1·821	·6532	1·531	·3521	2·840	·02727	36·68	·23
·24	·1341	7·459	·00726	137·7	·5586	1·790	·6613	1·512	·3629	2·755	·03026	33·04	·24
·25	·1420	7·042	·00836	119·6	·5680	1·761	·6692	1·494	·3736	2·677	·03345	29·90	·25
·26	·1501	6·664	·00957	104·5	·5772	1·733	·6768	1·478	·3842	2·603	·03682	27·16	·26
·27	·1583	6·319	·01091	91·70	·5861	1·706	·6843	1·461	·3946	2·534	·04039	24·76	·27
·28	·1666	6·003	·01237	90·87	·5949	1·681	·6915	1·446	·4049	2·470	·04416	22·65	·28
·29	·1750	5·714	·01396	71·66	·6035	1·657	·6986	1·432	·4151	2·409	·04812	20·78	·29
·30	·1836	5·448	·01569	63·74	·6119	1·634	·7055	1·418	·4252	2·352	·05229	19·12	·30
·31	·1922	5·202	·01757	56·92	·6201	1·613	·7122	1·404	·4353	2·298	·05667	17·65	·31
·32	·2010	4·974	·01960	51·02	·6282	1·592	·7188	1·391	·4452	2·246	·06125	16·33	·32
·33	·2099	4·764	·02179	45·88	·6361	1·572	·7252	1·379	·4550	2·198	·06605	15·14	·33
·34	·2189	4·567	·02416	41·39	·6439	1·553	·7315	1·367	·4648	2·152	·07107	14·07	·34
·35	·2281	4·385	·02671	37·45	·6516	1·535	·7377	1·356	·4744	2·108	·07630	13·11	·35
·36	·2373	4·214	·02943	33·98	·6591	1·517	·7438	1·345	·4840	2·066	·08175	12·23	·36
·37	·2466	4·055	·03235	30·91	·6665	1·500	·7497	1·334	·4935	2·026	·08743	11·44	·37
·38	·2561	3·905	·03547	28·19	·6738	1·484	·7555	1·324	·5030	1·988	·09334	10·71	·38
·39	·2656	3·765	·03880	25·78	·6810	1·468	·7612	1·314	·5124	1·952	·09947	10·05	·39
·40	·2752	3·633	·04234	23·62	·6881	1·453	·7668	1·304	·5216	1·917	·10584	9·448	·40
·41	·2850	3·509	·04610	21·69	·6951	1·439	·7723	1·295	·5309	1·884	·11245	8·893	·41
·42	·2948	3·392	·05010	19·96	·7019	1·425	·7777	1·286	·5400	1·852	·11929	8·383	·42
·43	·3047	3·282	·05434	18·40	·7087	1·411	·7831	1·277	·5491	1·821	·12637	7·913	·43
·44	·3148	3·177	·05883	17·00	·7154	1·398	·7883	1·269	·5582	1·792	·13369	7·480	·44
·45	·3249	3·078	·06357	15·73	·7220	1·385	·7934	1·260	·5672	1·763	·14125	7·079	·45
·46	·3351	2·984	·06858	14·58	·7285	1·373	·7985	1·252	·5761	1·736	·14908	6·708	·46
·47	·3454	2·895	·07386	13·54	·7349	1·361	·8035	1·245	·5850	1·710	·15715	6·363	·47
·48	·3558	2·811	·07943	12·59	·7412	1·349	·8084	1·237	·5938	1·684	·16547	6·043	·48
·49	·3663	2·730	·08529	11·73	·7475	1·338	·8133	1·230	·6025	1·660	·17405	5·745	·49

TABLE A.

Pressure, Temperature, and Volume of Air expanding without Gain or Loss of Heat—(*Continued*).

$\frac{p}{P}, \frac{t}{T}, \frac{V}{v}$	$\left(\frac{V}{v}\right)^{1\cdot408}$	$\left(\frac{v}{V}\right)^{1\cdot408}$	$\left(\frac{t}{T}\right)^{3\cdot451}$	$\left(\frac{T}{t}\right)^{3\cdot451}$	$\left(\frac{V}{v}\right)^{0\cdot408}$	$\left(\frac{v}{V}\right)^{0\cdot408}$	$\left(\frac{p}{P}\right)^{0\cdot29}$	$\left(\frac{P}{p}\right)^{0\cdot29}$	$\left(\frac{p}{P}\right)^{0\cdot71}$	$\left(\frac{P}{p}\right)^{0\cdot71}$	$\left(\frac{t}{T}\right)^{2\cdot451}$	$\left(\frac{T}{t}\right)^{2\cdot451}$	$\frac{p}{P}, \frac{t}{T}, \frac{V}{v}$
1	2	3	4	5	6	7	8	9	10	11	12	13	14
·50	·3768	2·654	·09144	10·94	·7537	1·327	·8180	1·222	·6112	1·636	·18289	5·468	·50
·51	·3875	2·581	·09791	10·21	·7598	1·316	·8227	1·215	·6199	1·613	·19198	5·209	·51
·52	·3982	2·511	·10470	9·551	·7658	1·306	·8274	1·209	·6285	1·591	·20134	4·967	·52
·53	·4091	2·445	·11181	8·944	·7718	1·296	·8320	1·202	·6371	1·570	·21096	4·740	·53
·54	·4200	2·381	·11926	8·385	·7777	1·286	·8365	1·196	·6456	1·549	·22085	4·528	·54
·55	·4310	2·320	·12705	7·871	·7836	1·276	·8409	1·189	·6540	1·529	·23101	4·329	·55
·56	·4420	2·262	·13521	7·396	·7893	1·267	·8453	1·183	·6625	1·510	·24144	4·142	·56
·57	·4532	2·207	·14372	6·958	·7951	1·258	·8497	1·177	·6708	1·491	·25215	3·966	·57
·58	·4644	2·153	·15261	6·553	·8007	1·249	·8540	1·171	·6792	1·472	·26313	3·800	·58
·59	·4757	2·102	·16189	6·177	·8063	1·240	·8582	1·165	·6875	1·455	·27439	3·645	·59
·60	·4871	2·053	·17156	5·829	·8119	1·232	·8624	1·160	·6957	1·437	·28593	3·497	·60
·61	·4986	2·000	·18163	5·506	·8174	1·223	·8666	1·154	·7039	1·421	·29775	3·359	·61
·62	·5101	1·960	·19211	5·205	·8228	1·215	·8706	1·149	·7121	1·404	·30985	3·227	·62
·63	·5218	1·916	·20302	4·926	·8282	1·207	·8747	1·143	·7203	1·388	·32225	3·103	·63
·64	·5335	1·875	·21435	4·665	·8335	1·200	·8787	1·138	·7284	1·373	·33493	2·986	·64
·65	·5452	1·834	·22614	4·422	·8388	1·192	·8827	1·133	·7364	1·358	·34790	2·874	·65
·66	·5571	1·795	·23837	4·195	·8441	1·185	·8866	1·128	·7445	1·343	·36116	2·769	·66
·67	·5690	1·757	·25107	3·983	·8493	1·178	·8904	1·123	·7524	1·329	·37473	2·669	·67
·68	·5810	1·721	·26424	3·785	·8544	1·170	·8943	1·118	·7604	1·315	·38858	2·573	·68
·69	·5931	1·686	·27789	3·599	·8595	1·163	·8981	1·114	·7683	1·302	·40274	2·483	·69
·70	·6052	1·652	·29203	3·424	·8646	1·157	·9018	1·109	·7762	1·288	·417·9	2·397	·70
·71	·6174	1·620	·30669	3·261	·8696	1·150	·9055	1·104	·7841	1·275	·43195	2·315	·71
·72	·6297	1·588	·32187	3·107	·8746	1·143	·9092	1·100	·7919	1·263	·44702	2·237	·72
·73	·6420	1·558	·33754	2·963	·8795	1·137	·9128	1·096	·7997	1·251	·46239	2·163	·73
·74	·6545	1·528	·35377	2·827	·8844	1·131	·9165	1·091	·8075	1·238	·47807	2·092	·74
·75	·6669	1·499	·37054	2·699	·8893	1·125	·9200	1·087	·8152	1·227	·49406	2·024	·75
·76	·6795	1·472	·38787	2·578	·8941	1·118	·9236	1·083	·8229	1·215	·51036	1·959	·76
·77	·6921	1·445	·40577	2·464	·8989	1·113	·9271	1·079	·8306	1·204	·52698	1·898	·77
·78	·7048	1·419	·42425	2·357	·9036	1·107	·9305	1·075	·8382	1·193	·54391	1·839	·78
·79	·7176	1·394	·44332	2·256	·9083	1·101	·9340	1·071	·8459	1·182	·56116	1·782	·79

TABLE A.

Pressure, Temperature, and Volume of Air expanding without Gain or Loss of Heat—(*Continued*).

$\frac{p}{P}$, $\frac{t}{T}$, $\frac{V}{v}$	$\left(\frac{V}{v}\right)^{1\cdot408}$	$\left(\frac{v}{V}\right)^{1\cdot408}$	$\left(\frac{t}{T}\right)^{3\cdot451}$	$\left(\frac{T}{t}\right)^{3\cdot451}$	$\left(\frac{V}{v}\right)^{0\cdot408}$	$\left(\frac{v}{V}\right)^{0\cdot408}$	$\left(\frac{p}{P}\right)^{0\cdot20}$	$\left(\frac{P}{p}\right)^{0\cdot29}$	$\left(\frac{p}{P}\right)^{0\cdot71}$	$\left(\frac{P}{p}\right)^{0\cdot71}$	$\left(\frac{t}{T}\right)^{2\cdot451}$	$\left(\frac{T}{t}\right)^{2\cdot451}$	$\frac{p}{P}$, $\frac{t}{T}$, $\frac{V}{v}$
1	2	3	4	5	6	7	8	9	10	11	12	13	14
·80	·7304	1·369	·46298	2·160	·9130	1·095	·9374	1·067	·8534	1·172	·57873	1·728	·80
·81	·7433	1·345	·48326	2·069	·9176	1·090	·9408	1·063	·8610	1·161	·59662	1·676	·81
·82	·7562	1·322	·50417	1·984	·9222	1·084	·9441	1·059	·8685	1·151	·61484	1·627	·82
·83	·7692	1·300	·52570	1·902	·9268	1·079	·9474	1·056	·8761	1·142	·63338	1·579	·83
·84	·7823	1·278	·54788	1·825	·9313	1·074	·9507	1·052	·8835	1·132	·65224	1·533	·84
·85	·7955	1·257	·57072	1·752	·9358	1·069	·9540	1·048	·8910	1·122	·67144	1·489	·85
·86	·8087	1·237	·59423	1·683	·9403	1·063	·9572	1·045	·8984	1·113	·69097	1·447	·86
·87	·8220	1·217	·61842	1·617	·9448	1·058	·9605	1·041	·9058	1·104	·71083	1·407	·87
·88	·8353	1·197	·64330	1·555	·9492	1·054	·9636	1·038	·9132	1·095	·73102	1·368	·88
·89	·8487	1·178	·66888	1·495	·9536	1·049	·9668	1·034	·9206	1·086	·75154	1·331	·89
·90	·8621	1·160	·69517	1·439	·9579	1·044	·9699	1·031	·9279	1·078	·77241	1·295	·90
·91	·8757	1·142	·72219	1·385	·9623	1·039	·9730	1·028	·9352	1·069	·79362	1·260	·91
·92	·8892	1·125	·74995	1·333	·9666	1·035	·9761	1·025	·9425	1·061	·81516	1·227	·92
·93	·9029	1·108	·77846	1·285	·9708	1·030	·9792	1·021	·9498	1·053	·83705	1·195	·93
·94	·9166	1·091	·80773	1·238	·9751	1·026	·9822	1·018	·9570	1·045	·85928	1·164	·94
·95	·9303	1·075	·83777	1·194	·9793	1·021	·9853	1·015	·9642	1·037	·88186	1·134	·95
·96	·9441	1·059	·86860	1·151	·9835	1·017	·9882	1·012	·9714	1·029	·90479	1·105	·96
·97	·9580	1·044	·90022	1·111	·9877	1·013	·9912	1·009	·9786	1·022	·92806	1·077	·97
·98	·9720	1·029	·93266	1·072	·9918	1·008	·9942	1·006	·9858	1·015	·95169	1·051	·98
·99	·9860	1·014	·96591	1·035	·9959	1·004	·9971	1·003	·9929	1·007	·97567	1·025	·99

TABLE B.

Mean Pressure of Air expanding without Gain or Loss of Heat.

Real Cut-off, or reciprocal of real ratio of expansion.	Ratio of mean total pressure during expansion, to pressure at commencement.	Ratio of mean total pressure during stroke, to initial pressure, when pressure at point of cut-off is equal to initial pressure.	Ratio of portion of mean total pressure due to expansion, to pressure at point of cut-off.
$\frac{1}{R}$	$\frac{2{\cdot}451 - 2{\cdot}451 \times \left(\frac{1}{R}\right)^{0{\cdot}408}}{R-1}$	$\frac{3{\cdot}451 - 2{\cdot}451 \times \left(\frac{1}{R}\right)^{0{\cdot}408}}{R}$	$\frac{2{\cdot}451 - 2{\cdot}451 \times \left(\frac{1}{R}\right)^{0{\cdot}408}}{R}$
1	2	3	4
·01	·0210	·0308	·0208
·02	·0399	·0591	·0391
·03	·0577	·0860	·0560
·04	·0747	·1117	·0717
·05	·0910	·1365	·0865
·06	·1068	·1604	·1004
·07	·1222	·1836	·1136
·08	·1371	·2061	·1261
·09	·1517	·2280	·1380
·10	·1659	·2493	·1493
·11	·1798	·2701	·1601
·12	·1935	·2903	·1703
·13	·2069	·3100	·1800
·14	·2201	·3293	·1893
·15	·2331	·3481	·1981
·16	·2458	·3665	·2065
·17	·2584	·3845	·2145
·18	·2708	·4020	·2220
·19	·2830	·4192	·2292
·20	·2499	·4360	·2360
·21	·3069	·4524	·2424
·22	·3186	·4685	·2485
·23	·3302	·4842	·2542
·24	·3416	·4996	·2596
·25	·3529	·5147	·2647
·26	·3641	·5295	·2695
·27	·3752	·5439	·2739
·28	·3861	·5580	·2780
·29	·3970	·5718	·2818
·30	·4077	·5854	·2854
·31	·4183	·5986	·2886
·32	·4288	·6116	·2916
·33	·4393	·6243	·2943
·34	·4496	·6367	·2967
·35	·4598	·6489	·2989
·36	·4700	·6608	·3008
·37	·4800	·6724	·3024
·38	·4900	·6838	·3038
·39	·4999	·6949	·3049
·40	·5097	·7058	·3058
·41	·5194	·7165	·3065
·42	·5291	·7269	·3069
·43	·5386	·7370	·3070
·44	·5481	·7470	·3070
·45	·5576	·7567	·3067

TABLE B.

Mean Pressure of Air expanding without Gain or Loss of Heat—(*Continued*).

$\frac{1}{R}$	$\frac{2{\cdot}451 - 2{\cdot}451 \times \left(\frac{1}{R}\right)^{0{\cdot}408}}{R-1}$	$\frac{3{\cdot}451 - 2{\cdot}451 \times \left(\frac{1}{R}\right)^{0\ 408}}{R}$	$\frac{2{\cdot}451 - 2{\cdot}451 \times \left(\frac{1}{R}\right)^{0{\cdot}408}}{R}$
1	2	3	4
·46	·5670	·7662	·3062
·47	·5762	·7754	·3054
·48	·5855	·7845	·3045
·49	·5947	·7933	·3033
·50	·6038	·8019	·3019
·51	·6128	·8103	·3003
·52	·6218	·8185	·2985
·53	·6307	·8264	·2964
·54	·6396	·8342	·2942
·55	·6484	·8418	·2918
·56	·6572	·8492	·2892
·57	·6659	·8563	·2863
·58	·6745	·8633	·2833
·59	·6831	·8701	·2801
·60	·6917	·8767	·2767
·61	·7002	·8831	·2731
·62	·7086	·8893	·2693
·63	·7170	·8953	·2653
·64	·7254	·9011	·2611
·65	·7337	·9068	·2568
·66	·7419	·9123	·2523
·67	·7501	·9176	·2475
·68	·7583	·9227	·2427
·69	·7664	·9276	·2376
·70	·7745	·9324	·2324
·71	·7826	·9369	·2269
·72	·7906	·9414	·2214
·73	·7985	·9456	·2156
·74	·8064	·9497	·2097
·75	·8143	·9536	·2036
·76	·8222	·9573	·1973
·77	·8300	·9609	·1909
·78	·8377	·9643	·1843
·79	·8455	·9675	·1776
·80	·8532	·9706	·1706
·81	·8608	·9736	·1636
·82	·8684	·9763	·1563
·83	·8760	·9789	·1489
·84	·8836	·9814	·1414
·85	·8911	·9837	·1337
·86	·8986	·9858	·1258
·87	·9060	·9878	·1178
·88	·9134	·9896	·1096
·89	·9208	·9913	·1013
·90	·9282	·9928	·0928
·91	·9355	·9942	·0842
·92	·9428	·9954	·0754
·93	·9500	·9965	·0665
·94	·9572	·9974	·0574
·95	·9645	·9982	·0482
·96	·9716	·9989	·0389
·97	·9788	·9994	·0294
·98	·9858	·9997	·0197
·99	·9929	·9999	·0099

quantities in columns (2), (4), (6), (8), (10), and (12), respectively. The use of the formulas and tables is illustrated in the following examples :

1. Suppose that air expands in an adiabatic curve, the initial pressure being 100 pounds per square inch, and the initial temperature 500° Fahrenheit. If the ratio of expansion is 4, find the pressures at 5 equidistant points of the expansion curve, the temperatures at these points, and the mean pressure during expansion.

As the air expands from a volume 1 to a volume 4, the ratios of expansion at the 5 equidistant points are, respectively, 1·6, 2·2, 2·8, 3·4, and 4, and the real cut-offs corresponding are,

$$\frac{1}{1\cdot6} = 0\cdot63,$$

$$\frac{1}{2\cdot2} = 0\cdot45,$$

$$\frac{1}{2\cdot8} = 0\cdot36,$$

$$\frac{1}{3\cdot4} = 0\cdot29,$$

$$\frac{1}{4} = 0\cdot25.$$

So that, by column (2), Table A, the mean pressures at these points are,

Cut-off 0·63, 100 × 0·5218 = 52·2 pounds per square inch.
" 0·45, 100 × 0·3249 = 32·5 " " " "
" 0·36, 100 × 0·2373 = 32·7 " " " "
" 0·29, 100 × 0·1750 = 17·5 " " " "
" 0·25, 100 × 0·1420 = 14·2 " " " "

The mean pressure during expansion for a cut-off 0·25 is, by column (2), Table B,

$$p_r = 100 \times 0\cdot3529 = 35\cdot3 \text{ pounds per square inch.}$$

The temperatures corresponding to these several points of cut-off, for an initial temperature of 500° + 461°·2 = 961°·2, are, by column (6), Table A,

		Absolute temperature, Fahrenheit.	Temperature on Fahrenheit scale.
Cut-off	0·63,	961°·2 × 0·8282 = 796°·1.	796°·1 − 461°·2 = 334°·9.
"	0·45,	961°·2 × 0·7220 = 694°·1.	694°·1 − 461°·2 = 232°·9.
"	0·36,	961°·2 × 0·6591 = 633°·5.	633°·5 − 461°·2 = 172°·3.
"	0·29,	961°·2 × 0·6035 = 580°·1.	580°·1 − 461°·2 = 118°·9.
"	0·25,	961°·2 × 0·5680 = 546°·0.	546°·0 − 461°·2 = 84°·8.

2. If air is compressed in an adiabatic curve from a pressure of 15 pounds per square inch to 150, what is the ratio of compression, and the mean pressure during compression?

The given ratio of pressures is $\frac{15}{150} = 0{\cdot}10$, and hence, by column (11), Table A, the ratio of compression is $5{\cdot}13$, and the corresponding cut-off, by column (10), is $0{\cdot}19$. The mean pressure during compression, for a cut-off of $0{\cdot}19$, is, by column (2), Table B,

$$p\,r = 150 \times 0{\cdot}2830 = 42{\cdot}5 \text{ pounds per square inch.}$$

3. If in an engine driven by compressed air, the mean pressure up to point of cut-off, clearance included, is 84 pounds per square inch, the pressure at point of cut-off 80, and the real cut-off $0{\cdot}33$, the mean total pressure during stroke is, by column (4), Table B,

$$P\,m = 84 \times 0{\cdot}33 + 80 \times 0{\cdot}2943 = 51{\cdot}3 \text{ pounds per square inch.}$$

TABLE C.

Weight and Volume of Dry Air at Atmospheric Pressure and Different Temperatures.

Temperature in degrees, Fahrenheit scale.	Weight of a cubic foot in pounds.	Volume of a pound in cubic feet.	Temperature in degrees, Fahrenheit scale.	Weight of a cubic foot in pounds.	Volume of a pound in cubic feet.	Temperature in degrees, Fahrenheit scale.	Weight of a cubic foot in pounds.	Volume of a pound in cubic feet.	Temperature in degrees, Fahrenheit scale.	Weight of a cubic foot in pounds.	Volume of a pound in cubic feet.
0	·08634	11·582	27	·08156	12·261	54	·07729	12·939	81	·07344	13·617
1	·08615	11·608	28	·08140	12·286	55	·07714	12·964	82	·07330	13·642
2	·08597	11·633	29	·08123	12·311	56	·07699	12·989	83	·07317	13·667
3	·08578	11·658	30	·08106	12·336	57	·07684	13·014	84	·07304	13·692
4	·08560	11·683	31	·08090	12·361	58	·07669	13·039	85	·07290	13·717
5	·08541	11·708	32	·08074	12·386	59	·07655	13·064	86	·07277	13·742
6	·08523	11·733	33	·08057	12·411	60	·07640	13·089	87	·07264	13·767
7	·08505	11·758	34	·08041	12·436	61	·07625	13·114	88	·07250	13·792
8	·08487	11·783	35	·08025	12·461	62	·07611	13·140	89	·07237	13·818
9	·08469	11·808	36	·08009	12·487	63	·07596	13·165	90	·07224	13·843
10	·08451	11·834	37	·07993	12·512	64	·07582	13·190	91	·07211	13·868
11	·08433	11·859	38	·07977	12·537	65	·07567	13·215	92	·07198	13·893
12	·08415	11·884	39	·07961	12·562	66	·07553	13·240	93	·07185	13·918
13	·08397	11·909	40	·07945	12·587	67	·07539	13·265	94	·07172	13·943
14	·08379	11·934	41	·07929	12·612	68	·07524	13·290	95	·07159	13·968
15	·08362	11·959	42	·07913	12·637	69	·07510	13·315	96	·07146	13·993
16	·08344	11·984	43	·07897	12·662	70	·07496	13·340	97	·07133	14·018
17	·08327	12·009	44	·07882	12·687	71	·07482	13·366	98	·07121	14·043
18	·08309	12·034	45	·07866	12·713	72	·07468	13·391	99	·07108	14·069
19	·08292	12·060	46	·07851	12·738	73	·07454	13·416	100	·07095	14·094
20	·08275	12·085	47	·07835	12·763	74	·07440	13·441	101	·07083	14·119
21	·08258	12·110	48	·07820	12·788	75	·07426	13·466	102	·07070	14·144
22	·08241	12·135	49	·07805	12·813	76	·07412	13·491	103	·07058	14·169
23	·08224	12·160	50	·07789	12·838	77	·07399	13·516	104	·07045	14·194
24	·08207	12·185	51	·07774	12·863	78	·07385	13·541	105	·07033	14·219
25	·08190	12·210	52	·07759	12·888	79	·07371	13·566	106	·07020	14·244
26	·08173	12·235	53	·07744	12·913	80	·07358	13·592	107	·07008	14·270

TABLE C.

Weight and Volume of Dry Air at Atmospheric Pressure and Different Temperatures—(*Continued*).

Temperature in degrees, Fahrenheit scale.	Weight of a cubic foot in pounds	Volume of a pound in cubic feet.	Temperature in degrees, Fahrenheit scale.	Weight of a cubic foot in pounds.	Volume of a pound in cubic feet.	Temperature in degrees, Fahrenheit scale.	Weight of a cubic foot in pounds.	Volume of a pound in cubic feet.	Temperature in degrees Fahrenheit scale.	Weight of a cubic foot in pounds.	Volume of a pound in cubic feet.
108	·06996	14·295	135	·06679	14·973	161	·06400	15·626	187	·06143	16·279
109	·06983	14·320	136	·06668	14·998	162	·06389	15·651	188	·06134	16·304
110	·06971	14·345	137	·06657	15·023	163	·06379	15·676	189	·06124	16·329
111	·06959	14·370	138	·06645	15·048	164	·06369	15·701	190	·06115	16·354
112	·06947	14·395	139	·06634	15·073	165	·06359	15·726	191	·06105	16·379
113	·06935	14·420	140	·06623	15·098	166	·06349	15·751	192	·06096	16·404
114	·06923	14·445	141	·06612	15·123	167	·06339	15·776	193	·06087	16·429
115	·06911	14·471	142	·06601	15·149	168	·06329	15·802	194	·06077	16·455
116	·06899	14·496	143	·06590	15·174	169	·06318	15·827	195	·06068	16·480
117	·06887	14·521	144	·06580	15·199	170	·06308	15·852	196	·06059	16·505
118	·06875	14·546	145	·06569	15·224	171	·06298	15·877	197	·06050	16·530
119	·06863	14·571	146	·06558	15·249	172	·06289	15·902	198	·06041	16·555
120	·06851	14·596	147	·06547	15·274	173	·06279	15·927	199	·06031	16·580
121	·06839	14·621	148	·06536	15·299	174	·06269	15·952	200	·06022	16·605
122	·06828	14·646	149	·06526	15·324	175	·06259	15·977	201	·06013	16·630
123	·06816	14·671	150	·06515	15·350	176	·06249	16·002	202	·06004	16·655
124	·06804	14·697	151	·06504	15·375	177	·06239	16·028	203	·05995	16·681
125	·06793	14·722	152	·06494	15·400	178	·06230	16·053	204	·05986	16·706
126	·06781	14·747	153	·06483	15·425	179	·06220	16·078	205	·05977	16·731
127	·06770	14·772	154	·06473	15·450	180	·06210	16·103	206	·05968	16·756
128	·06758	14·797	155	·06462	15·475	181	·06200	16·128	207	·05959	16·781
129	·06747	14·822	156	·06452	15·500	182	·06191	16·153	208	·05950	16·806
130	·06735	14·847	157	·06441	15·525	183	·06181	16·178	209	·05941	16·831
131	·06724	14·872	158	·06431	15·550	184	·06172	16·203	210	·05933	16·856
132	·06713	14·897	159	·06420	15·576	185	·06162	16·228	211	·05924	16·881
133	·06701	14·923	160	·06410	15·601	186	·06153	16·254	212	·05915	16·907
134	·06690	14·948									

III. Weight and Volume of Dry Air.

a, Volume of a pound in cubic feet, at absolute pressure P (in pounds per square inch), and absolute temperature T (in Fahrenheit degrees).

$$V = \frac{T}{2{\cdot}7093 \times P} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20).$$

b. Weight of a cubic foot in pounds.

$$W = \frac{1}{V} = 2{\cdot}7093 \times \frac{P}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21).$$

Table C is calculated from these formulas, making P constant, $= 14{\cdot}7$. For any other pressure, p, it is only necessary to multiply W or divide V by $\frac{p}{P}$. For example, the weight of a cubic foot of air at a temperature of 150° Fahrenheit, and pressure of 45 pounds per square inch, is, by Table C,

$$W = 0{\cdot}06515 \times \frac{45}{14{\cdot}7} = 0{\cdot}19944.]$$

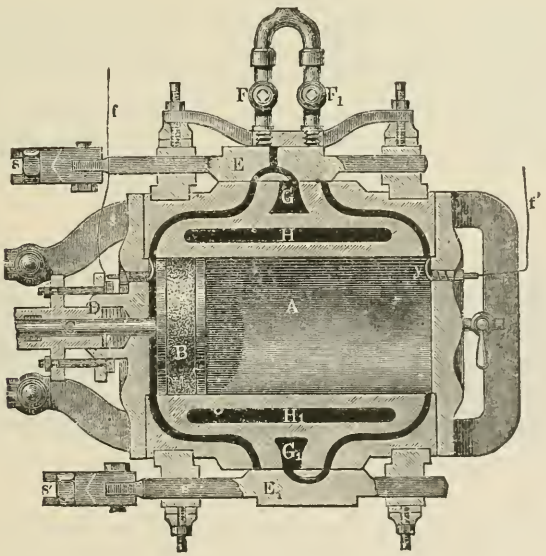
§ **518.—Gas Engines.**—Gas engines have not as yet met with more success in practice than hot-air engines. They also require for equal power a much greater amount of fuel. We may distinguish thus far three systems of gas engines—viz., the system of Lenoir, of Hugon, and of Otto and Langen.

In all these, the moving power is supplied by a kindled gas mixture, consisting of ordinary illuminating gas and ten to forty times as much air. The two first engines are double-acting; the gas mixture is admitted to the cylinder alternately upon each side of the piston, lighted, and the piston driven forward by the explosion. The third system is single-acting; the gas being admitted upon one side only of the piston, and exploded, the explosion being used not as the moving force, but in order to cause a partial vacuum when the atmosphere acts to produce motion. In such an engine, accordingly, the return of the piston is caused by the atmosphere, and the action is precisely the same as for the atmospheric steam engine (§ 439), and may, therefore, be called an atmospheric gas engine. As to the

lighting of the gas, this is effected in Lenoir's engine by the spark from a Rhumkorff's coil, while in the engines of Hugon and Otto and Langen it is effected by an ordinary gas flame.

The Lenoir gas engine has the general appearance of an ordinary steam engine with horizontal cylinder. It has, like the Corliss engine (Fig. 770), four gas ways and two distribution valves, by which the ways are alternately opened and closed, two at a time. The arrangement and action of such an engine are seen from Fig. 813. A is the cylinder, B the piston, and C the piston rod, which transfers the motion to an ordinary crank, shaft, and fly-wheel. E and E_1 are the two distribution valves moved by eccen-

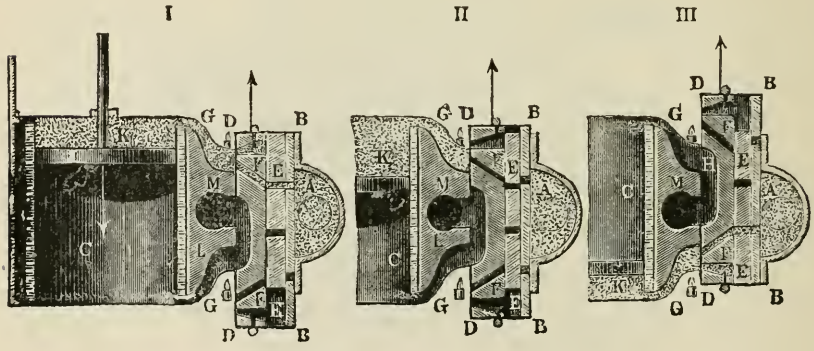
Fig. 813.



trics, F and F_1 the conducting pipes for the gas, and G and G_1 passages communicating with the air, by which the air is brought into union with the gas, and the products of combustion escape. In the position shown in the figure, the outer air enters at G, and soon afterwards gas enters through F, both pass through the left passage to the left

side of the piston *B*, while the products of combustion escape through G_1. If the slide *E* moves to the right, *G* and F_1 communicate with the right-hand passage, the mixture enters on the right of the piston and is exploded, while the products of combustion escape through G_1, the slide E_1 having moved to the left. To ignite the gas, we have isolated platinum or copper wires *x*, *y*, with their points turned towards the side of the cylinder. Upon ignition, a part of the oxygen of the air unites with the carbon of the carbonic acid and another portion with the hydrogen of the gas, forming water, and the heat generated takes effect partly as work upon the piston and partly upon the water which circulates around the cylinder at *H*, in order to prevent over-heating. Lenoir's gas engine is specially suited as a motor for small machines of from ½ to 2 horse-power, and uses nearly 2½ cubic meters of gas per hour per horse-power.

FIG. 814.

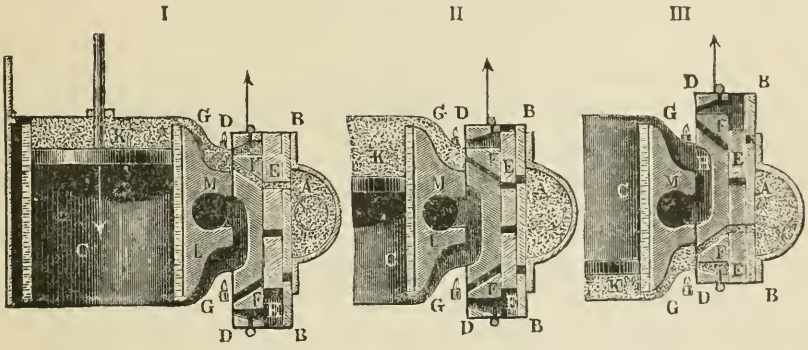


In Hugon's gas engine, of which Fig. 814, I., II., III., gives a representation, the gas mixture is ignited by two burners carried by the distribution valve, which in certain positions of the valve pass before two other fixed burners.

A small quantity of water is introduced at each stroke, by which not only the necessary cooling for the preservation of the machine is effected, but also the delivery is increased. The cylinder is also enclosed by running water

to prevent overheating. In the figure, C is the cylinder with the piston K, A the gas chamber into which the mixture is forced by a pump, under the pressure of a column of water from 0·6 to 0·7 metres high, $B\,B$ is the cut-off, and $D\,D$ the distribution valve. Between both slides, we have the sliding partition $E\,E$ moving with $B\,B$. Ordinary circular eccentrics move the slides. The distribution valve $D\,D$ has, besides the ordinary ports, two passages F, F, in which the movable gas burners are situated. As

FIG. 815.



these passages pass before the permanent burners $G\,G$, they are lighted and ignite the mixture in the cylinder C.

In the position shown in Fig. 815, I., fresh gas flows from A above the piston K; if the slide now moves up a little to II., the gas is cut off from A and ignited, and the expansive force drives the piston K down, while the products of combustion pass out through $L\,M$, below the piston. If the slide occupies the position in III., the reverse takes place, and the piston is forced upwards, while the products of combustion escape through $H\,M$.

§ 519.—**The Atmospheric Gas Engine.**—In spite of the cooling of the cylinder by surrounding it with cold water, and by injecting water into it, the gases depart from Hugon's engine with a high temperature of about 186°, so that

there is considerable loss of work. In order to avoid this, or at least diminish this effect, in the Otto-Langen engine the piston during the explosion is unloaded, and the work of the gas is only that required to raise the weight G of the piston, and to overcome its inertia. If F is the piston area, p_1 the mean value of the gas pressure during the explosion, q the back pressure of the atmosphere, and s_1 the travel of the piston during the explosion, the maximum piston velocity being v, we have the explosion work,

$$A = \frac{G\,v^2}{2\,g} = [F(p_1 - q) - G]\,s_1.$$

When the maximum velocity is attained, the moving force $F(p_1 - q) - G$ of the piston is zero, and hence the inner gas pressure,

$$p_1 = q + \frac{G}{F}.$$

For further motion of the piston, we have $p_1 < q + \frac{G}{F}$ and hence the impelling force is negative. The piston velocity then gradually decreases, and when the work inherent in the piston $A = \frac{G\,v^2}{2\,g}$ is consumed by the negative force, the piston comes to rest. If p_2 is the mean gas pressure, and s_2 the travel while the velocity diminishes from v to zero, we have

$$A = \frac{G\,v^2}{2\,g} = [F(q - p_2) + G]\,s_2;$$

hence

$$[F(q - p_2) + G]\,s_2 = [F(p_1 - q) - G]\,s_1,$$

and

$$\frac{s_2}{s_1} = \frac{F(p_1 - q) - G}{F(q - p_2) + G}.$$

After the space s_2 has been passed through, the piston

rod is connected with the fly-wheel shaft, which, by the descent of the piston, receives the work,

$$A = P s_2 = [F(q - p_2) + G]\, s_2,$$

due to the atmosphere. At the end of this distance, the force P is equal to the force $F(q - p_3)$, together with the weight G, or

$$P = F(q - p_3) + G,$$

hence

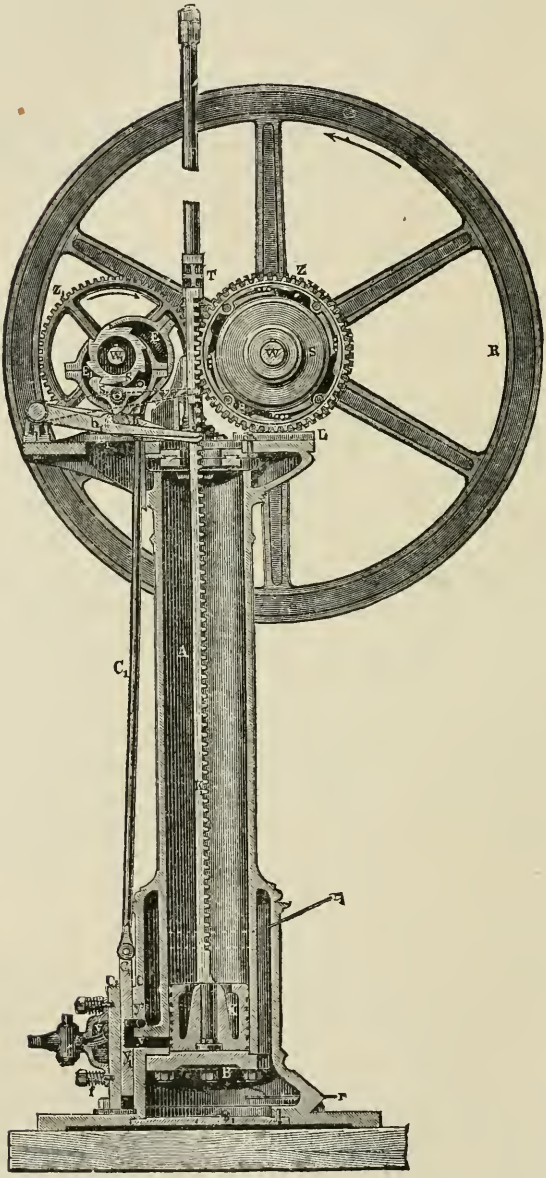
$$p_3 = q + \frac{G}{F} - \frac{P}{F} = p_1 - \frac{P}{F}.$$

The piston finally passes through a small distance, during which it forces out the gases of combustion.

The arrangement of an Otto-Langen gas engine is shown in Fig. 816. The cylinder A is fastened to a foot plate B, and surrounded by a jacket C with foot plate B_1, containing cold water made to circulate through the tubes r and r_1. The piston K has a rod with straight rack K_1, which engages with the toothed wheel Z upon the axis of the fly wheel. In order that the axis W may experience no resistance during the upward motion of the rack, the wheel Z is not fixed to it, but slides upon the disc S. Between Z and S are wedges and rollers, which, upon the descent of the piston, engage the disc and turn the axis W, while during its ascent they move freely and allow the wheel Z to move in the opposite direction, without interfering with the motion of W.

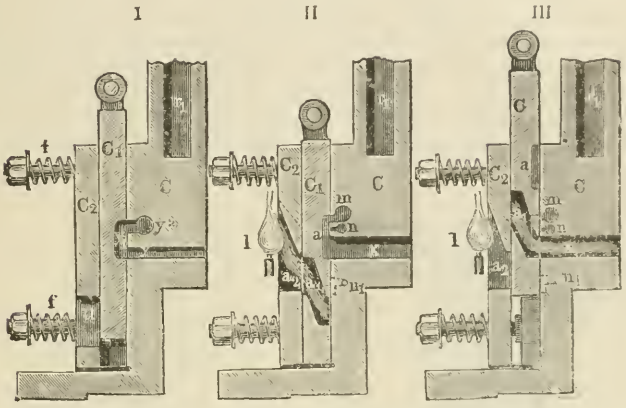
The valve gear consists essentially of a slide C_1, which, by means of a rod, eccentric, ratchet wheel, etc., is connected with a shaft W_1, made to revolve by gear wheels z, z_1. The vertical sections I, II, III, in Fig. 817, show the slide valve in three different positions. In the central position I, while the piston reaches the end of its stroke, the used gas escapes through y, y_1, and y_2 to a pipe furnished with a ball valve. If the valve takes the position II, the space below the piston fills with gas which enters

Fig. 816.



through m and n, while a portion passes through n_1 to the chamber a_1, and is ignited by the gas jet l. Finally, in III,

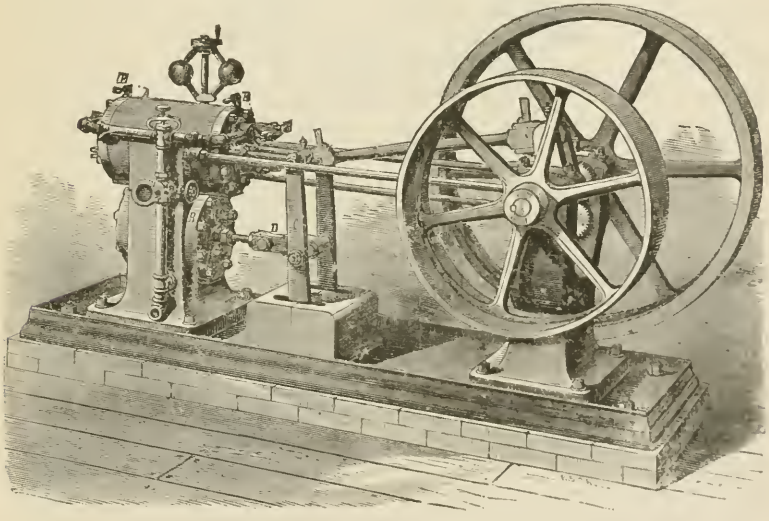
Fig. 817.



a_1 communicates with x, and the gas below the piston is ignited.

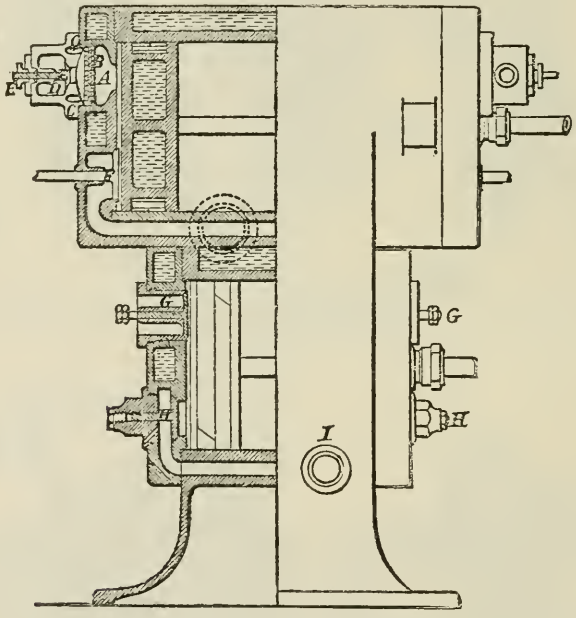
[The Brayton hydro-carbon engine, using the vapor from crude petroleum, is represented in Figs. 817*a* and 817*b*, which, with the accompanying description, were published in *Engineering* for February 16th, 1877:

Fig. 817*a*.



"In Fig. 817*a*, *A* is the working cylinder and *B* the air pump. Instead, however, of having a crosshead and slides, a parallel motion is provided for the piston as follows: *C* is a lever, the lower end of which is a radial foot, the circle being struck off the centre of the crosshead to which the arm or lever *C* is pivoted by a journal. The radial foot of *C* rests upon a pathway parallel with the bore of the cylinder, hence as the piston crosshead moves the radial foot rolls along the pathway. This motion, it will be observed, is practically frictionless; it is imperative, however, that the engine runs with the fly-wheel revolving in a direction away from the cylinder, so that the angular pressure of the connecting-rod shall always keep the radial foot

FIG. 817*b*.



down upon the pathway. The method of forming the combustible compound is shown in Fig. 817*b*, in which it is shown in the sectional view that the cylinder cover is provided with a chamber which performs three functions, the first to vaporize the petroleum, the second to insure safety, and the third to maintain continuous combustion, and thus avoid the necessity of lighting the vapor at each stroke. In this figure, *A* represents the combustion chamber in which the vapor burns continuously, and *B* the safety device which is composed of perforated plates with diaphragms of wire gauze between them, which, on the principle of the Davy lamp, renders it impossible for the flame in the chamber to pass through. At *C* is shown an annular groove packed with felt or sponge, into which the petroleum is fed by a small pump. A small

jet of air is also introduced into the fibrous material while it is moistened with the petroleum. As the supply of this jet is constant and under considerable pressure, the result is that the petroleum is forced out in the form of spray, which is spread over and absorbed by the meshes of wire gauze. The air under pressure delivered by the air pump passes in volume from the air chest *D* through the valve *E*, to fill the cylinder. This volume of air (which is under a pressure varying from 30 pounds to 75 pounds, according as regulated to suit the duty) in passing through the gauze takes up the spray, and mechanically evaporates it as it enters the combustion chamber. The small air jet is carbonized to a degree rendering its combustion continuous, as previously indicated.

"To start the engine, the two plugs *E E*, in Fig. 817*a*, are taken out of the cylinder, and the small pump is worked by hand through means of the small hand wheel shown on the top of the governor; the small air jet is then let on, and a match is applied to the holes from which the plugs were removed; so soon as the combustion takes place, the plugs are reinserted and the supply of air from the reservoir is turned on, whereupon the engine starts instantly.

"The motive power then is produced by the whole products of combustion acting upon the piston. The point of cut-off is regulated by the point at which the valve *E*, Fig. 817*b*, closes. The exhaust takes place through the valve *F*, which is operated by a positive motion, all the valves being worked from one shaft, and the valve *E* being attached to the governor.

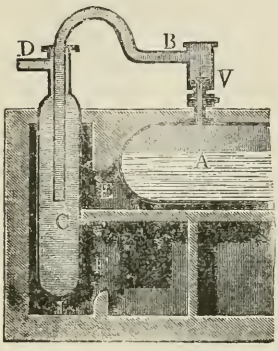
"The inlet valves to the air pump are shown at *G G*, connected together by a rod and working automatically. The air-pump discharge valves are shown at *H H*, the hole *I* being for the purpose of attaching a pipe to an independent air reservoir, which is required in some cases. Suppose, for example, an engine is sometimes required to perform for a short period a duty much above its average allotted duty, then when running under the lesser load, air may be compressed and stored in a suitable receptacle, and when the load is excessive the air pump may be relieved or partially relieved of duty and the air stored may be used. To provide sufficient air pressure to start the engine in the morning, the frame of the engine contains an air reservoir, the hole *I* being plugged up, unless the extra air reservoir above referred to is employed.

"The small pump which supplies the petroleum deserves notice from its perfection of action. The plunger is but about $\frac{3}{16}$ of an inch in diameter, its stroke being adjustable from $\frac{1}{16}$ to $\frac{1}{2}$ inch, by which means the supply of oil to suit any speed and power of engine is regulated, and with a given length of stroke the same number of drops of oil will be pumped, whether the engine is running at its slowest—that is, 80 revolutions per minute—or its calculated 180 revolutions per minute.

"When the engine is kept steadily at work, it is stated to consume about a gallon of crude petroleum per horse-power per day of twelve hours, this consumption being slightly exceeded in engines of three horse-power and less."]

§ 520.—Engines with Superheated Steam.—In recent times, the air engine has been applied to steam also. The steam is not led direct from the boiler to the cylinder, but passes first to a special vessel, the superheater, and is there, by the application of heat, superheated (§ 382). Fig. 818 shows the main features of a steam boiler with superheater, according to Chaigneau and Bichon. *A* is the rear end of the boiler, *C* the superheater, *B* and *D* pipes leading from one to the other. The superheater is heated by the hot air from the flue *E*, which has to pass around it before it can enter the chimney at *F*. A valve, easily adjusted, in the pipe *B* regulates the tension in the superheater, so that the tension of the steam in the boiler exceeds but little that in the superheater, and the action of the latter consists principally in the expansion of the steam.

Fig. 818.



If p is the steam tension, and V the steam volume used per stroke, ε the expansion ratio with which the engine works, we have, according to § 480, the work per stroke,

$$A = V p\,(1 + log.\ nat.\ \varepsilon).$$

If, now, this volume V becomes V_1 in the superheater, without p being changed, the capacity for work is

$$A_1 = V_1 p\,(1 + log.\ nat.\ \varepsilon),$$

and hence

$$\frac{A_1}{A} = \frac{V_1}{V} = \frac{1 + \delta t_1}{1 + \delta t}.$$

For the generation of the steam quantity $V\gamma$, the heat required is, approximately,

$$W = 630\, V\gamma \quad (§\,401),$$

and to convert this into superheated steam, the heat is

$$W_1 = 0{\cdot}480\,(t_1 - t)\,l'\gamma,$$

assuming that the specific heat of steam is 0·480. Hence the ratio of the heat required for the superheated to that required for saturated steam, is

$$\frac{W + W_1}{W} = 1 + \frac{0{\cdot}480\,(t_1 - t)}{630} = 1 + 0{\cdot}000762\,(t_1 - t),$$

and therefore the ratio of the efficiency of the engine with superheated steam to that with saturated steam, is

$$\frac{\eta_1}{\eta} = \frac{A_1}{A} \cdot \frac{W}{W + W_1} = \frac{1}{1 + 0{\cdot}000762\,(t_1 - t)} \cdot \frac{1 + \delta\, t_1}{1 + \delta\, t}$$

$$= \frac{1 + 0{\cdot}00367\, t_1}{[1 + 0{\cdot}000762\,(t_1 - t)]\,(1 + 0{\cdot}00367\, t)}.$$

For example, for $t = 120°$ and $t_1 = 300°$,

$$\frac{\eta_1}{\eta} = \frac{2{\cdot}001}{1{\cdot}137 \times 1{\cdot}4404} = \frac{2{\cdot}001}{1{\cdot}633} = 1{\cdot}25.$$

The delivery is, therefore, 25 per cent greater in the first case than in the second.

From experiments upon such an engine at the Paris Exposition, in 1855, this ratio was found 1·58.

The application of superheated steam has recently found wide acceptance, especially in Elsass, as appears from an article in the "Bulletin de la Soc. Ind. de Mulhouse, Avril et Mai, 1867." The superheating apparatus consists of a system of cast-iron tubes, in which the steam taken from the steam dome of an ordinary boiler is heated to 220° C. Recent experiments have been made by Prof. Hartig, in Dresden, upon the delivery of an engine with superheated steam, which also appears to show the advantage of the method. (See Civilingenieur, Jahrg. XIII.,

Heft 3.) The boiler was Romminger's patent, constructed in Dresden, and consisted of a cast-iron frame containing 14 wrought-iron pipes, 25 millimeters in diameter, and 1·6 meters long, by which the feed water was almost instantaneously converted into steam.

The Wethered Bros., in Baltimore, use, instead of simple superheated steam, a mixture of 1 part saturated and 3 parts superheated steam, and thus prevent the drying effects of the heat upon the oil, packing, etc. For this purpose, besides the ordinary steam pipe which conducts the saturated steam to the steam chamber, a spiral steam pipe passes through the smoke chamber, and thus conducts superheated steam to the steam chamber also.

Engines have also been used in which the condensation of the exhaust steam is made to generate steam from another liquid, which is applied to a second engine. Since sulphuric ether is converted into steam at 37·8° (§ 372), and for equal temperatures has a much higher tension than water steam (§ 392), it is especially suited to such engines. Here belong the engines of Trembley. (See "Annales des Mines," 1853, T. 4, also "Polytech. Centralblatt," 1854.)

Finally, we have, in recent times, engines with regenerated steam, where the steam, after it has performed its work, is heated anew (regenerated) and used again. To this class belong the engine of Siemens as also of Seguin. In such machines, the steam is alternately superheated and brought back to the saturated condition. In order to obtain a regular action, it is necessary to have two such engines united.

Upon Siemens' engine, see Cosmos, "Revue encyclopédique," 1855, also Dingler's "Polytechn. Journal," 1855. Upon Seguin's, see "Le Génie industrielle, par Armengaud," T. XIII., 1857.

Closing Remark.—The literature of the steam engine is so extensive that it is impossible to give here a full list. We cannot call attention especially to the numerous articles and treatises, but must confine ourselves to the larger works and those writings which are distinguished by originality. A treatise which contains the recent advances of the steam engine is that of

R. Schmidt, "Die Fortschritte in der Construction der Dampfmaschine während 1854 bis 1857, und während 1857 bis 1862," 2 vols., Leipzig, 1857 and 1862.

Still valuable are Tredgold's, as also Farey's, "Treatise on the Steam Engine," especially the French translation of the first work by Mellet, which appeared in 1828 under the title, "Traité des machines à vapeur, etc." Barlow's "Treatise on the Manufactures and Machinery of Great Britain" is a condensed treatise, possessing more especially historical interest only. In accordance with the present standpoint are, "A Treatise on the Steam Engine, etc., by the Artizan Club," edited by J. Bourne, 5th edition, London, 1861; "Catechism of the Steam Engine," by Bourne, 1865; also "Traité sur les machines à vapeur, par Bataille et Jullien." The first section of this work is a mere translation of the English work. The second section, which treats of the construction of the steam engine, has special practical value. We may also notice here, "Handbuch über den Bau, die Aufstellung, Behandlung, etc., der Dampfmaschinen," from the French of Grouvelle, Jaunez and Jullien, Weimar, 1848. Especially to be recommended, from a theoretical standpoint, is the second edition of Pambour's "Théorie des machines à vapeur," Paris, 1844. A German translation is given by Crelle in his "Journal der Baukunst," Bd. 23, etc. The best theoretical work upon the steam engine is the third part of "Leçons de Mécanique pratique, par A. Morin," Paris, 1846. This contains also extracts from the interesting article by Thomas Wicksteed, "On the Cornish Engines, etc." Formulæ, tables, and rules for the calculation of the steam engine will be found in Redtenbacher's "Resultate über den Maschinenbau." Special information upon heat, steam, and the steam engine is given in Redtenbacher's "Maschinenbau," Bd. II., Mannheim, 1863.

Bernoulli's "Handbuch der Dampfmaschinenlehre," 5th edition, Stuttgart, 1865, rewritten by Prof. Böttcher, of Chemnitz, is especially to be recommended to those who wish only general information. So also Rühlmann's "Allgemeine Maschinenlehre," Bd. I., especially valuable for its literary and historical information. Especially worthy of notice for its thoroughness is the work of Verdam, "Die Grundsätze, nach welchen alle Arten von Dampfmaschinen zu beurtheilen und zu behandeln sind," German by Schmidt. New theoretical views upon the action of steam, by Clapeyron and Holzmann, will be found in the article of the first upon the motive force of heat in Poggendorff's "Annalen," Bd. 59, and in the treatise of the latter, "Ueber die Wärme und Elasticität der Dämpfe und Gase." Upon the application of the theory of heat to the steam engine, by Clausius, see Poggendorff's Annalen, Bd. 97. Here also belongs the article of Rankine, "On the Mechanical Action of Heat," in the "Philosophical Magazine," Vol. VII., 1854. Tyndall, "Heat as a Mode of Motion."

The mechanical theory of heat will be found treated especially in the articles by R. Clausius, Braunschweig, 1864 and 1867; Zeuner's "Grundzüge der mechanischen Wärmetheorie," Leipzig, 1866; "Manual of the Steam Engine and other Prime Movers," by W. J. Macquorn Rankine, London and Glasgow, 1859; "Théorie mécanique de la chaleur, par

G. A. Hirn, seconde édition, Paris, 1865." Here also belongs the "Theorie der Dampfmaschinen," by Gustav Schmidt, Freiberg, 1861; as also "Die Dampfmaschinenberechnung mittels praktischer Tabellen und Regeln," by Josef Hrabák, Prag, 1869.

Good drawings and descriptions of new engines will be found in the "Sammlung von Zeichnungen einiger ausgeführten Dampfkessel und Dampfmaschinen," by Rottebohm, Berlin, 1841, and of old machines in the treatise by Severin, "Beiträge zur Kenntniss der Dampfmaschinen," Berlin, 1826 ("Abhandlung der königl. Deputation der Gewerbe"). We may also recommend Reech, "Mémoire sur les machines à vapeur," Paris, 1844, also Alban, "Die Hochdruckdampfmaschine," Rostock, 1843. Further, "The Steam Engine, etc.," by Hodge, New York, 1840, and the "Catéchisme du mécanicien à vapeur ou traité des machines à vapeur, etc.," par E. Paris, Paris, 1850. More recently published, Jul. Gaudry, "Traité élément. et prat. des machines à vapeur," 2 vols., Paris, 1856. To be recommended to practical men, "Der Führer des Machinisten," by Scholl, Braunschweig, 1864, 6th edition; "Anleitung für Anlage und Wartung der stationären Dampfkessel," by Marin, Brünn, 1859. Numerous other contributions upon steam generation will be found cited at the close of the third chapter. We may notice also in this connection, "Les applications de la chaleur, etc.," by Valerius, Bruxelles, 1867, 2d ed. Also, "Der Indicator und seine Anwendung," by Rosenkranz, Berlin, 1868.

The steam engine is extensively treated in G. Weissenborn's "American Engineering, embracing various Branches of Mechanics," New York, 1861, with 52 plates. Upon steam and gas engines at the last Paris Exposition, 1867, see "Die Motoren der Pariser Weltaustellung, 1867," by Bergrath Prof. Jenny, Wien, 1868; also Oppermann, "Visite d'un Ingenieur à l'exposition universelle de 1867;" "Revue de l'exposition de 1867: mines, métallurgie, chimie, mécanique, etc., par Noblet, Paris et Liège, 1868." From the Transactions of the Verein für Gewerbefleiss in Preussen, in separate reprint, "Die atmosphärische Gaskraftmaschine, von Otto und Langen, Berlin, 1868." The hot-air engines of Windhausen and Huch, as well as of Roper, are discussed by Delabar in Dingler's Journal, Bd. 187.

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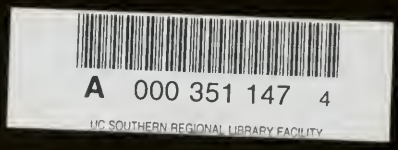
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